

NEW MATH FOR LIFE ACTUARIES

The fact

The model used in the technique of the Life Actuary is built on

- i) probabilities of insured events, e.g. death, survival, disablement, just to mention the most common examples for such events,
- ii) time value of money.

The model has been in use since more than 200 years (1762 Old Equitable) hence its practicability has been proved. Beyond that it has been the basis for insurance legislation all over the world. Hence the model is iron cast by law to be the right one.

The challenge

All models are wrong. Still we all know that they can nevertheless serve their purpose if they are applied with care. That means, if we add what the practitioners call “experience” and what the academics call “understanding of the limitations of the model” they become very valuable and useful tools. The question hence arises by **how much** a useful model may be wrong. My point is that for **dealing with the time value of money** (item ii) above the classical actuarial technique is nowadays so far off economic reality that it needs to be **fundamentally revised**. Here is a program for revision. The ideas are not new. One finds them in several scientific papers, some also published in the ASTIN Bulletin. My aim is to communicate these ideas more widely such that they will be applied and used.

- 1) Recognize that Life Assurance is always expressed in **units**. In private insurance (social insurance needs another definition of units) these units can always be thought of as units of a (possibly dynamic) portfolio. Such units can contain guarantees, one only needs to add the appropriate (European) options to the portfolio. As a special case, if the units are defined as the unit of the local currency, the portfolio is made up by Zero Coupon bonds plus options for the contractual guarantees.
- 2) Understand that in the language of units **one unit always remains one unit**. This has the great advantage that we can use the classical actuarial formulae at interest rate zero. This is the first discovery: **The technical interest rate** which good actuaries have always handled with suspicion **has disappeared!** The second discovery is even more convenient: **The present value** (in local currency) defined as the value of a corresponding (called replicating) portfolio can always (in principle) **be read in today's newspaper!** (at least if it has a reasonable financial section).

By the way, the time needed to explain the “numeraire” you can gain by omitting “compound interest”.

- 2) To the actuarial consultants it should be obvious how to use the proposed approach for calculating Embedded Value. This would then also provide a common basis for such calculations hence lead to identical results — independent of who of us performs this calculation.
- 3) It is not my intention to make any proposals regarding legal and/or accounting rules. They rely on a process of consensus hence changes will always take time. Still from a risk management point of view one has to notice that the numeraire which has value 1 at time zero and value $(1 + i)^n$ at each time n is — theoretically **not** replicable.

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PS My aim was to communicate ideas. Some fine points have been omitted on purpose — e.g. how to deal with surrender values. Such guarantees having the character of an American option can also be dealt with by the approach as outlined in my editorial, but need a technically different treatment than those guarantees which can be replicated by European options.