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A Class of Supercuspidal Representations of $G_2(k)$

Gordan Savin

Abstract. Let H be an exceptional, adjoint group of type E_6 and split rank 2, over a p-adic field k. In this article we discuss the restriction of the minimal representation of H to a dual pair $PD^\times \times G_2(k)$, where D is a division algebra of dimension 9 over k. In particular, we discover an interesting class of supercuspidal representations of $G_2(k)$.

Introduction

Let k be a p-adic field. Let $\mathfrak{h}$ be an exceptional, adjoint Lie algebra of type E_6 and split rank 2, over k. Its restricted root system is of type G_2. The long root spaces are one-dimensional, and the short root spaces admit the structure of a division algebra D of dimension 9 over k. Let $PD^\times = D^\times/k^\times$. It acts on $\mathfrak{h}$, trivially on the long root spaces, and by conjugation on the short root spaces ($\cong D$). Let H be the corresponding algebraic group of adjoint type. The centralizer of $PD^\times$ is $G_2(k)$, the simple split group of type G_2. In fact $PD^\times \times G_2(k)$ is a dual reductive pair in H.

Let Π be the minimal representation of H. It is the smallest (in a well defined sense, see [MS]), non-trivial representation of H. Since $PD^\times$ is compact, we can write

$$\Pi|_{PD^\times \times G_2(k)} = \bigoplus_\pi \pi \otimes \Theta(\pi) \tag{0.1}$$

where the sum runs over irreducible, smooth representations π of $PD^\times$. A conjectural description of this correspondence is given in [GS2]. In this article we refine this conjecture and present some evidence. We show that $\Theta(\pi)$ is supercuspidal if $\pi \neq 1$, and we determine the leading part of its character expansion. In particular, all $\Theta(\pi)$ are degenerate, *i.e.*, do not have Whittaker functionals.

More precisely, let $\mathfrak{g}_2(k)$ be the Lie algebra of $G_2(k)$, and $\overline{\mathcal{O}}_{sr} \subset \mathfrak{g}_2(\bar{k})$ the subregular nilpotent orbit. Then $\overline{\mathcal{O}}_{sr} \cap \mathfrak{g}_2(k)$ breaks up as a union

$$\overline{\mathcal{O}}_{sr} \cap \mathfrak{g}_2(k) = \bigcup_E \mathcal{O}_E \tag{0.2}$$

of subregular G-orbits, parametrized by isomorphism classes of separable cubic algebras E over k [HMS]. The structure of nilpotent G-orbits is given in Figure 1, where $\mathcal{O}_{\text{short}}$ and

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$$\mathcal{O}_{\mathrm{reg}}$$

$$/ \quad | \quad \backslash$$

$$\mathcal{O}_{E_1} \quad \mathcal{O}_{E_2} \quad \cdots$$

$$\backslash \quad | \quad /$$

$$\mathcal{O}_{\mathrm{short}}$$

$$|$$

$$\mathcal{O}_{\mathrm{long}}$$

$$|$$

$$\{0\}$$

Figure 1

$\mathcal{O}_{\mathrm{long}}$ are orbits of non-zero vectors in the short and the long root spaces, respectively. Since $\Theta(\pi)$ is degenerate, its leading part of the character expansion will be

(0.3) $$\sum_E c_E \hat{\mu}_{\mathcal{O}_E},$$

where $\mu_{\mathcal{O}_E}$ is a $G_2(k)$-invariant measure on $\mathcal{O}_E$, and $\hat{\mu}_{\mathcal{O}_E}$ its Fourier Transform as in [MW]. We show that

(0.4) $$c_E = \dim \pi^{E^\times},$$

if $E \subset D$ (this happens precisely when E is a field), and 0 otherwise.

1 A Construction of $\mathfrak{h}$

The algebra $\mathfrak{h}$ can be described in terms of a $\mathbb{Z}/3\mathbb{Z}$-gradation. To explain this, let $\mathfrak{a}$ be a simple Lie algebra together with a $\mathbb{Z}/3\mathbb{Z}$-gradation

(1.1) $$\mathfrak{a} = \mathfrak{a}_{-1} \oplus \mathfrak{a}_0 \oplus \mathfrak{a}_1.$$

Then a Killing form $\kappa\langle\ ,\ \rangle$ on $\mathfrak{a}$, restricts to a Killing form $\langle\ ,\ \rangle_0$ on $\mathfrak{a}_0$, and gives an $\mathfrak{a}_0$-invariant pairing

(1.2) $$\langle\ ,\ \rangle_{00} \colon \mathfrak{a}_{-1} \times \mathfrak{a}_1 \to k.$$

In particular, $\mathfrak{a}_{-1} \cong \mathfrak{a}_1^*$ as $\mathfrak{a}_0$-modules. Also, it induces an $\mathfrak{a}_0$-invariant skew trilinear form $\langle\ ,\ ,\ \rangle$ on $\mathfrak{a}_1$ by

(1.3) $$\langle X, Y, Z\rangle = \kappa\langle X, [Y, Z]\rangle.$$

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Now it is easy to check that the Lie bracket on $\mathfrak{a}$ is completely determined by $\langle\,,\,\rangle_0$, the pairing (1.2), and the skew form (1.3).

We now give a construction of $\mathfrak{h}$ following these ideas. Let D be a division algebra of rank 9 over k. Let Let N and Tr denote the reduced norm and trace of D. Let D^0 be the set of traceless elements in D. Define

$$\mathfrak{h}_0 = \mathrm{sl}_3(k) \oplus D^0 \oplus D^0, \tag{1.4}$$

with a Killing form

$$\langle (a,b,c),(x,y,z)\rangle_0 = \mathrm{Tr}(ax) + \mathrm{Tr}(by) + \mathrm{Tr}(cy), \tag{1.5}$$

where $\mathrm{Tr}(ax)$ is the ordinary trace of a 3×3 matrix. Let

$$\begin{cases} V = ke_1 \oplus ke_2 \oplus ke_3 \\ V^* = ke_1^* \oplus ke_2^* \oplus ke_3^* \end{cases} \tag{1.6}$$

be the standard representation of $\mathrm{sl}_3(k)$ and its dual. Put $D^* = D$, and define

$$\mathfrak{h}_1 = V \otimes D \quad \text{and} \quad \mathfrak{h}_{-1} = V^* \otimes D^* \tag{1.7}$$

with a pairing

$$\langle e_i \otimes d, e_j^* \otimes d^* \rangle_{00} = \delta_{ij}\,\mathrm{Tr}(dd^*), \tag{1.8}$$

where δ_{ij} is the Kronecker symbol. Let $x, y \in D^0$, and $z \in D$. Then

$$A_{x,y}(z) = xz - zy \tag{1.9}$$

defines a representation of a Lie algebra $D^0 \oplus D^0$ on D. This, with the standard action of $\mathrm{sl}_3(k)$ on V, defines an action of $\mathfrak{h}_0$ on $\mathfrak{h}_1$. The action of $\mathfrak{h}_0$ on $\mathfrak{h}_{-1}$ is now defined as well, since we require that the form (1.8) be $\mathfrak{h}_0$-invariant.

Let

$$(a,b,c) = N(a+b+c) - N(a+b) - N(b+c) - N(c+a) + N(a) + N(b) + N(c) \tag{1.10}$$

be a symmetric tri-linear form on D, and

$$\langle\,,\,,\,\rangle' \colon V \times V \times V \to \wedge^3 V = k \cdot e_1 \wedge e_2 \wedge e_3 \cong k, \tag{1.11}$$

a skew-form on V. Then

$$\langle\,,\,,\,\rangle = \langle\,,\,,\,\rangle' \otimes (\,,\,,\,), \tag{1.12}$$

defines a skew-symmetric form on $\mathfrak{h}_1$. Since

$$3(xz - zy, z, z) = \big(\mathrm{Tr}(x) - \mathrm{Tr}(y)\big)(z,z,z) \tag{1.13}$$

for any x, y and $z \in D$, it follows that $\big(A_{x,y}(z), z, z\big) = 0$. This implies that the skew-form (1.12) is $\mathfrak{h}_0$-invariant. The construction is now complete.

2 Some Structure of $\mathfrak{h}$

We first give some explicit brackets in $\mathfrak{h}$. Let 1 be the identity element of D, and e_{ii} be a diagonal 3×3 matrix with 1 at the i-th place and 0 elsewhere. Then

$$\begin{cases} [e_i \otimes 1, e_j \otimes 1] = \pm 2e_k^* \otimes 1 \\ [e_i \otimes 1, e_i^* \otimes 1] = 3e_{ii} - (e_{11} + e_{22} + e_{33}) \text{ in } \mathrm{sl}(3). \end{cases} \tag{2.1}$$

In the first formula, $\pm$ is the sign of permutation (i, j, k) of $(1, 2, 3)$.

Let D^0 be diagonally embedded in $D^0 \oplus D^0 \subset \mathfrak{h}$. Since $A_{x,x}(z) = 0$ for all x in D^0 if and only if z is in the center of D, it follows that the centralizer of D^0 in $\mathfrak{h}$ is

$$\mathfrak{g}_2(k) = V^* \oplus \mathrm{sl}_3(k) \oplus V. \tag{2.2}$$

The formulas in (2.1) imply that this is a simple Lie algebra of type G_2. Conversely, the centralizer of $\mathfrak{g}_2(k)$ in $\mathfrak{h}$ is D^0. Indeed, the centralizer of $\mathrm{sl}_3(k)$ is $\mathfrak{h}_0$. In addition, $A_{x,y}(1) = 0$ if and only if $x = y$. This shows that

$$D^0 \times \mathfrak{g}_2(k) \tag{2.3}$$

is a dual reductive pair in $\mathfrak{h}$.

Let

$$s_1 = \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \quad \text{and} \quad s_2 = \begin{pmatrix} 1 & & \\ & 0 & \\ & & -1 \end{pmatrix} \tag{2.4}$$

be in $\mathrm{sl}_3(k) \subset \mathfrak{g}_2(k) \subset \mathfrak{h}$. Define

$$\mathfrak{h}_i(j) = \{x \in \mathfrak{h} \mid [s_i, x] = jx\}. \tag{2.5}$$

The structure of $\mathfrak{h}_i(j)$ can easily be computed from the $\mathbb{Z}/3\mathbb{Z}$-gradation of $\mathfrak{h}$. In particular, $\mathfrak{p}_i = \mathfrak{m}_i \oplus \mathfrak{n}_i$ are parabolic subalgebras. Here

$$\mathfrak{m}_i = \mathfrak{h}_i(0) \quad \text{and} \quad \mathfrak{n}_i = \bigoplus_{j>0} \mathfrak{h}_i(j). \tag{2.6}$$

The unipotent radical $\mathfrak{n}_1$ is a 3-step nilpotent Lie algebra, and $\mathfrak{n}_2$ is a 2-step nilpotent Lie algebra. The center $\mathfrak{z}_2$ of $\mathfrak{n}_2$ is 1-dimensional, and

$$\mathfrak{n}_2/\mathfrak{z}_2 = \mathfrak{h}_2(1) = k \oplus D \oplus D^* \oplus k^*. \tag{2.7}$$

Note that we have isomorphisms

$$\begin{cases} \mathfrak{m}_1 \cong \mathrm{gl}_2(k) \oplus D^0 \oplus D^0 \\ \mathfrak{m}_2 \cong \mathrm{gl}_2(D). \end{cases} \tag{2.8}$$

Analogously, s_1 and s_2 define two maximal parabolic subalgebras in $\mathfrak{g}_2(k)$:

$$\begin{cases} \mathfrak{q}_1 = \mathfrak{l}_1 \oplus \mathfrak{u}_1 \\ \mathfrak{q}_2 = \mathfrak{l}_2 \oplus \mathfrak{u}_2. \end{cases} \tag{2.9}$$

Their structure is quite analogous to the structure of the corresponding algebras of $\mathfrak{h}$: replace D by k in formulas (2.7) and (2.8).

3 Minimal Representation Π

Let $\mathcal{O}$ be the ring of integers in k, and $\mathfrak{p} = (p)$ the maximal ideal of $\mathcal{O}$. Also, let R be the maximal order in D, and $\mathfrak{m} = (\varpi)$ the maximal ideal of R. Note that $\mathbb{E} = R/\mathfrak{m}$ is a cubic extension of $\mathbb{F} = \mathcal{O}/\mathfrak{p}$.

First, we describe a special maximal compact subgroup of H. Let $\mathfrak{k}$ be an $\mathcal{O}$-lattice in $\mathfrak{h}$ defined by

$$\begin{cases} \mathfrak{k}_0 = \mathrm{sl}_3(\mathcal{O}) \oplus R^0 \oplus R^0 \\ \mathfrak{k}_1 = V_{\mathcal{O}} \otimes_{\mathcal{O}} R \text{ and } \mathfrak{k}_{-1} = V^*_{\mathcal{O}} \otimes_{\mathcal{O}} R^* \end{cases} \tag{3.1}$$

where $V_{\mathcal{O}}$ and $V^*_{\mathcal{O}}$ are the standard $\mathcal{O}$-lattices in V and V^*, and $R^* = R \subset D = D^*$.

Let $\mathfrak{k}'$ be a lattice defined by

$$\begin{cases} \mathfrak{k}'_0 = \mathrm{sl}_3(\mathfrak{p}) \oplus \{(x, y) \mid x, y \in R^0, x \equiv y \mod(\mathfrak{m})\} \\ \mathfrak{k}'_1 = V_{\mathcal{O}} \otimes_{\mathcal{O}} \mathfrak{m} \text{ and } \mathfrak{k}_{-1} = V^*_{\mathcal{O}} \otimes_{\mathcal{O}} \mathfrak{m}^* \end{cases} \tag{3.2}$$

where $\mathfrak{m}^* = \mathfrak{m} \subset R = R^*$.

Let $\mathbb{V}$ and $\mathbb{V}^*$ be the reductions mod $\mathfrak{p}$ of $V_{\mathcal{O}}$ and $V^*_{\mathcal{O}}$. Since $[\mathfrak{k}, \mathfrak{k}'] \subseteq \mathfrak{k}'$, and $\mathfrak{p}\mathfrak{k} \subset \mathfrak{k}' \subset \mathfrak{k}$, it follows that

$$\mathfrak{k}/\mathfrak{k}' = \mathbb{V}^* \otimes \mathbb{E}^* \oplus \left(\mathrm{sl}_3(k) \oplus \mathbb{E}^0\right) \oplus \mathbb{V} \otimes \mathbb{E}, \tag{3.3}$$

where $\mathbb{E}^0$ is the set of traceless elements in $\mathbb{E}$, is a Lie algebra over $\mathbb{F}$. In fact, it is a simple Lie algebra of type D_4^3 [HMS].

Let K be the stabilizer of $\mathfrak{k}$ in H. It is the special maximal compact subgroup. Let K' be the subgroup of K stabilizing the lattice $\mathfrak{k}'$. Since $[\mathfrak{k}, \mathfrak{k}'] \subseteq \mathfrak{k}'$, K' is a normal subgroup of K. The quotient K/K' is a semidirect product of $D_4^3(q)$, and its group of outer automorphisms $\Gamma \cong \mathbb{Z}/3\mathbb{Z}$ generated by the conjugation action of ϖ.

Let $\pi_{\min}$ be the "reflection" representation of $D_4^3(q)$. It is the smallest non-trivial unipotent representation [C, p. 478], its dimension is $q^5 - q^3 + q$. Let Π be the unique representation of H such that the K/K'-module $\Pi^{K'}$ is isomorphic to $\pi_{\min}$.

Theorem 3.4 **(Rumelhart [R])** *The representation Π is minimal. This means that the character expansion of Π is given by*

$$\hat{\mu}_{\mathcal{O}_{\min}} + c\hat{\mu}_{\{0\}}$$

where $\mathcal{O}_{\min}$ is the minimal non-trivial nilpotent orbit [CM], and c some constant.

4 Conjectures

Let π'_I be the unique degenerate discrete series representation of $G_2(k)$ with one-dimensional space of Iwahori-fixed vectors [B]. Let $\pi'[\nu^a]$, $a = 1, 2$, be the unipotent supercuspidal representations of $G_2(k)$ induced from the unipotent cuspidal representations $G_2[\nu^a]$ [C, p. 478] of $G_2(q)$. In [GS2] we have introduced a conjecture describing the correspondence between representation of $PD^\times$ and $G_2(k)$:

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Conjecture 4.1

(1) Representations $\Theta(\pi)$ are irreducible.
(2) $\Theta(\pi_1) \cong \Theta(\pi_2)$ only if $\pi_1 \cong \pi_2$.
(3) $\Theta(1) = \pi_I'$, and $\Theta(\pi)$ is supercuspidal if $\pi \neq 1$.
(4) $\Theta(\chi_D) = \pi'[\nu]$, and $\Theta(\chi_D^2) = \pi'[\nu^2]$.

The unramified character χ_D of $PD^\times$ will be specified in the last section.

In Section 6 we shall prove the statements (3) and (4) of this conjecture.

5 Tools

In order to prove the statements (3) and (4) we need some technical results.

Proposition 5.1 *Let $N_1 \supset U_1$ and $N_2 \supset U_2$ be the unipotent radicals of maximal parabolic subgroups of H and $G_2(k)$. We have the following equalities of Jacquet modules.*

$$\begin{cases} \Pi_{N_1} = \Pi_{U_1} \\ \Pi_{N_2} = \Pi_{U_2}. \end{cases}$$

Proof We shall first prove the second statement. Recall that N_2 is a two-step nilpotent group, and let Z_2 be its one-dimensional center (it is also the center of U_2). Let $\bar{N}_2$ be the opposite unipotent radical, and $\bar{Z}_2$ its center. The Killing form on $\mathfrak{h}$ induces a non-degenerate pairing $\langle\cdot,\cdot\rangle$ between N_2/Z_2 and $\bar{N}_2/\bar{Z}_2$. Thus, every one-dimensional character of N_2/Z_2 is of the form

$$\psi_y(x) = \psi\big(\langle x, y\rangle\big)$$

for some $\bar{x}$ in $\bar{N}_2/\bar{Z}_2$, and ψ a given non-trivial additive character of k. If Π_{U_2} is not equal to Π_{N_2}, then there exists a non-trivial character $\psi_{\bar{x}}$ such that

$$\psi_{\bar{x}}|_{U_2} = 1 \quad \text{and} \quad (\Pi_{U_2})_{N_2,\psi_{\bar{x}}} \neq 0.$$

Since Π is minimal, $\bar{x}$ has to lie in the smallest non-trivial M_2-orbit in $\bar{N}_2/\bar{Z}_2$. On the other hand, $\bar{x}$ has to lie in the orthogonal complement of U_2/Z_2 in $\bar{N}_2/\bar{Z}_2$. It can be checked that these two sets have empty intersection. This is a contradiction, and the second statement follows.

The first statement can be checked analogously. In fact, if Z_1 is the center of N_1 (it is also the center of U_1), then a stronger statement

$$\Pi_{N_1} = \Pi_{Z_1}$$

is true. The proposition is proved.

Corollary 5.2

$$\begin{cases} \Pi_{U_1} = (\pi_I')_{U_1} \\ \Pi_{U_2} = (\pi_I')_{U_2}. \end{cases}$$

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Proof Note that π_I' is unique representation of $G_2(k)$ such that, up to a twist by an unramified character, $(\pi_I')_{U_1}$ is a Steinberg L_1-module, and $(\pi_I')_{U_1}$ is a trivial L_1-module. The same is true for Π: up to a twist by an unramified character, Π_{N_1} is a Steinberg M_1-module, and Π_{N_2} is a trivial M_2-module. The corollary now follows from Proposition 5.1 (note that L_1 is the sole non-compact factor of M_1, hence the Steinberg representation of M_1 restricts to the Steinberg representation of L_1).

Let (x, y, z) be the symmetric tri-linear form on D defined by (1.10). Let x be in D, and λ in k. Then

$$\mathrm{Char}_x(\lambda) = (\lambda - x, \lambda - x, \lambda - x) \tag{5.3}$$

is called a *characteristic polynomial* of x. Its leading coefficient is 6 (since $(1, 1, 1) = 6$).

Recall from [GS1], that characters of U_2 are parametrized by cubic polynomials. We have the following fundamental result [GS1, Ch. VI] and [HMS].

Proposition 5.4 *Let P be a cubic polynomial with the leading coefficient 6, and ψ_P the corresponding character of U_2. Then*

$$\Pi_{U_2,\psi_P} = \mathcal{C}_c^\infty(\omega_P)$$

where

$$\omega_P = \{x \in D \mid \mathrm{Char}_x = P\}.$$

Examples 5.5 (1) If $P(\lambda) = 6\lambda^3$, then $\omega_P = 0$, and $\Pi_{U_2,\psi_P} = \mathbb{C}$.

(2) If $P(\lambda) = 6\lambda^2(\lambda - 1)$, then $\omega_P = \varnothing$, and $\Pi_{U_2,\psi_P} = 0$.

(3) If $E = k[\lambda]/(P)$ is a cubic separable algebra, then $\omega_P = \varnothing$ unless E is a field, in which case

$$\Pi_{U_2,\psi_P} = \mathcal{C}_c^\infty(D^\times/E^\times).$$

Just as in [HMS] the first example imples that Π has no Whittaker vectors for $G_2(k)$. In particular, $\Theta(\pi)$ are degenerate. The third example is a consequence of the following two facts; any cubic field E is contained in D, and any two regular elements in D with the same characteristic polynomial are conjugated. Also, if E is a field, then the third example implies that

$$\Theta(\pi)_{U_2,\psi_P} \cong \pi^{E^\times}. \tag{5.6}$$

This is equivalent to (0.5) by [MW].

6 Proofs

In this section we shall prove the parts (3) and (4) of Conjecture 4.1. Recall from [HMS] that under the action of $\Gamma \times G_2(q)$ the reflection representation $\pi_{\min}$ decomposes as

$$1 \otimes \phi_{1,3''} \oplus \chi_D \otimes G_2[\nu] \oplus \chi_D^2 \otimes G_2[\nu^2] \tag{6.1}$$

for a choice of the cubic character χ_D of Γ. Here $\phi_{1,3''}$ is a unipotent representation of $G_2(q)$ [C, p. 478].

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It is the minimal K-type of π_I'. This and Corollary 5.2 immediately imply that π_I' is a direct summand of $\Theta(1)$, and $\pi'[\nu^a]$ is a direct summand of $\Theta(\chi_D^a)$, $(a = 1, 2)$ (note that Γ is a quotient of $PD^\times$, hence χ_D is the unramified character mentioned in Conjecture 4.1).

Calculations of the previous section, compared with results of [HMS] where $\Theta(\chi_D^a)_{U_2,\psi_P}$ have been computed, show that

$$\dim\bigl(\pi'[\nu^a]\bigr)_{U_2,\psi_P} = \dim\bigl(\Theta(\chi_D^a)\bigr)_{U_2,\psi_P} \tag{6.2}$$

for any P. This implies that the complements of $\pi'[\nu^a]$ in $\Theta(\chi_D^a)$, $(a = 1, 2)$, are trivial (for example, they have trivial character expansion). Also, the results of [HMS] combined with calculations in the Grothendieck group of representations of $G_2(k)$, show that

$$\dim(\pi_I')_{U_2,\psi_P} = \dim\bigl(\Theta(1)\bigr)_{U_2,\psi_P} \tag{6.3}$$

for any P defining a cubic separable algebra. Since $(\pi_I')_{U_1}$ is a generic L_1-module, it follows that $(\pi_I')_{U_2,\psi_P} \neq 0$ for $P(\lambda) = 6\lambda^3$. In particular, we again have an equality in (6.3) for all P, and $\pi_I' = \Theta(1)$ follows. This proves the parts (3) and (4) of Conjecture 4.1 (cuspidality of $\Theta(\pi)$ if $\pi \neq 1$ follows from Corollary 5.2).

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Department of Mathematics
University of Utah
Salt Lake City, Utah 84112
U.S.A.
email: savin@math.utah.edu