ON THE INTERNAL STRUCTURE OF MAIN-SEQUENCE STARS

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We present the main results of a study of the observed internal structure constants, k_2, for a wide set of eclipsing binaries. From the analysis of the variations in relative positions of the eclipses and the comparison with different theoretical models, we could deduce that the discrepancy, previously reported by several authors between theory and observations, is no longer supported. Moreover, a strong correlation has been found between the evolution of the parameter k_2 and the gravity at the surface of the star, g.

Some forty years ago, Z. Kopal (1940) tried to get observational information about the internal density concentrations of the stars by means of a study of apsidal motion in close binaries. Since that time, this kind of approach has represented one of the few tests for the different theoretical stellar structure models. It is well known that changes in the observed orbital parameters (in particular the position of the periastron) are related to dynamical perturbations arising from rotation, tides and even the presence of a third body. As a consequence, since the perturbing potential can be computed and the orbital movements accurately measured for eclipsing systems, some information about the internal structure of the stars can in principle be obtained.

A detailed agreement between observations and theoretical models is obviously required, but even recent studies (Monet, 1980) show an important systematic deviation between both. On the other hand, the low accuracy of available observational data made impossible any further research on the physical reasons for such a situation. In order to diminish the problem inherent in the accuracy of the apsidal motion determinations, we have developed a new procedure

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already explained in a previous paper (Giménez, 1981). Enhancing the importance of such a rediscussion of the observations from the theoretical point of view, apsidal motion periods have been claimed as a test for the validity of different types of opacities (Stothers, 1974).

To keep our data base homogeneous, we have only used the displacements of minima in eclipsing binaries for the determination of apsidal motions. A catalogue of 113 candidates was obtained from different sources and all of them were fully investigated. To avoid false or highly biased determinations, several conditions were imposed to the systems to consider them as definite candidates. These rectrictions are related to the photometric behaviour of the stars as well as their dynamical and evolutionary status and have been already discussed (Giménez, 1981). The final result is a set of 55 eclipsing systems (Giménez and Delgado, 1980) for which a list of some 1500 times of minimum has been compiled. This list represents in fact the primary source of information for our study.

The analysis of the final catalogue of 55 eclipsing binaries with detectable apsidal motion, proceeded in two directions:

a) Determination of absolute dimensions, and

b) Obtention of internal structure constants.

The method applied to the analysis of apsidal motions includes the determination of anomalisitic period and orbital eccentricity combining frequency and time domains. The equation relating orbital parameters with times of minimum is,

$$T = T_o + P_s E + \frac{\theta P}{2\pi} + \frac{P}{\pi} \sum_{n=1}^{\infty} (-\beta\ )^n (1+ \sqrt{1-e^2})\ \sin nv$$

where T_o is initial epoch, P_s the sidereal period and P the anomalistic period, while $\underline{v}$ is the true anomaly, $\underline{e}$ the eccentricity and θ the phase of conjunction. β is a known function of the eccentricity (Brouwer and Clemence, 1961). An example of the method applied can be found in the system Y Cyg (Giménez and Costa, 1980). Further details on the equations and procedure will be soon published elsewhere.

To obtain absolute dimensions, as well as estimations of the age, Hejlesen (1980) evolutionary tracks have been used following the method currently applied by the group of Copenhagen (Andersen et al., 1979).

The internal structure constants, k_2, for each system (weighted mean of both components) were obtained after correction for relativistic apsidal motion and higher order terms (k_3 and k_4). The observed values of log k_2 are represented in figure 1 with respect to the logarithm of the mass (also expressed by the weighted mean value). Zero-age main sequence theoretical models are shown for comparison according to the computations by Mathis (1967), Cisneros-Parra (1970), Stothers (1974) and Semeniuk and Paczynski (1968). These models adopted the following opacities and chemical compositions:

Authors	Opacities	X	Z
Stothers (S)	Carson	0.730	0.020
Mathis (M)	Cox & Stewart	0.739	0.021
Cisneros-Parra (C-P)	"	0.602	0.044
Semeniuk & Paczynski (S-P)	"	0.602	0.044

The computations by Semeniuk and Paczyński are only given for 4 and 16 solar masses but include evolved models and we have represented them by dashed lines.

A mere inspection of the diagram shows that almost all the observed values of log k_2 are below the ZAMS theoretical models. The few exceptions were re-investigated and it was found that either a very close companion was present (therefore increasing the speed of periastron revolution) or the masses were too low to ignore the effects of convection in tidal evolution. The mean errors of the determination of log k_2 range from 0.03 to 0.10 while those of log g are between 0.02 and 0.06. Consequently, it can be confirmed that the systematic deviation of the observed values of the density concentrations from the theoretical models is real and must be due to some physical fact. Actually, we know that given a particular model, the distribution of mass and therefore the value of k_2 is a function,

$$k_2 = k_2(m,X,t)$$

where m is the mass, X the chemical composition and t the age of the star. A detailed study of the influence of different values of the chemical composition in the computed ZAMS configurations, lead to the conclusion that, within a reasonable range, changes in the hydrogen and metal content can not explain differences in log k_2 larger than 0.1 approximately. Thence, observed deviations in figure 1, for a given mass and theoretical model, must be related to the evolutionary status of the system.

The age t is a difficult parameter to measure accurately enough without involving theoretical evolutionary tracks. For this reason, we have adopted a directly observable element in eclipsing binaries very sensitive to changes

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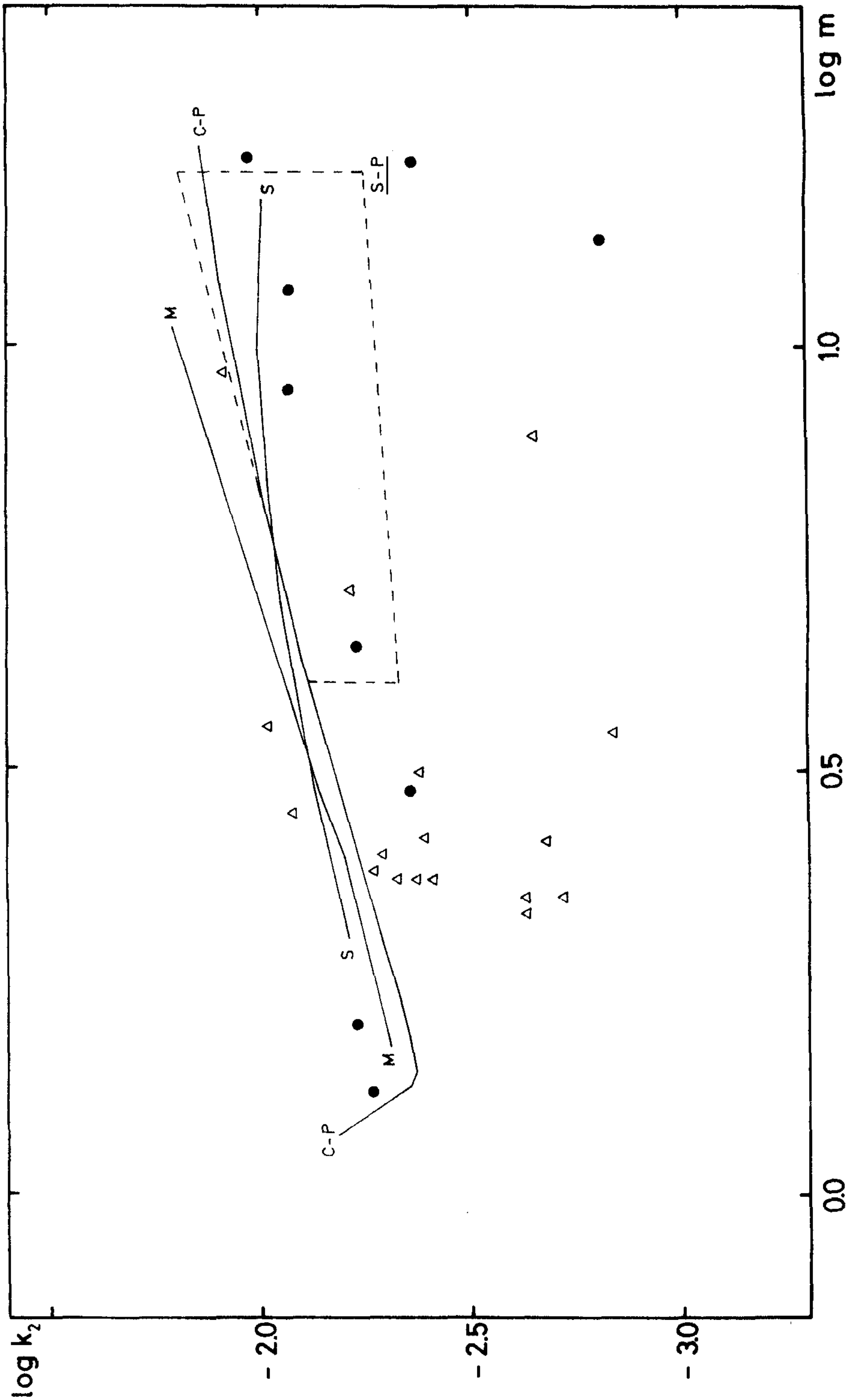

Figure 1

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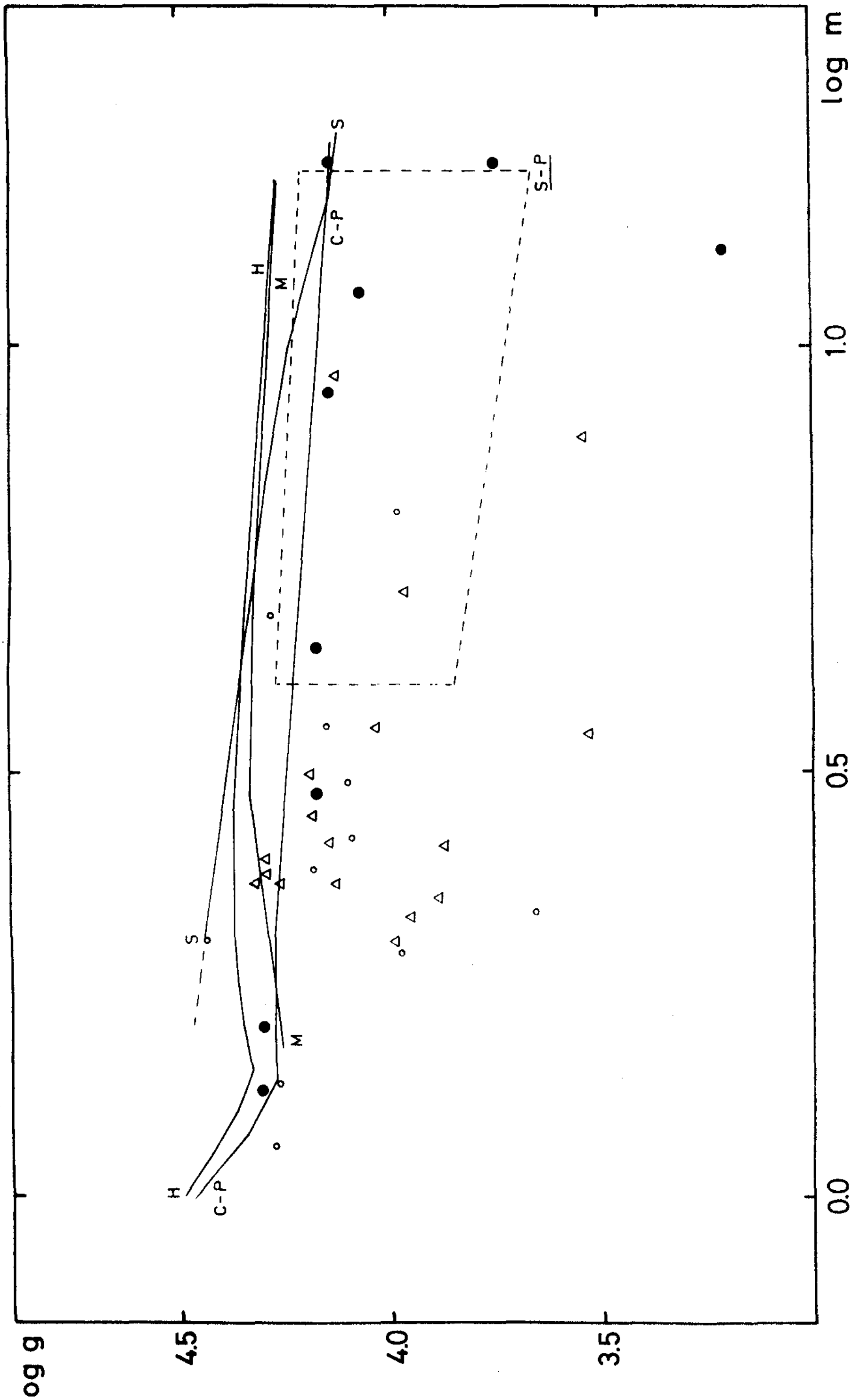

Figure 2

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of age: the radius, R. For comparison with $\Delta\log k_2$, where Δ represents the difference between observed and theoretical ZAMS values, we have used,

$$\Delta\log g = -2.\Delta\log R$$

In figure 2, we have represented the observed values of log g together with theoretical computations like in figure 1. Models by Hejlesen (H) for X = 0.700 and Z = 0.020 are also shown for comparison. It can be clearly seen that the same general behaviour of figure 1 is present. All the observed points are below the ZAMS values as it is expected due to the evolution during the main sequence or consumption of the hydrogen in the core (increasing radius implies decreasing surface gravity). Besides, the few discordant points in the $\log k_2$-log m plane do not show the same extreme positions in agreement with the above given suggestions that do not have any influence in the evaluation of R.

The comparison of the observed minus ZAMS values for $\log k_2$ and log g is shown in figure 3 for a particular set of models. Filled dots indicate larger weight and similar representations have been made for each type of theoretical models also separating the systems in groups of different quality and degree of evolution. The straight line in figure 3, represents the best linear fitting for all the points. Nevertheless, for our purposes, it is better to make separate fittings only for the main-sequence systems, not including those after consumption of the hydrogen in the core. The obvious reason is the functional behaviour of the density distribution for stars after the TAMS (non-monotonically decreasing). The results obtained for the slope of figure 3 by means of least-squares are summarized in the following table,

Models	Group ●	Groups ●&o	Mean value
Cisneros-Parra	1.09 ± 0.04	0.95 ± 0.06	1.02 ± 0.04
Stothers	1.04 0.09	0.96 0.06	1.00 0.06
Mathis	0.99 0.04	0.95 0.06	0.97 0.04
Mean value	1.04 0.04	0.95 0.04	1.00 0.03

Within their mean errors, all the possible representations gave a value of approximately 1 for the slope of the relation between $\Delta\log k_2$ and $\Delta\log g$. Small differences could be very well explained in terms of a slight dependence on the total mass of the system. The group of higher weight(●) is also that of higher mean mass and, therefore, it seems that $\Delta k_2/\Delta R$ is slightly faster for more massive systems. Using figure 3, we have represented the deviation of the

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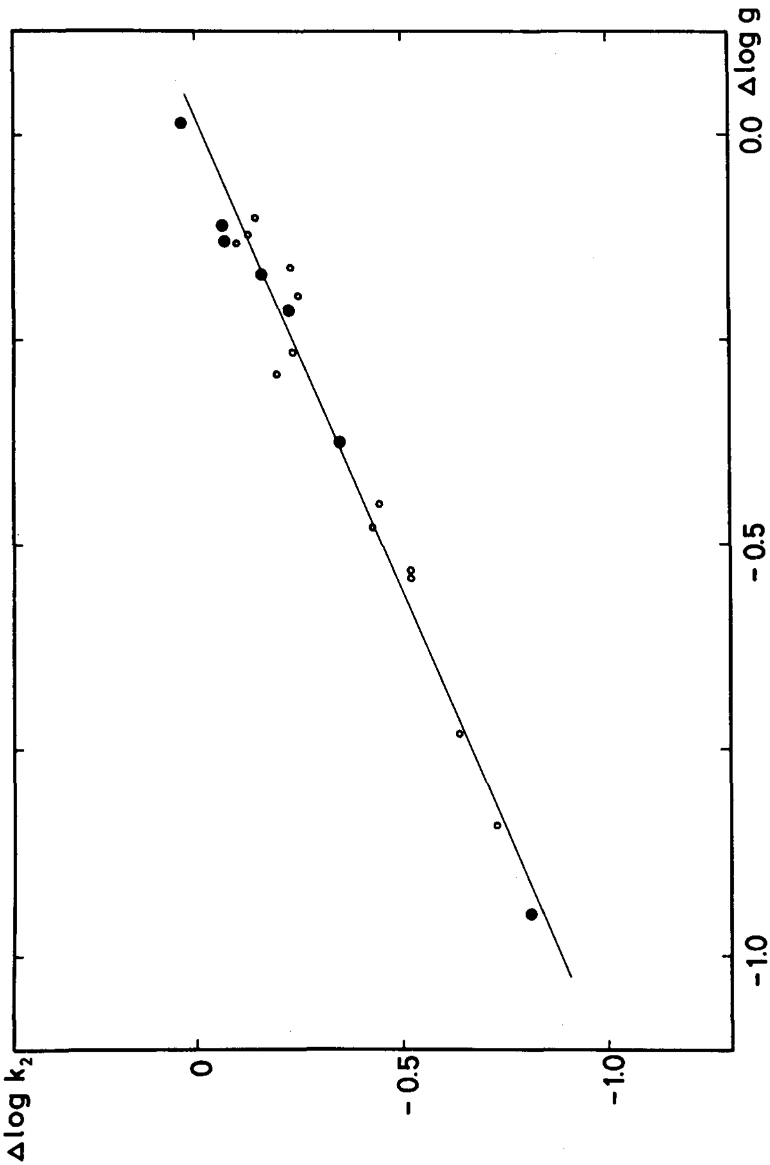

Figure 3

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points with respect to the best fitting versus log m and a significant although small correlation could be detected. After removing this "effect of mass" the dispersion in figure 3 was considerably diminished.

We have seen that, in general, the observed values of the internal structure constants in eclipsing binaries are in agreement with theoretical predictions for the main sequence provided that they evolve in such a way that,

$$\Delta \log k_2 = c.\Delta \log g$$

where c is approximately 1. After consumption of the hydrogen in the core, the value of c seems to reach a lower slope close to 0.8.

In order to see if the suggested evolutionary behaviour of k_2 is also shown by theoretical models, we have to integrate the internal density concentrations for evolved configurations using Radau's differential equation. Unfortunately this is still something to be done for a wide range of masses and ages but some data are available from the literature for particular cases (Stothers, 1974 or Odell, 1974). For these few examples, we have found the same linear relation between $\Delta \log k_2$ and $\Delta \log g$ with a slope close to 1 although slightly depending on the adopted opacity tables. However, more high-accuracy observational data are still needed to be able to select empirically the "best" opacities.

With respect to the zero value of the already discussed linear fittings, it is clear that for the ZAMS, $\Delta \log k_2 = 0$. Since small displacements (up and down) of the points in figure 3 are possible by changing the adopted chemical composition (essentially the metal abundance, Z), we have an extra procedure to check the best values.

As a final remark, it is important to notice that assuming c to be exactly 1, the simple relation found implies that, during the main sequence, the internal density distributions of the stars evolve in such a way that the function $k_2.R^2$ remains constant. According to this conclusion, we have plotted the observed systems in the $\log(k_2R^2)$-log m plane, independent of time, together with the predicted values for the theoretical models computed by Cisneros-Parra and Stothers as shown in figure 4. This diagram supports the suggestion that Carson's and Cox-Stewart's opacities "bracket" the real values (Söderhjelm, 1976) but we have to be very cautious in this conclusion. In fact, some

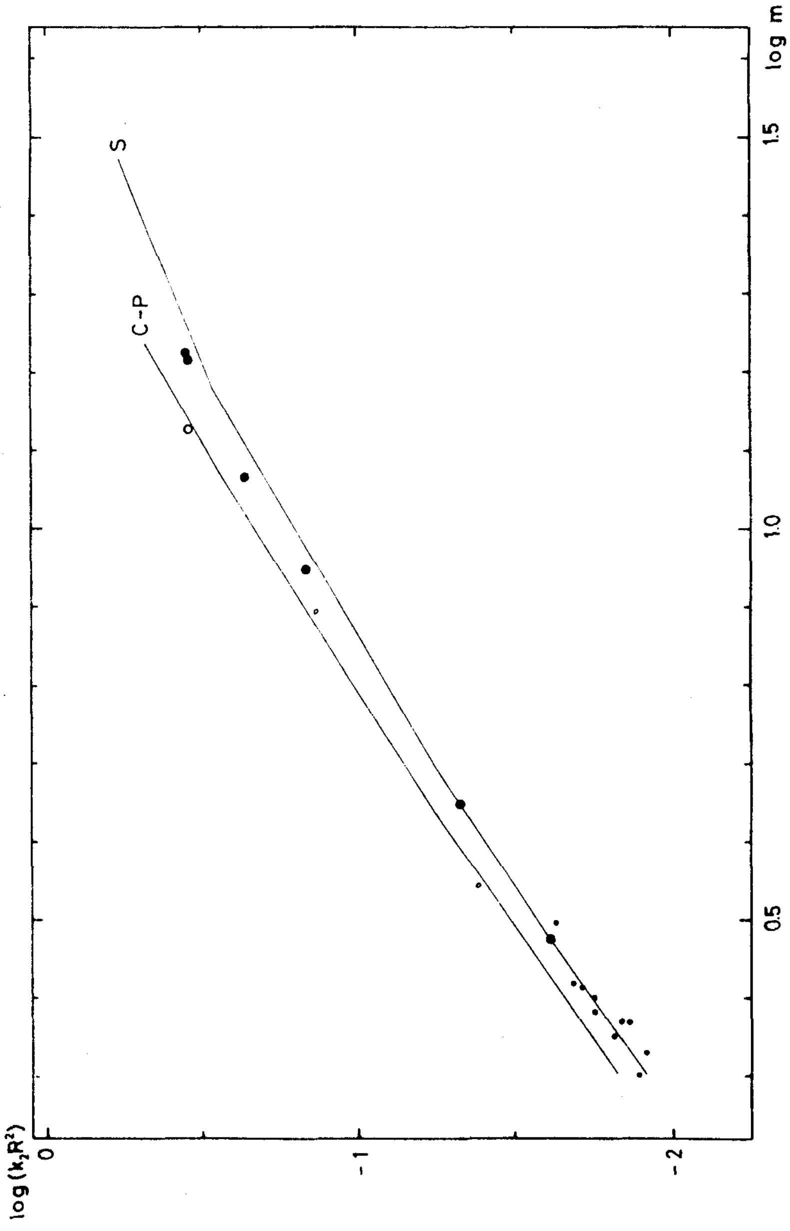

Figure 4

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small changes in the chemical compositions could well reverse the picture and, although it is clear that observations agree well with the models, nothing can be established about the opacities for the moment. Again, like other astrophysical tests, a marginal confirmation of the computations by Carson is shown, but the mean errors of $\log(k_2R^2)$ are much larger than the differences between the C-P and S curves. Finally, representing in the same diagram some observed systems outside the main sequence (open circles), the statement c = 1 is no longer valid and the points move upwards since the slope in figure 3 is smaller.

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