Session II

From the Milky Way to the Local Group

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

Advancing the Physics of Cosmic Distances
Proceedings IAU Symposium No. 289, 2012
Richard de Grijs, ed.

doi:10.1017/S1743921312021205

RR Lyrae stars: prime calibrators of the first rung of the distance ladder

Carla Cacciari

INAF–Osservatorio Astronomico, Bologna, Italy
email: carla.cacciari@oabo.inaf.it

Abstract. RR Lyrae variables are the primary standard candles for old stellar populations, and the traditional first step in the definition of the distance scale. Their properties are known on the basis of well-established physical concepts and their calibration is based on several empirical methods. Both aspects are critically reviewed, and their application as distance indicators within the Galaxy and the Local Group are discussed, also in view of the observing facilities that will be available in the near future.

Keywords. stars: evolution, stars: pulsations, stars: variables: other, stars: distances, cosmology: distance scale

1. Introduction

Soon after RR Lyrae (RRL) stars were discovered in Galactic globular clusters (GCs), it became clear that their apparent (absolute) luminosity was nearly constant, and hence they could be good standard candles (for a review, see Smith 1995). The division of GCs into two groups according to the mean period of their RRL fundamental pulsators (0.55 and 0.65 days; Oosterhoff 1939, 1944) was later found to correspond to a division in metallicity as well (Arp 1955; Kinman 1959), and this observational evidence could be explained by a difference in intrinsic mean RRL luminosity of approximately 0.2 mag between the two groups. This property is not an ensemble property caused by different period distributions in different clusters, but it applies to the individual stars in both GCs and the field (Preston 1959; Sandage 1982, 1993).

Therefore, it was natural to search for a relation between intrinsic luminosity and metallicity, assuming a linear dependence of the form $M_V = a + b$[Fe/H], which turned out to be a rather good approximation until very recently, given the accuracy of the data.

There is abundant literature, both on the theoretical (stellar evolution) definition of this relation and on its calibration based on different methods. For more detailed and complete reviews, the interested reader is referred to, among many others, Smith (1995), Cacciari & Clementini (2003), Sandage & Tammann (2006), Catelan (2009) and de Grijs (2011).

2. Requirements for a standard candle

According to Aaronson & Mould (1986), good standard candles should have the following attributes: (i) a sound physical basis; (ii) quantitative (and not subjective) measurables; (iii) a high intrinsic luminosity, spanning a luminosity range which is as small as possible and with minimal dependence on other parameters. In addition, they should preferably be numerous and easily recognizable objects. Below, we discuss how well RRL stars meet these requirements.

Number and identification: RRL variables are numerous in Population II environments (i.e. in most stellar systems): there are presently ∼3000 known RRLs in Galactic GCs

(Clement *et al.* 2001) and several thousand in the field (Samus *et al.* 2012). They are found among stars of A2–F6 spectral types, and are easily recognizable because of their large optical amplitude variations (approximately 0.2–2.0 mag), their periods (∼0.2–1.2 days) and their typical light-curve shapes. They mostly belong to two radial-pulsation groups, the fundamental-mode pulsators (RRab, with periods longer than ∼0.45 days and amplitudes larger than ∼0.5 mag) and those pulsating in the first overtone mode (RRc, with periods shorter than ∼0.45 days and amplitudes smaller than ∼0.5 mag). Given the known ratio between the fundamental and first overtone periods, the 'fundamentalized' period of an RRc star can be computed via the relation $\log P_0 = \log P_1 + 0.128$. The light-curve shapes are correspondingly different: RRc curves are nearly sinusoidal, whereas RRab light curves are asymmetric, with a sharp rise to maximum light ($\Delta\phi \sim 0.10$–0.15), a slower decline ($\Delta\phi \sim 0.50$) and a well-defined minimum ($\Delta\phi \sim 0.35$–0.40). RRL variables with special characteristics, such as Blažhko stars, double-mode and higher-mode pulsators, are ignored here, because they are irrelevant for the purpose of this paper.

High luminosity: Our knowledge of the astrophysical properties of RRL stars is quite reliable and robust, since it is based on the well-established theories of stellar evolution and pulsation (for a discussion about the uncertainties associated with stellar evolution models, see VandenBerg, this volume). Thus, we know from stellar evolution theory that RRL variables are low-mass (∼0.6–0.8 $M_\odot$) core-helium-burning stars which are found within the pulsational instability strip (IS) in the horizontal-branch (HB) region of the Hertzsprung–Russell diagram. The distribution of stars on the zero-age horizontal branch (ZAHB) depends primarily on metallicity, but other factors also play an important role, including age, mass loss and helium abundance. The IS is the region of the HB covering the temperature range of approximately 5500–8500 K, and stellar evolution theory tells us that ZAHB stars in the IS have nearly constant luminosity in the range $\log L/L_\odot \simeq 1.75$–$1.60$, depending on HB morphology (Sandage 2010; and references therein). Therefore, RRL stars are indeed rather luminous objects and potentially very good standard candles.

Small luminosity range: In a simple stellar population (i.e. at any given age and chemical composition) only $\lesssim$10% of the HB lifetime is spent on the ZAHB, and the remaining ∼90% is spent at brighter luminosities, evolving off the ZAHB towards the asymptotic giant branch.

Post-ZAHB evolution produces a cosmic scatter of order 0.1–0.2 mag depending on HB morphology, which in turn depends mainly on the metallicity, [Fe/H]. This scatter has been estimated empirically and parameterized by Sandage (1993) as $\Delta V(\text{ZAHB–HB}) = 0.05[\text{Fe/H}] + 0.16$, and semi-empirically by Ferraro *et al.* (1999) as $\Delta V(\text{ZAHB–HB}) = 0.236[\text{M/H}] + 0.106[\text{M/H}]^2 + 0.193$, who interpreted the colour–magnitude diagrams of 61 Galactic GCs with the help of theoretical models.

Therefore, it is important to account for this intrinsic scatter when comparing the observed mean magnitude of an ensamble of RRLs in a GC (which is equivalent to the observed mean magnitude of the HB) with the corresponding theoretical quantity which refers to the ZAHB. In the absence of any information about metallicity, it is common practice to apply a constant correction of ∼ 0.08–0.10 mag.

Luminosity dependence on metallicity [Fe/H]: In stellar populations with different metallicities (but similar age and helium abundance, Y), HB stellar models predict a dependence of the visual HB luminosity M_V(HB) (usually taken at a reference point near the middle of the IS, i.e. $\log T_{\rm eff} = 3.85$) on metallicity. This is exactly the relation referred to in Section 1, which was originally defined based on empirical evidence and provided the basis for Population II distance determinations. Past theoretical and empirical determinations of this relation showed linear or quadratic shapes (for recent summaries,

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

Table 1. Examples of recent empirical and theoretical M_V–[Fe/H] relations for RRL stars. The values of M_V (HB = RR) are computed at [Fe/H] = −1.5 dex.

Relation	Ref.	Type	M_V (HB)
M_V (RR) = 0.84 + 0.214[Fe/H]	Clementini *et al.* (2003)	Empirical	0.52
M_V (HB) = 0.941 + 0.348[Fe/H] + 0.040[Fe/H]^2	Pietrinferni *et al.* (2006)	Theor.	0.51
M_V (HB) = 0.909 + 0.250[Fe/H]	Pietrinferni *et al.* (2006)	Theor.	0.53
M_V (RR) = 1.179 + 0.548[Fe/H] + 0.108[Fe/H]^2	Catelan *et al.* (2004)	Theor.	0.60
M_V (RR) = 1.109 + 0.600[Fe/H] + 0.140[Fe/H]^2	Sandage (2006)	Semi-emp.	0.52
M_V (RR) = 1.576 + 1.068[Fe/H] + 0.242[Fe/H]^2	Sandage & Tammann (2006)	Semi-emp.	0.52
M_V (HB) = 0.89 + 0.25[Fe/H] [1]	Federici *et al.* (2012)	Semi-emp.	0.52

Note:
[1] This relation, obtained from the analysis of 43 GCs in M31, is calibrated based on the GC system of the Milky Way, and leads to a distance estimate to M31 of $(m - M)_0$ (M31) = 24.42 ± 0.06 mag, which is in very good agreement with the most recent determination based on Cepheids (Riess *et al.* 2012).

see Catelan 2009; Federici *et al.* 2012). In addition, the zero points could differ by as much as ∼0.3 mag, as shown by Gallart *et al.* (2005, their fig. 9), generally because of different calibrations. Examples of these relationships obtained during the past decade are given in Table 1. All, except the relation of Catelan *et al.* (2004), are calibrated using, or consistent with, a distance modulus to the Large Magellanic Cloud (LMC) of $(m - M)_0 = 18.54$ mag. Typical rms errors in these M_V (HB) determinations are often less than ∼0.05 mag, and we are now in a situation where systematic (calibration) errors dominate.

Luminosity dependence on helium abundance Y: In stellar systems of similar age and metallicity, but different helium abundance, Y, stellar evolution models predict that ZAHB loci and HB evolutionary tracks become bluer and brighter with increasing Y (Sweigart & Catelan 1998; Caputo 2012). The dependence of the HB luminosity on helium content is much stronger than on any other parameter thus far considered, i.e. $\Delta M_{\rm bol}(\text{HB})/\Delta Y \sim -4.5$ mag (Catelan 2009).

Typical cases are GCs hosting multiple stellar populations, where helium-abundance differences were proposed to explain, e.g., the multiple main sequences in ω Cen (Piotto *et al.* 2005) or the strong HB 'second-parameter' characteristics of NGC 2808 (D'Antona & Caloi 2004), NGC 6388 and NGC 6441 (Busso *et al.* 2007). Multiple-population GCs are now being found in such large numbers so as to suggest that this is a common rather than an exceptional situation (for a review, see Gratton *et al.* 2012).

Summary of requirements: On the basis of empirical optical evidence and stellar evolution theory, we can say that RRL stars are potentially good standard candles, because they are bright, numerous and easily recognizable objects. However, a few factors need to be taken into account to improve the precision with which their intrinsic luminosity is presently known:

- Off-ZAHB evolution produces a scatter $\Delta M_V \sim 0.08$–0.10 mag, which can be parameterized as a function of metallicity;
- Metallicity variations produce a scatter $\Delta M_V/\Delta[\text{Fe/H}] \sim 0.20$–$0.25$ mag;
- Helium-abundance variations produce a scatter $\Delta M_V/\Delta Y \sim -4.5$ mag.

Therefore, based on stellar evolution theory, there is a [Fe/H]–Y–evolution degeneracy, in the sense that evolution, lower metallicity and higher helium abundance each contribute to make the stars brighter, and the individual effects cannot be disentangled unless additional, independent information is available.

Obvious sources of improvement are better/different empirical data, e.g., very accurate [Fe/H] and [α/Fe] measurements, as well as measures or estimates of helium abundances (if possible). The use of near-infrared (mostly K-band) photometry can also greatly

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

help in several respects, including (i) better definition of the mean observed magnitudes, because K-band light curves have nearly sinusoidal shapes and much smaller amplitudes; (ii) lower sensitivity to metallicity and reddening effects; and (iii) lower sensitivity to off-ZAHB evolution and mass spreads (although very small) across the IS.

From a theoretical perspective, the most important help comes from stellar pulsation theory, which characterizes the RRLs on the complementary basis of their pulsation properties.

3. Stellar Pulsation

RRL variables were first identified as radially pulsating stars by Shapley (1914), and the pulsation mechanism was later identified as the opacity-driven κ mechanism in H and He partially ionized atmospheric layers (for a detailed review, see Christy 1966). Pulsation and evolution models were then used to provide the first link between the physical (mass, luminosity, temperature) and pulsation (period) properties of RRL stars by van Albada & Baker (1971). Their well-known and widely used relation has recently been updated using nonlinear, nonlocal, time-dependent convective pulsation models (Caputo 2012):

$$\log P_{\rm f} = 11.276 + 0.858 \log L - 0.659 \log M - 3.420 \log T_{\rm eff} + 0.010Y + 0.013 \log Z.$$

How can the additional information provided by the RRL pulsation properties help disentangle the metallicity–helium–evolution degeneracy? An example is shown in fig. 15 of Cacciari *et al.* (2005): in the GC M3, where there is no evidence of any significant helium-abundance spread (but for a different view, see Caloi & D'Antona 2008), the RRL variables which define a locus in the period–amplitude plane at longer periods than the main RRL distribution are also brighter, and this can be due only to lower metallicity or off-ZAHB evolution. However, in the period–ϕ_{31} plane, where ϕ_{31} is the parameter of the Fourier light-curve decomposition that best traces metallicity, these 'long-period' variables fit the main relationship very well. Therefore, a metallicity difference is not the cause of their high luminosity. If it is indeed confirmed that there is no significant helium-abundance spread in M3, then only off-ZAHB evolution remains to account for this spread. As far as the helium content is concerned, the period–amplitude relation is not a good diagnostic to constrain it, as also shown by Bono *et al.* (2011, their fig. 1).

Infrared data.

Infrared (mostly K-band) photometry, along with the RRL pulsation period, provides an essential contribution to the definition of a reliable and accurate period–luminosity–metallicity relation, thanks to the properties of the K-band data that we have mentioned before. By observing eight Galactic GCs at 2.2 μm, Longmore *et al.* (1986, 1990) found that "a large majority of the RRL stars within any GC lie on a very well-defined relationship between $\log P$ and 2.2 μm magnitude. The observed relationship agrees very well with that expected from pulsation theory ..." This result was further confirmed by Nemec *et al.* (1994, see their fig. 3) based on the K-band data of $\sim$1200 RRL stars in GCs and fields of the Milky Way, the Magellanic Clouds, and a few Local Group dwarf galaxies.

Examples of these relationships obtained during the past decade are given in Table 2. Much like in the visual range, the total error in these M_K(RR) determinations is typically $\sim$0.10 mag, most of which is systematic.

4. Calibration methods

The absolute calibration of the RRL luminosity has been pursued on the basis of various methods, the most popular and widely used of which are briefly described below (for a more comprehensive review, see Cacciari & Clementini 2003).

Table 2. Examples of empirical and theoretical P–L_K–Z relations for RRL stars. The values of M_K(RR) are computed at [Fe/H] = −1.23 dex and [α/Fe] = 0.0 dex, $\log Z = -3.0$, $Y = 0.23$ and $\log P = -0.3$.

Relation	Ref.	Type	M_K(RR)
$M_K(\mathrm{RR}) = -0.482 - 2.071 \log P_f + 0.167 \log Z$	Bono (2003)	Theor.	−0.36
$M_K(\mathrm{RR}) = -0.597 - 2.353 \log P_f + 0.175 \log Z$	Catelan *et al.* (2004)	Theor.	−0.42
$M_K(\mathrm{RR}) = -1.00 - 2.71 \log P_f + 0.12[\mathrm{Fe/H}]$	Del Principe *et al.* (2006)	Emp.	−0.34
$M_K(\mathrm{RR}) = -1.07 - 2.38 \log P_f + 0.08[\mathrm{Fe/H}]$[1]	Sollima *et al.* (2008)	Emp.	−0.45
$M_K(\mathrm{RR}) = -1.05 - 2.11 \log P_f + 0.05[\mathrm{Fe/H}]$	Borissova *et al.* (2009)	Emp.	−0.48
$M_K(\mathrm{HB}) = -0.74 - 2.32 \log P_f + 0.18([\mathrm{Fe/H}] + \alpha) - 0.53Y$	Caputo (2012)	Theor.	−0.39

Note:
[1] The metallicity in this relationship is on the Carretta & Gratton (1997) scale.

The Baade–Wesselink moving-atmospheres method: This method has been applied to approximately 30 field RRL stars by various groups. The results are summarized and critically reviewed by Fernley *et al.* (1998). The value of M_V(RR) at the reference metallicity [Fe/H] = −1.5 dex is 0.68 ± 0.10 mag. However, when the method was re-applied to RR Cet with several empirical and theoretical improvements by Cacciari *et al.* (2000), the M_V of this star became 0.11 mag brighter. It is clear that the results from the Baade–Wesselink method can be affected by significant systematic errors, as well as inaccuracies.

Statistical parallaxes: This method is based on analysis of radial velocities and proper motions of a sample of stars which are assumed to be dynamically homogeneous, i.e. drawn from a single velocity distribution. It was applied to field RRL stars with *Hipparcos* proper motions and the result was $M_V(\mathrm{RR}) = 0.75 \pm 0.13$ mag at [Fe/H] = −1.6 dex (Popowski & Gould 1998). However, the recent application of this method to a sample of 247 RRc variables has produced a much brighter result, $M_V(\mathrm{RR}) = 0.52 \pm 0.11$ mag at [Fe/H] = −1.6 dex (Kollmeier *et al.* 2012).

GC main-sequence fitting to local subdwarfs: This method is aimed at obtaining a best match between the main sequence of a GC and subdwarfs of the same metallicity for which accurate distances are known, thus allowing one to derive the cluster's distance (and hence that of its RRL stars). It has been applied to nine GCs using 56 local subdwarfs for which good *Hipparcos* parallaxes were available (but they had not yet been corrected following van Leeuwen's 2007 revision of the *Hipparcos* data reduction). The result is $M_V(\mathrm{RR}) = 0.45 \pm 0.12$ mag at [Fe/H] = −1.5 dex, which becomes $M_V(\mathrm{RR}) = 0.55 \pm 0.12$ mag if the result is normalized to $(m - M)_0(\mathrm{LMC}) = 18.54$ mag (Carretta *et al.* 2000).

Trigonometric parallaxes; RR Lyr: Trigonometric parallax is the most direct and powerful method of distance determination, but its application is limited to nearby objects for which the parallax can be measured with sufficient accuracy and precision (see Lindegren, this volume).

RR Lyr is the nearest and only RRL star with a good *Hipparcos* parallax: $\pi = 3.46 \pm 0.64$ mas, i.e. $(m-M)_0 = 7.30$ mag ±18% (van Leeuwen 2007). Therefore, $M_V(\mathrm{RR\,Lyr}) = 0.37$ mag, assuming the same value for the extinction, $A_V = 0.13$ mag, as adopted by Benedict *et al.* (2011), who derived the star's *Hubble Space Telescope (HST)*-based parallax, $\pi = 3.77 \pm 0.13$ mas, i.e. $(m-M)_0 = 7.13$ mag ±3.4%, and hence $M_V(\mathrm{RR\,Lyr}) = 0.54$ mag.

On the other hand, Catelan & Cortés (2008) used Stroemgren photometry, based on which they estimated $A_V = 0.048$ mag, and they found that RR Lyr is evolved and hence overluminous by 0.064 ± 0.013 mag. Therefore, $M_V(\mathrm{RR\,Lyr}) = 0.52$ or 0.69 mag using either the *Hipparcos* or the *HST* parallaxes, respectively.

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

So, even in this most favourable case, the discrepancy between the best available astrometric results is $\Delta(m-M)_0 = 0.17$ mag, to which one must add other sources of error such as, e.g., the reddening. This calls for caution as regards the use of RR Lyr as standard candle, and in general stresses that the absolute luminosity calibration of RRL stars still needs to be improved by all possible means.

5. The future: *Gaia* and other surveys

As referred to in Section 2, improvement in the next decade is expected to come from stellar evolution and pulsation theories, as well as from observations.

Gaia will provide a complete census of Galactic RRL stars, their distances and their physical, chemical and dynamical properties.† These data will allow us to fully characterize the various RRL subpopulations (bulge, disk, halo, streams) in the Milky Way, and hence correctly define their properties as distance (and population) indicators.

Several other photometric, spectroscopic and astrometric surveys are planned in the near future. These additional data, in synergy with *Gaia*'s, represent a formidable tool for a dramatic improvement of our knowledge of the RRL stars and their use as fundamental calibrators of the distance scale.

5.1. *Expectations from Gaia*

Considering only the trigonometric distances provided by *Gaia*'s astrometric data, a simple simulation has been performed, using 6739 field RRL stars from the GEOS database.‡ The following approximations have been assumed: (i) the photographic magnitudes, p, have been transformed to $V = p - 0.15$ mag; (ii) the B magnitudes have been transformed to $V = B - 0.30$ mag; (iii) the H magnitudes have been transformed to $V = H - 0.08$ mag; (iv) $M_V = 0.52$ mag for all stars; (v) zero reddening; and (vi) the end-of-mission *Gaia* astrometric performance σ_π/π for F–G stars. The results of this simulation are shown in Fig. 1 (left): for approximately 5% of the RRL stars, i.e. those brighter than $V \sim 11.3$ mag and at distances as far away as 1.5 kpc, the distance can be determined to better than 1%, for ∼15% of the stars (at $\leqslant$ 4 kpc) to better than 5% accuracy, and for ∼26% of the stars (at $\leqslant$5.7 kpc) to better than 10% accuracy. According to Eyer & Cuypers (2000), *Gaia* is expected to detect and measure up to $\sim (4\text{–}5) \times 10^4$ bulge and $\sim 7 \times 10^4$ halo RRL stars.

The 157 Galactic GCs in the Harris (1996, 2010) catalogue have been used to perform a similar simulation, assuming for each of them the catalogue values of V(HB) and reddening, the cluster distance accuracy σ_π/π from the mean of 10^3 red giant branch (RGB) star measurements, the average magnitude of a RGB star as $V(\mathrm{RGB}) = V(\mathrm{HB}) - 0.5$ mag and the end-of-mission *Gaia* astrometric performance σ_π/π for K–M stars. Finally, we assume that the internal distance spread for the individual stars is negligible with respect to the cluster distance. The results of this simulation are shown in Fig. 1 (right): for approximately 49% of the clusters (within ∼16 kpc), the distance to the cluster, and hence to its RRL stars, can be determined to better than 1%, for ∼75% of the clusters (within ∼25 kpc) to better than 3%, and for ∼84% of the clusters (as distant as ∼33 kpc) to better than 5%.

Finally, *Gaia* will greatly improve the GC main-sequence fitting calibration method. Metal-poor subdwarfs are faint, with $M_V = 5\text{–}7$ mag, and only a few tens of these stars

† The description of the *Gaia* mission and its expected astrometric, photometric and spectroscopic performance can be found at
`http://www.rssd.esa.int/index.php?project=GAIA&page=Science_Performance`.

‡ `http://dbrr.ast.obs-mip.fr/`

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

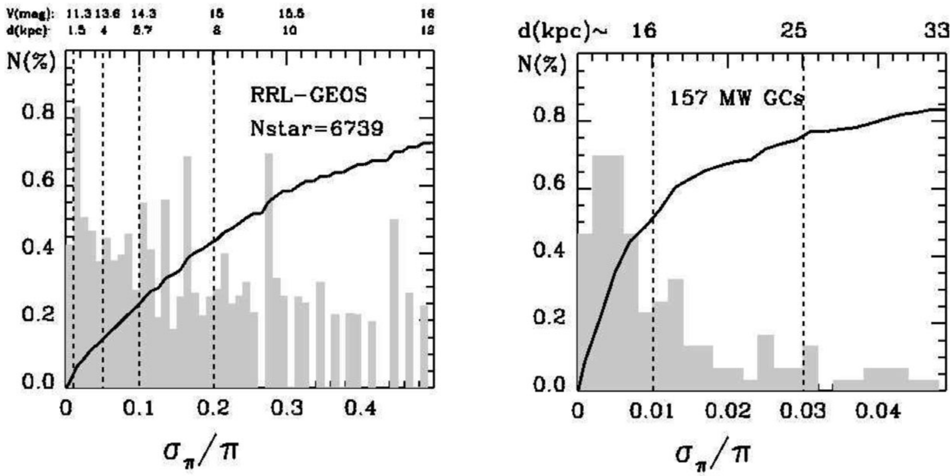

Figure 1. (left) Galactic-field RRL stars. The solid line shows the simulated cumulative distributions of the *Gaia* parallax accuracy corresponding to the shaded histogram. (right) Galactic GCs. Same as in the left panel, assuming as cluster distance the mean of 1000 RGB stars (see text for details).

within 100 pc have sufficiently good *Hipparcos* parallaxes to be useful for GC distance determination. *Gaia* will measure accurate (∼2–5%) parallaxes for such stars located at up to 1 kpc, increasing their number by a factor $\sim 10^3$ with finer metallicity sampling.

5.2. *RRLs in the Local Group*

From the average of a few million bright-star parallaxes, *Gaia* is expected to derive the mean distance (without considering depth) to the LMC and the Small Magellanic Cloud with an accuracy of ∼0.5 and 1.5%, respectively. The RRL stars will be very important, not only to trace the internal structure of the Clouds, but especially to verify the calibration obtained for the Galaxy using additional data from different stellar populations, aiming at the definition of a universal period–luminosity–metallicity relationship.

In the near future, the distance to M31 will be derived increasingly accurately by several means, including through *Gaia*'s proper motions of the brightest stars, combined with an adequate kinematic model. RRL stars in M31 are beyond the reach of *Gaia*, but, as for the Magellanic Clouds, other forthcoming photometric and spectroscopic surveys will be able to provide essential complementary information for the calibration of RRL stars, and thus establish the first step of the cosmic distance scale on a very firm and reliable basis.

Acknowledgements

Support from the Istituto Nazionale di Astrofisica (INAF) and the Agenzia Spaziale Italiana (ASI) under contract I/058/10/0 is gratefully acknowledged.

References

Aaronson, M. & Mould, J. 1986, *ApJ*, 303, 1
Arp, H. C. 1955, *AJ*, 60, 317
Benedict, G. F., McArthur, B. E., Feast, M. W., *et al.* 2011, *AJ*, 142, 187
Bono, G. 2003, *Lect. Notes Phys.*, 635, 85
Bono, G., Dall'Ora, M., Caputo, F., *et al.* 2011, *Carnegie Obs. Astrophys. Ser.*, 5, 1

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press

Borissova, J., Rejkuba, M., Minniti, D., *et al.* 2009, *A&A*, 502, 505
Busso, G., Cassisi, S., Piotto, G., *et al.* 2007, *A&A*, 474, 105
Cacciari, C., Clementini, G., Castelli, F., & Melandri, F. 2000, *Astron. Soc. Pac. Conf. Ser.*, 203, 176
Cacciari, C. & Clementini, G. 2003, *Lect. Notes Phys.*, 635, 105
Cacciari, C., Corwin, T. M., & Carney, B. W. 2005, *AJ*, 129, 267
Caloi, V. & D'Antona, F. 2008, *ApJ*, 673, 847
Caputo, F. 2011, *Ap&SS*, 341, 77
Carretta, E. & Gratton, R. G. 1997, *A&AS*, 121, 95
Carretta, E., Gratton, R. G., Clementini, G., & Fusi Pecci, F. 2000, *ApJ*, 533, 215
Catelan, M., Pritzl, B. J., & Smith, H. A. 2004, *ApJS*, 154, 633
Catelan, M. & Cortés, C. 2008, *ApJ*, 676, L135
Catelan, M. 2009, *Ap&SS*, 320, 261
Christy, R. F. 1966, *ARA&A*, 4, 353
Clement, C. M., Muzzin, A., Dufton, Q., *et al.* 2001, *AJ*, 122, 2587
Clementini, G., Gratton, R., Bragaglia, A., *et al.* 2003, *AJ*, 125, 1309
D'Antona, F. & Caloi, V. 2004, *ApJ*, 611, 871
de Grijs, R. 2011, *An Introduction to Distance Measurement in Astronomy*, Wiley
Del Principe, M., Piersimeni, A. M., Storm, J., *et al.* 2006, *ApJ*, 652, 362
Eyer, L. & Cuypers, J. 2000, *Astron. Soc. Pac. Conf. Ser.*, 203, 71
Federici, L., Cacciari, C., Bellazzini, M., *et al.* 2012, *A&A*, 544, A155
Fernley, J., Carney, B. W., Skillen, I., *et al.* 1998, *MNRAS*, 293, L61
Ferraro, F. R., Messineo, M., Fusi Pecci, F., *et al.* 1999, *AJ*, 118, 1738
Gallart, C., Zoccali, M., & Aparicio, A. 2005, *ARA&A*, 43, 387
Gratton, R. G., Carretta, E., & Bragaglia, A. 2012, *A&ARev*, 20, 50
Harris, W. E. 1996, *AJ*, 112, 1487 (2010 edition; `http://www.physics.mcmaster.ca/Globular.html`)
Kinman, T. D. 1959, *MNRAS*, 119, 538
Kollmeier, J. A., Szczygiel, D. M., Burns, C. R., *et al.* 2012, *ApJ*, submitted (arXiv:1208.2689)
Longmore, A. J., Ferneley, J. A., & Jameson, R. F. 1986, *MNRAS*, 220, 279
Longmore, A. J., Dixon, R., Skillen, I., Jameson, R. F., & Ferneley, J. A. 1990, *MNRAS*, 247, 684
Nemec, J. M., Linnell Nemec, A. F., & Lutz, T. E. 1994, *AJ*, 108, 222
Oosterhoff, P. T. 1939, *Obs.*, 62, 104
Oosterhoff, P. T. 1944, *Bull. Astron. Inst. Neth.*, 10, 55
Pietrinferni, A., Cassisi, S., Salaris, M., & Castelli, F. 2006, *ApJ*, 642, 797
Piotto, G., Villanova, S., Bedin, L. R., *et al.* 2005, *ApJ*, 621, 777
Popowski, P. & Gould, A. 1998, *ApJ*, 506, 259
Preston, G. W. 1959, *ApJ*, 130, 507
Riess, A. G., Fliri, J., & Valls-Gabaud, D. 2012, *ApJ*, 745, 156
Samus N. N., Durlevich O. V., Kazarovets E. V., *et al.* 2012, *General Catalog of Variable Stars (GCVS database 2012)*, CDS B/gcvs
Sandage, A. 1982, *ApJ*, 252, 553
Sandage, A. 1993, *AJ*, 106, 719
Sandage, A. 2006, *AJ*, 131, 1750
Sandage, A. 2010, *ApJ*, 722, 79
Sandage, A. & Tammann, G. 2006, *ARA&A*, 44, 93
Shapley, H. 1914, *ApJ*, 40, 448
Smith, H. A. 1995, *Cambridge Astrophys. Ser.*, 27
Sollima, A., Cacciari, C., Arkharov, A. A. H., *et al.* 2008, *MNRAS*, 384, 1583
Sweigart, A. V. & Catelan, M. 1998, *ApJ*, 501, L63
van Albada, T. S. & Baker, N. 1971, *ApJ*, 169, 311
van Leeuwen, F. 2007, *Hipparcos, the new reduction of the raw data*, Dordrecht: Springer

https://doi.org/10.1017/S1743921312021205 Published online by Cambridge University Press