PROBLEMS FOR SOLUTION

P. 146. (i) Let $n_1 < n_2 < \ldots$ be an infinite sequence of integers such that $\sigma(n_i) - n_i$ is a constant, where $\sigma(n)$ is the sum of the divisors of n. Prove that each n_i is prime.

(ii) For each $k \geq 1$, show that there exist integers $n_1 < n_2 < \ldots < n_k$, none of which is a prime, such that $\sigma(n_i) - n_i$ is constant.

P. Erdős

P. 147. Let p be a prime with $p \equiv 1 \pmod 3$. Prove that $(x+1)^p - x^p - 1 \equiv 0 \pmod{p^3}$ has at least two solutions in the range $1 \leq x \leq p - 1$.

H.A. Heilbronn, University of Toronto

P. 148. Let X be a locally separable connected metric space. Prove that X is separable. Is this true if X is not metric?

J. Marsden, University of California, Berkeley

SOLUTIONS

P. 136. Find a topological space X which is T_o and such that Y' fails to be closed for at least one subset Y of X. (Here Y' denotes the set of all accumulation points of Y.)

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Solution by J. Marsden, University of California, Berkeley

Let $X = \{x_1, x_2, \ldots\} \cup \{x\}$ with topology $\{U_n = \{x_k : k \geq n\} \cup \{x\}\}$. This space is T_o but not T_1. Let $Y = \{x\}$. Then $Y' = \{x_1, x_2, \ldots\}$ which is not a closed set.