A CHARACTERISATION OF A CERTAIN CLASS OF CONVOLUTION ALGEBRAS

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In (6) Taylor has introduced the notion of a convolution measure algebra. In the same paper he constructed a canonical embedding of an arbitrary, semi-simple commutative convolution measure algebra A into the algebra $M(S)$ of all bounded, regular Borel measures on a compact semigroup S. This embedding has the properties that A is $\sigma(M(S), C(S))$-dense in $M(S)$, that if μ is in A and ν is absolutely continuous with respect to μ, then ν is in A, and that the set $A^\wedge$ of non-zero complex homomorphisms of A can be identified with the set $S^\wedge$ of continuous semicharacters of S where $h \in A^\wedge$ is identified with $\chi \in S^\wedge$ by the equation

$$h(\mu) = \int \chi(x) d\mu(x) \quad (\mu \in A). \tag{1}$$

Throughout this paper we shall use the abbreviation CMA to stand for a commutative, semi-simple Banach algebra. The semi-group S mentioned above is usually known as the *structure semigroup* of A; S is a compact commutative topological semigroup for which $S^\wedge$ separates the points of S.

It is of interest to attempt to classify all CMA's by means of Taylor's embedding. In this paper we shall do this for a very simple class, namely those CMA's which have discrete spectra. A study of such CMA's was initiated in (3), and the results here follow on from that paper. The contents of this paper are briefly as follows. Firstly, we ask the question as to how far a CMA is determined by its structure semigroup. In other words, we wish to locate A within $M(S)$, if possible. The only class of algebras for which this has been done are those algebras whose structure semigroup is a group. A complete classification of such algebras was given by Taylor in (7) and (8). This turns out to be relatively simple for algebras with totally disconnected (in particular, discrete) spectrum, but a lot of deep work is involved in the general case. In this paper we offer a generalisation of the former case. Theorem 1 provides a complete answer to the question as to what subalgebras of $M(S)$ can occur as CMA's with discrete spectra. In Theorem 2 we complete the picture by characterising those semigroups which are structure semigroups of such CMA's. The author is very grateful to the referee who provided some important improvements and corrections to the original draft of the paper.

Let A be a CMA with structure semigroup S. We shall always regard A as a subalgebra of $M(S)$. Whenever there can be no ambiguity we shall not distinguish between continuous semicharacters of S and the non-zero complex

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