Deriving the Derivative

Benjamin Lee, *The New School*

ABSTRACT
The anthropology of finance has been dominated by ethnographic field research and the performativity thesis advanced by Donald Mackenzie and Michel Callon. This essay takes a different tack and proposes a semiotic framework to look at the central concept in derivative finance, that of volatility. Volatility has also been increasingly important in our contemporary culture and politics of volatility, suggesting that the implications of the concept touch upon far more than just finance. I trace the development of the Black-Scholes model for pricing options from its initial use as a foundation for a "physics of finance" to its current use to calculate the "implied volatility" of trillions of dollars of derivative contracts on a daily basis. At the same time, the use of Black-Scholes to calculate implied volatility violates one of the fundamental presuppositions of the model, and I argue that instead of being part of a "physics of finance," Black-Scholes now functions more like the discourse-indexical component of a "leaky grammar of prices."

Derivative finance and the linguistic turn developed independently of one another. It wasn't until Donald Mackenzie's *An Engine, Not a Camera* (2006) that the issue of the performativity of finance was raised, but even that path-breaking work didn't use any tools of linguistic or semiotic analysis. Austin's discussion of performativity inspired linguists, continental philosophers, and literary scholars and became an important part of the linguistic turn and continues to be of contemporary relevance in the work of Judith Butler and others. Although not noticed at the time, some of the issues raised by the linguistic turn, particularly performativity, suggest the possibility of a "semiotics of finance" and more particularly, a "semiotics of volatility" that are relevant to the understanding of contemporary capitalism.

This article examines several developments in derivative finance and the linguistic turn that make a semiotics of volatility possible. First, the grammatical structuring of language and particularly explicit performatives, stock prices,

Contact Benjamin Lee at The New School, Room 903, 65 Fifth Ave., New York, NY 10011 (leeb@newschool.edu)

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Brownian motion, and even Peirce's account of semiosis turn out to share the fractal property of "recursive self-similarity." Even though there would seem to be a big difference between the fractal structure of performatives and stock price movements (or other fractal phenomena such as snowflakes, mountain ranges, and coastlines), the formal similarity suggests areas of complementary synergy, especially around the issue of volatility.

Second, the efficient market hypothesis, which is widely seen as a condition for Black-Scholes, makes the randomness of information a key variable in the behavior of stock prices and thus adds a hermeneutic dimension to finance; since individuals and markets receive overlapping information, there should be an information-inflected connection between individual decision making and market behavior. This "semiotic mediation" is provided by a third factor, the exploration of "intensionality" in the linguistic turn. Building upon Frege's discovery of quantification theory, the philosophers of language discovered that indexicality, modality, and the verbal expression of speaking, thinking, and feeling were intensional: they introduce contexts in which reference shifts and the truth-functional intersubstitution of identities (the logical hallmark of mathematics and science) fail. This structure also connects the contingency of pricing with volatility, which in the form of standard deviation, measures the magnitude or range of the alternative prices. These counterfactual alternatives are interpreted as representing different possible states of the world, each of which is associated with a probability of occurrence; the performative expression of buying and selling at a particular price mediates the probabilistic interpretation of the modality of pricing and the subjectivity of mental acts, states, and feelings, thereby connecting the "inner" subjectivity of individual decision making and the market behavior of prices.

The fourth factor is the evolution of the use of Black-Scholes to calculate "implied volatility," which makes it function more like an indexical discourse grammar than the heat transfer equation that it is formally equivalent to. The Black-Scholes formula is a differential equation in which the major unknown variable is the volatility of the underlying stock. Since it is identical to a well-known heat equation in physics, it was thought to be a crucial part of a "physics of finance" that the new financial mathematics would herald in. The success of Black-Scholes produced more liquid and complete options markets, with the result that people would invert the formula and then insert the market price of the option to calculate the "implied volatility" of the underlying stock as the market's estimate of the future volatility of the stock. This process is known as "calibration" and introduces an indexical element (the market price) to the model; calculating implied volatility is the predominant contemporary use of Black-Scholes. When combined with another feature of the Black-Scholes equation, dynamic replication, which mimicks linguistic co-reference, Black-Scholes as a physics equation turns into a simulacrum of an indexicalized discourse grammar. The result is that options with different strike prices but the same expiration date will have different implied volatilities, violating a presupposition of Black-Scholes that the volatility of the underlying stock should be constant, which raises the question about the "scientific" status of finance—how can you have a physics when its basic model is constantly violated?

Finally, the fractal property of recursive self-similarity creates an internal connection among efficient markets, Black-Scholes, performativity, and semiosis that also allowed anthropological linguists to see ritual and religion as performatively structured in which the ritual events are microcosms of the social macrocosms that they invoke and instantiate; the performative construction of social imaginaries would be extended from ritual to the "imagined communities" of nationalism and the market and would touch upon such cultural issues as ethos, worldview, and ideology. The present discussion of neoliberalism has focused on the expansion of market thinking to what previously money couldn't buy; but while authors often see a connection between neoliberalism and finance, they overlook the internal connection between markets, information, and volatility that are redefining what the market is in neoliberalism.

In Honor of Michael Silverstein

I was a graduate student in Michael's classes in the early 1970s and then ran the Center for Psychosocial Studies, which was at the heart of the "linguistic turn" in anthropology for the next decade. It was a heady experience—among my classmates were Charles Briggs, Joe Errington, Bill Hanks, Rick Parmentier, and Greg Urban; my center colleagues included Maya Hickmann, John Lucy, Elizabeth Mertz, and James Wertsch. Michael was then a newly minted PhD fresh from a junior fellowship at Harvard but also, more importantly for me, a graduate of Stuyvesant High School, just a few blocks from where I grew up in New York City. When I entered Chicago the linguistic turn was beginning, and the joint PhD program in anthropology and linguistics effectively gave students training in both disciplines—I remember sitting in on Language and Culture as well as Jim McCawley's seminar on Montague grammars. At the same time, modern finance was being developed at the Business School and operationalized at the Chicago Options Exchange (with the help of Milton Friedman), but the "formal-substantive controversy" prevented the anthropology department from developing a similar program in economics that would provide formal training in economics the way they did in linguistic anthropology. At the same time the lack of formal economic training meant that anthropologists who might want to do field work in "finance" were handicapped by their lack of understanding of what the "natives" were talking about.

Although I hoped to expand upon the literary "fieldwork" I did in *Talking Heads* (1997), I watched comparative literature shrink with the demise of the "Gang of Four" at Yale. I then turned to the intricacies of derivative finance and struggled to overcome the formal-substantive divide that had haunted my training; the mathematics of finance was formidable, and it took a couple of years just to get up to the level of mathematical fluency needed to follow derivations from forty years ago; it would seem that nothing could be further from the linguistic semiotics I started out with than the path I've now taken.

This wall began to be breached with the development of the "performativity of finance" thesis in the sociology of finance by Donald Mackenzie and Michel Callon, along with the anthropological work of figures such as Knorr-Certina, Karen Ho, and Caitlin Zaloom. My colleague Arjun Appadurai took the "performative" thesis in a very different direction in his very interesting book on promising and derivatives, *Banking On Words* (2016), and Ed LiPuma and I came out with *Derivatives and the Globalization of Risk* (2004) over a decade ago. In fact, my present work takes a different track, through the intricacies of the Black-Scholes model for pricing options and my current explorations of volatility. But this opportunity to honor Michael's work strangely brought me back to themes I raised for literary analysis in *Talking Heads* through a detour that has included Bergson's account of duration and Deleuze's cinema work (a fascinating mix of Bergson and Peirce). What I'd like to do in this article is to sketch how the evolution of Black-Scholes, especially the development of what is called "implied volatility" and "dynamic replication," has brought out the "semey" side of finance by highlighting aspects of its implementation that bring it closer to a pragmatic indexical discourse grammar rather than the foundation of a "physics of finance" it was originally thought to be.

Some Background

The early 1970s were an especially exciting period to be a graduate student in linguistic anthropology, as the linguistic turn was in full bloom. The Black-Scholes equation was published by Chicago professors Fischer Black and Myron Scholes in 1973. Of course, I didn't know about Black-Scholes or that the Chicago Options Exchange was also founded that year, giving birth to derivative finance. Over forty years later, Black-Scholes is still the most used options pricing model, pricing trillions of dollars of derivatives every day; the net value of derivatives is ten times that of the global gross domestic product—over a quadrillion dollars. Black-Scholes is a differential equation that is formally identical with a heat transfer formula in physics, which contributed to the idea of a "physics of finance"; the model was quickly used to identify mispricings as deviations from which the quick and the bold could make arbitrage profits.

Derivative finance developed along two fronts. There was an internal development of the technical aspects of finance, which would lead to Black-Scholes as a model not only for pricing options but for financial innovation in general, as it was used to create new financial instruments out of existing ones. The second trajectory was the social and political effects of the tremendous new wealth created by the expansion of derivatives. In the twenty-five years from 1980 to 2005, finance expands its share of US corporate profits from under 10 percent to over 40 percent, overlapping with the rise of wealth and income inequality that Thomas Piketty describes in *Capital in the Twentieth Century* (2014). This growth was catalyzed by neoliberal policies of financial deregulation mainly under the guidance of Alan Greenspan; the result was a twenty-year period of low macroeconomic volatility known as the "Great Moderation" that ended with the financial crisis of 2007–8. It's not difficult to see the impact of this history in the "politics of volatility" of Donald Trump, which contrasted with the prudent neoliberal risk management style of Hillary Clinton, itself a legacy from her husband's collaboration with Greenspan.

The key discovery of Black-Scholes is how to access and price volatility, and it is the expansion of derivative finance that plays a significant role in the social and political transformations of the last thirty years. In *Talking Politics* (2003) and *Creatures of Politics* (2012), Silverstein extended semiotic analysis to political messages by looking at their pragmatics and metapragmatics and gave us a glance at what a "semiotics of politics" might look like. Ulrich Beck's work on "risk society" (1986) foregrounded how the management of risk and uncertainty has played an increasingly important role in contemporary capitalism. Since the early seventies, the discovery of volatility has become a crucial factor in the development of capitalism not only in finance, but also in postmodern culture and the arts. The rise of derivative finance gives us an unusual opportunity to create a "semiotics of volatility" that would draw together the microanalysis of the technical aspects of derivatives with their larger social impact.

Building upon the cultural analyses of the Frankfurt School, David Harvey suggested in his *The Condition of Postmodernity* (1990) that postmodern cultural forms emerged in the early seventies (he actually gave the date of 1972!) out of the volatility created by the collapse of the post-Fordist economy under the pressure of the rise of flexible accumulation led by the "Asian tigers." The year 1973 was also the birth of derivative finance, with the publication of the Black-Scholes equation, the founding of the Chicago Options Exchange, the end of Bretton Woods, and the Arab oil crisis, which created huge amounts of speculative capital searching the globe for new areas and financial instruments to invest in. The decline of Fordism created new forms of economic and social volatility as vertically integrated production gave way to the centrifugal forces of flexible accumulation; it was out of the ruins of the Fordist inner city that skate boarding, hip hop, punk, graffiti, and rap would emerge as new social forms of "risking together" that still command contemporary relevance.

Postmodern was a catchall term that covered a range of social and cultural forms from art and architecture to literary criticism and philosophy. Many of the properties attributed to postmodernism, such as the focus on fragmentation, ephemerality, and the resistance to grand narratives, would seem to be subsumed under the idea of nondirectional volatility, and volatility is often seen as a characteristic of postmodernism. However, in Marxist accounts volatility is seen as the product of the dynamics of labor and value-based production. For example, Harvey saw the "postmodern condition" as developing out of the "time-space compression" of flexible accumulation that increased volatility by compressing the turnover time of capital. Flexible accumulation was seen as reorganizing production and circulation and not creating a new form of wealth. For Harvey and most Marxists, derivatives did not produce "value" and were seen as "fictitious capital."

The breakthrough of derivative finance and Black-Scholes is that what is priced is volatility, which is accessed by delta hedging and dynamic replication. Whether this is a new form of wealth ("abstract risk" versus "abstract labor time") depends on what one thinks counts as a source of value. Black-Scholes shows how to price volatility through "replication"—the price of the option is derived from the "value" or volatility of a different financial instrument, that is, its underlying stock. Emanuel Derman, one of the first "quants" with Fischer Black at Goldman Sachs in the mid-1980s, has pointed out that Black-Scholes introduced a new principle of innovation in finance that guided the explosive growth of derivative finance over the next several decades: "Before Black and Scholes and Merton no one had even guess that you could manufacture an option out of simpler ingredients. Anyone's guess for its value is as good as anyone else's; it was strictly personal. The Black-Scholes Model . . . revolutionized modern finance. Using Black and Scholes's insight, trading houses and dealers could value and sell options on all sorts of securities, from stock to bonds to currencies by synthesizing the option out of the underlying security" (2011, 176).

It is hard to underestimate the effects of the discovery of volatility. In finance, it will lead to the unprecedented expansion of derivatives to where their present notional value is ten times the size of the global economy. Derivative capitalism will fuel the rise of neoliberalism whose deregulation of derivatives will catalyze their growth and creates the new "risk management" techniques used during the "Great Moderation" from the 1980s until the financial crisis of 2008. The Great Moderation will also overlap accelerating wealth and income inequality that starts in the United States and Europe in the mid-1980s.

The story of the Black-Scholes model starts in 1973 with its publication and its immediate use on the floor of the Chicago Options Exchange. As Mackenzie relates, the very success of Black-Scholes made options markets more transparent and liquid; options prices became closer and closer to the prices predicted by the model (2006). Since price transparency and liquidity are presuppositions for the functioning of Black-Scholes (increased trading also lowered transaction costs in the direction of the model's assumption of minimal transaction expenses), the increasing use of the formula "created" the presuppositions that made it true and the "spread" between theoretical and market prices narrowed; although not the instantaneous indexical performativity of an explicit linguistic performative, the act of using the model created conditions that increasingly fulfilled the model's presuppositions and made representation and practice align—hence the performative thesis.

Mackenzie's use of performativity was to criticize the model of finance as a kind of physics and to introduce a constructivist element to economics—economic categories not only modeled human behavior but were used to produce behavior that conformed to that described by the models. Although Mackenzie and Callon have inspired an explosion of ethnographic work on finance, most of this research has overlooked the linguistic analysis of performativity and its anthropological use to analyze ritual. The performative analysis of ritual was developed at Chicago by Michael and Stanley Tambiah and has subsequently become almost "settled science" (Silverstein 2001, 606) in linguistic anthropology. Austin's original account of linguistic performativity (1962) was saturated with ritual—his examples of explicit performatives were a wedding vow, a ship christening, a bequeathal, and a bet. The ritual setting embeds the performative event within an encompassing social totality whose "force" helps to bring about an instance of what is invoked; the macrocosm becomes instantiated in the microcosm. In some cases, the social totality would be a cosmological social imaginary as in Tambiah's analysis of a Thai royal ceremony as an "indexical-icon" of its divine counterpart or the Andersonian construction of the "we, the people" of modern nationalisms; of course the most relevant for economics would be the performative construction of "the market" from the contractual promises of its participants.

Mackenzie's story stops with the stock market crash of October 19, 1987, when global markets declined more than 20 percent in a single day. After Black Monday, the options market developed what has become known as the "volatility smile," whose appearance would violate a fundamental presupposition of the pricing model. The success of Black-Scholes brought about more liquid and transparent prices for all sorts of options, and it became possible to run Black-Scholes "backward" to calculate what was known as "implied volatility." In the early uses of Black-Scholes, the major unknown was the volatility of the underlying stock, which was estimated by its "historical volatility." Plugging in the historical volatility of the stock and solving the Black-Scholes equation would result in the theoretical option price. However, as liquid market prices for a full range of options became available, the market price of the option would be inserted into an "inverted" formula and the "implied volatility" of the stock would be calculated. This would be the market's estimate of the future volatility of the stock, the volatility needed to make the Black-Scholes model "work" given the market price of the option.

Implied volatilities are what are now used in options markets—calculating implied volatilities is the basic use of Black-Scholes—and the volatility index, or VIX, is the implied volatility of the S&P 500. But after Black Monday, options on the same stock with different strike prices but the same expiration turned out to have different implied volatilities, which when graphed produced a "smile" rather than a straight line that constant volatility would imply. The basic assumption of Black-Scholes is that stock volatility is constant, so the predominant use of Black-Scholes violates the fundamental assumption of Black-Scholes—so much for a "physics of finance" in which a basic presupposition of the model is falsified every time it is used. The volatility smile continues to the present, and Mackenzie has called the appearance of the smile an example of "counter-performativity," in which the spread between theoretical and market prices begins to increase.

But what is the effect of inverting Black-Scholes and calculating implied volatility? It would seem that the most common use of the Black-Scholes model, which is to calculate implied volatility, violates a fundamental assumption of the model that the volatility for the underlying stock was constant. It would seem to foreclose the issue of whether a "physics of finance" was possible—how would you have a physics whose basic model violated one of its fundamental assumptions? Indeed, Elie Ayache, author of *The Blank Swan* (2010) and *The Medium of Contingency* (2015) and former options trader, summarizes the paradox:

> More specifically, if, from the market price of the call option and by inverting the Black-Scholes model, we infer volatility σ_1 the first day, volatility σ_2 the second day, and volatility σ_3 the third (this occurs in order to put in place dynamic replication), this, in itself, is *already* the sign that the Black-Scholes model is not valid, because the Black-Scholes model assumes that volatility is constant! . . . This phenomenon, called the *volatility smile*, is indeed the rule rather than the exception in the options market. Not mentioning that the sole purpose of the activity of writing derivatives and valuing them is to trade them, a principle we can now rephrase as: "The only purpose of framing derivatives in a given representation and model is to unsettle that representation and model." If implied volatility is followed through all its implications we find that it perpetually leads to the devastation of its concept. (2005, 32–33)

The insertion of the market price of an option moves the model to the indexical real time of trading, what is known in finance as "calibration." It's at this point that the formal semantics of Black-Scholes meets its pragmatic counterpart and Black-Scholes begins to taken on a semiotic form. The equation is like the formal part of a grammar that contains an indexical component (the insertion of the market price of the option) that constantly calibrates itself to the moment of speaking; when combined with a property of Black-Scholes known as "dynamic replication," this has the effect of making Black-Scholes into a discourse-indexical "grammar" in which each successive price is linked to its predecessor.

The Grammar of Black-Scholes

How does this grammar work, and what are its implications? Black-Scholes is an equation for pricing options. A derivative, or contingent claim, is a financial instrument that derives its value from something else (it may be another financial instrument such as a stock but can be anything that can be given a price). An *option* is the right but not the obligation to buy or sell an asset at a fixed price (the strike price) at a particular time in the future. For example, buying a \$5 call on Apple stock might give the buyer the right but not the obligation to buy Apple at \$120/share in six months; a *put* is the right to sell and a bet that the stock will go down in price. If the stock is now at \$100 and at \$120 at expiration in six months, then the profit on the call is $\$120 - 100 - 5 = \$15/$share, or a 200 percent return on the \$5 investment in buying the call. Buying the stock at \$100 would give a return of 15 percent; stock options are leveraged bets on the performance of the underlying. The loss on a call is limited to the price of the call; if Apple drops to \$90 at expiration, the call owner simply refuses to exercise the call and buy Apple at the strike price of \$120. There is thus an asymmetry in the payoffs of a call; its downside is limited to the price of the call, and its upside is theoretically unlimited, to wherever the stock price might go. This asymmetry is mathematically known as "convexity," and it's the value added by an option ("time and the right to choose"[1]) compared with that of the stock. The problem is to calculate the value of that convexity.

Since the extra value of an option is the convexity created by its strike price and expiration time, the mathematical problem is to figure out the value of the option at expiration and work backward—that is, calculate its present value by discounting its future value to the present (sort of like running compound interest backward from the future to the present). The value of optionality and the passage of time interact to arrive at the value of the option at any moment. If the strike price of a call is lower than that of the stock price, then the call is "in the money"—at that point in time, exercising the option would result in a profit—buy it at the strike price and sell it at the market price. If it's close to expiration, a call that is deeply out of the money will have little value because there is little time left for the stock to increase; similarly, a call that is "deep" in the money will already reflect that gain as there is little time for the stock price to decrease. This requires a model of price movements of the underlying stock from the present to expiration that balances "time decay" and convexity.

For a variety of technical reasons, the model for stock price movements is taken from the "random" behavior of Brownian movement; stock prices are assumed to follow a lognormal path, an insight first formulated by Louis Bachelier in 1900, which remains one of the foundations of options pricing theory. Figure 1 shows the expected value of a stock (the "drift") as a straight line (the "certainty model") and a jagged line representing geometric Brownian motion. Figure 2 combines the drift portion ($\mu\Delta t$ = the drift, μ = expected return), the random component ($\sigma\varepsilon\sqrt{\Delta t}$; σ = standard deviation, ε = standard normal), and the normal distribution of stock prices moving forward in time. Figure 2 includes the "shock factors" that trigger price movements, which are random

1. From a class lecture by Emanual Derman (n.d.).

but follow a normal distribution; in the efficient market hypothesis, these shocks would represent the dynamic input of new information that causes stock prices to behave randomly.

The original discovery of Brownian motion was that of the seemingly random movements of pollen particles suspended in water first noticed by the botanist Robert Brown. In 1904 Einstein published a paper explaining these movements in terms of water molecules bombarding the pollen particles, thereby producing their "random walk" behavior. The efficient market hypothesis assumes that stock prices are determined by information, and that the information is random—the information randomly "bombards" traders, which leads to the random behavior of stock prices. In the efficient market model, the "shock" would be the random arrival of new information that would then trigger a new normally distributed stock price. While the Brownian motion represents the random or stochastic portion of the behavior of pollen particles while the drift in Brownian motion might be due to a current in the water in which the pollen particles were suspended; the important drift factor in options pricing will be the risk-free interest rate.

Combining the linear and random portions of stock price behavior, we get the following equation in which the change in stock price (dS_t) is a function of time (t), two constants, the expected rate of return of stock (μ) and its volatility or standard deviation (σ), and a Brownian process (B_t):

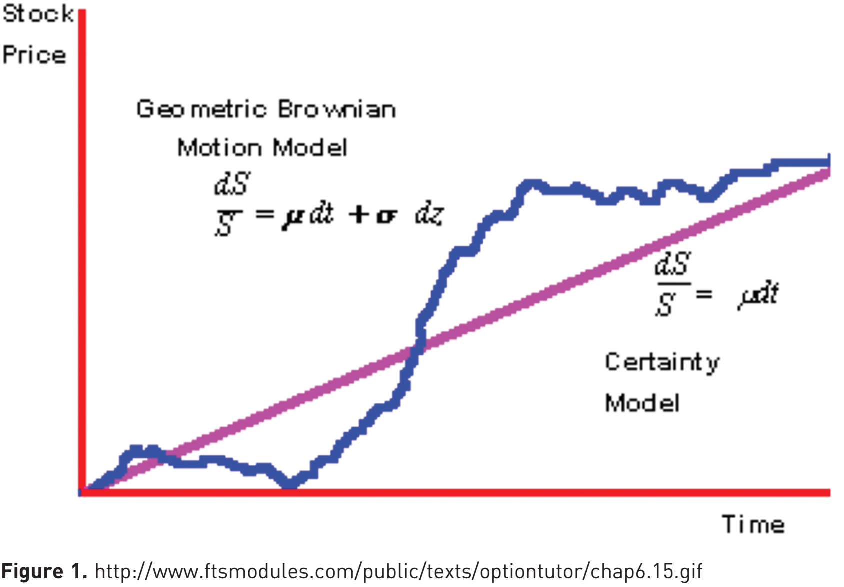

Figure 1. http://www.ftsmodules.com/public/texts/optiontutor/chap6.15.gif

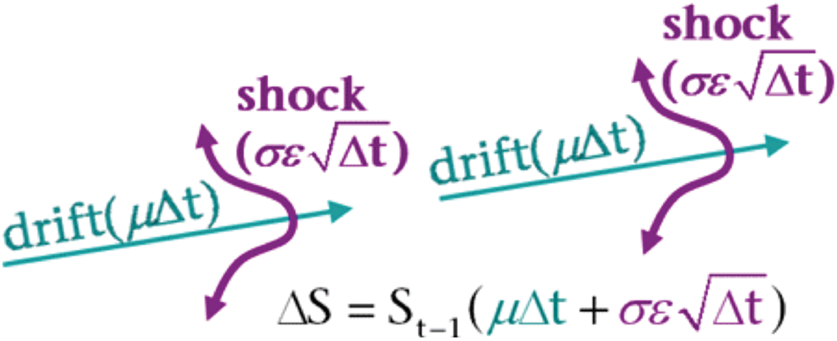

Figure 2. http://www.investopedia.com/articles/07/montecarlo.asp

$$d(S_t) = \underbrace{\mu S_t dt}_{\text{drift}} + \underbrace{\sigma S_t dB_t}_{\text{random}}$$

This differential equation states that a change in a stock's price contains a drift component $\mu S_t dt$ and a random component $\sigma S_t dB_t$ that follows Brownian motion. Whereas the drift component is linear, smooth (nonjagged), and continuous, the Brownian component is nonsmooth but continuous. One of the properties of Brownian motion is that any portion or segment has the same formal properties of the whole, so that even an infinitesimal segment still has a jagged nonsmooth appearance, which makes the Riemann-Stieltjes integral of standard calculus inapplicable. The result is that although $\mu S_t dt$ is treatable by traditional calculus, the Brownian portion requires stochastic calculus that can handle its nonsmooth or jagged behavior. The most popular version of stochastic calculus used for financial purposes is Ito's calculus, which shows that any function with that takes an Ito process as its object and has the same variables is itself an Ito process with a linear and stochastic component. Since options are functions on stocks, an equation can be written using the same variables that commensurates the option function with the stock price function; the resulting option function will also have drift and random components (it will also be an Ito process). If we take the stock price function above, a call function (C) could be written that would look like:

$$dC(S_t) = \underbrace{\left(\mu S_t \frac{\partial C}{\partial S} + \frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2}\right) dt}_{\text{drift}} + \underbrace{\sigma S \frac{\partial C}{\partial S} dB_t}_{\text{random}}$$

From this formula it is now possible to derive the Black-Scholes differential equation for an option (V replaces C in the above formula) in which the expected return of the stock drops out and the only unknown parameter is the volatility of the stock:

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

1) $\partial V/\partial t$ = time decay, how much the option value (V) changes if the stock price doesn't change (t = time); V replaces C in the call function
2) $\frac{1}{2}\sigma^2 S^2 (\partial^2 V/\partial S^2)$ = convexity term, how much a hedged position makes on the average from stock moves; V = option price, σ = volatility of the stock, S = stock price
3) $rS(\partial V/\partial S)$ = drift term allowing for the growth in the stock at the risk-free rate
4) $-rV$ = the discounting term, since the payoff is received at expiration but you are valuing the option now (Wilmott 2009, 130–31)

The key term here is (2), the convexity term, which refers to a "hedged position" that is the fundamental breakthrough of derivative pricing and contains the ideas of delta hedging and dynamic replication. Convexity is a mathematical measure (Jensen's inequality) of the asymmetry of the payoffs—the maximum upside is unlimited while the maximum loss is the price of the option. In the standard mathematics of portfolio theory, the expected return of a stock is the mean price movement adjusted for some time period, usually a year; graphically it is represented by a straight line and is a first derivative. Convexity is a second derivative compared to the linearity of the standard expected return calculations in portfolio theory. Convexity is measured by $\partial^2 V/\partial S^2$, the second derivative or the change in the option price given a change in delta, which is how much the option price changes with a change in the stock price (the first derivative), so it's how much the option price changes with a change in delta.

The idea behind the delta hedge and dynamic replication is described by Fisher Black in his article "How We Came Up with the Option Formula":

> Suppose there is a formula that tells how the value of a call option depends on the price of the underlying stock, the exercise price and the maturity of the option, and the interest rate.
>
> Such a formula will tell us, among other things, how much the option value changes when the stock price changes by a small amount within a

> short time. Suppose that the option goes up about $.50 when the stock goes down $1.00. Then you can create a hedged position by going short two option contracts and long one round lot of stock.
>
> Such a position will be close to riskless. For small moves in the stock in the short run, your losses on one side will be mostly offset by gains on the other side. If the stock goes up, you will lose on the option but make it up on the stock. If the stock goes down, you will lose on the stock but make it up on the option.
>
> At first, you create a hedged position by going short two options and long one stock. As the stock price changes, and as the option approaches maturity, the ratio of option to stock needed to maintain a close-to-riskless hedge will change. To maintain a neutral hedge, you will have to change your position in the stock, your position in the option, or both.
>
> As the hedged position will be close to riskless, it should return an amount equal to the short-term interest rate on close-to-riskless securities. This one principle gives us the option formula. It turns out that there is only one formula for the value of an option that has the property that the return on hedged position of option and stock is always equal to short-term interest rate. (1989, 4)

Creating the hedged position by selling two options for each share of stock is the "delta hedge;" dynamic replication is rehedging in the face of price changes and time decay. The delta hedge and its continuous updating remove any exposure to price movements and directional risks in the underlying asset. The convexity of optionality also produces the possibility of making money from volatility; whether the stock price goes up or down, there is a positive gain from volatility; taking Black's example, if the risk-free delta-hedged portfolio consists of two short option contracts and one long share of stock, because of the asymmetry of convexity, when the stock goes up in price, the long position of one share of stock will go up more than the two short positions will go down, yielding a net profit; if the stock goes down, the short position in the option will go up more than the long position in the stock goes down. The holder of a delta-hedged position can make money regardless of whether the stock goes up or down as long as he beats time decay.

Delta hedging and dynamic replication are written into the "grammar" of Black-Scholes; calculating the option price depends on hedging and replicating until the expiration date of the option, with each step in the price process dependent only on the previous one in a potentially unending process of dynamic replication. Black-Scholes shows that if you want to access and price volatility, you need to hedge out or neutralize directional risk; as the asset price changes, you need to continuously reset the hedge (delta hedging) in a process (dynamic replication) that ends only at the expiration, or "death," of the option. Hedging out or neutralizing directional risk creates a "volatility spread" within which the instantaneous volatility and the option prices move; in the case of Black-Scholes, delta hedging shrinks the "volatility spread" to the riskless rate of interest because all directional risk has been eliminated.

In the Black-Scholes formula, delta hedging and convexity interact to produce one of the principal components of the price of an option; the other major factor, time decay, $\partial V/\partial t$, reflects the change in value as the option approaches expiration. The convexity term also depends only on the volatility of the underlying asset; the Black-Scholes model makes no mention of the expected return of the stock, a result that initially surprised Black and Scholes; but the delta hedge "neutralizes" directional risk, leaving volatility and the risk-free interest rate as the principal factors in derivative pricing. The delta hedge also sets up a risk-neutral portfolio that can be evaluated at the risk-free interest rate; the risk neutrality of the portfolio allows the application of a whole set of mathematical models such as martingale theory to derivative pricing. But perhaps most importantly, the delta hedge introduces into finance the notion of replication; the Black-Scholes formula computes the unknown current value of a stock option by "replicating" it in terms of the underlying stock and a risk-free bond, whose values are known.

Since the Black-Scholes equation was formally equivalent to a standard heat transfer equation in physics, when it first appeared it wasn't surprising that people in finance thought a "physics of finance" was in the making. Indeed, the initial uses of Black-Scholes was to look for discrepancies between market prices and the theoretical prices and bet upon them as arbitrage opportunities. The rapid spread of Black-Scholes improved liquidity and price transparency, and the spread between theoretical-model and market prices began to tighten and options markets began their unprecedented expansion.

The Implications of Implied Volatility

The growing popularity of Black-Scholes also meant that as the breadth and liquidity of options prices grew there became available a full range of options at various strike points and expiration dates. People also began to invert the Black-Scholes formula and insert the now liquid market prices of options as parameters to calculate the "implied volatility" of the underlying stock. Today, the prices of options are given as implied volatilities, and the dominant use of Black-Scholes is to calculate implied volatilities. The final irony is that options on the same underlying stock often yield different implied volatilities for the underlier, which violates a fundamental assumption that the volatility of the underlier is constant.

The problem of the calibration to indexicality was immanent in Black-Scholes. In its "physics of finance" interpretation, the equation is like that of the heat transfer equation, which describes the diffusion of heat in a perfectly insulated metal bar. If extreme heat is applied to the bar at some point, the heat diffuses across the bar through the Brownian movements of heat molecules, whose distribution follows a log-normal pattern, thus completing the connection with stocks, which are also thought of as moving in a Brownian manner. Volatility is the only unknown parameter in the Black-Scholes equation and immediately involves Black-Scholes in a paradox. "Actual volatility is a measure of the amount of randomness in a financial quantity at any point in time. It's what Desmond Fitzgerald calls the 'bouncy, bouncy.' It's difficult to measure, and even harder to forecast but it's one of the main inputs into option-pricing models. . . . It's difficult to measure since it is defined mathematically via standard deviations, which requires historical data to calculate. Yet actual volatility is not a historical quantity but an instantaneous one" (Wilmott 2009, 162). The variance of stocks is calculated by computing the mean of a set of price movements over some period of time (usually a year) and then summing the squares of the difference between the mean and each price movement; the standard deviation is the square root of the variance. What financial analysts want is the instantaneous volatility, but they can only approximate it with a historical volatility (the standard deviation for some fixed time period).

Delta hedging and dynamic replication are present in the original equation, but implied volatility is not; its calculation requires inverting the formula and inserting the market price of the option. It moves the model into real time, to the point where it makes contact with the instantaneous volatility of the event of trading, which exposes a nonquantitative dimension of volatility, captured in this quote from Elie Ayache describing his experience market making in which the quantitative dimension of volatility as embodied in Black-Scholes interacts with a qualitative experience of volatility to produce the event of trading: "Through the dynamic delta-hedging and the anxiety that it generates (Will I execute it right? When to rebalance it, etc.), the market-maker penetrated the market. He penetrated its volatility and he could now feel it in his guts. In a word, he became a *dynamic trader*. He now understood—not conceptually, but through his senses, through his body—the inexorability of time decay, the pains and joys of convexity" (2008, 36–37). The quote invokes the key components of Black-Scholes: dynamic replication, delta hedging, and (implied) volatility, which come together in the qualitative volatility and indexical time of the act of trading. "Dynamic delta hedging" eliminates directional risk, exposing the trader to the anxiety of a "pure" volatility that changes prices as he rehedges to maintain a risk-free position; calibration and implied volatilities keep him tied to the real time of trading in which the market maker is constantly weighing the upside of convexity against the ravages of time decay. The Bloomberg machines map the movement of implied volatilities that the market maker tracks as he "surfs" the volatility wave. The unfolding of volatility infuses his attention and consciousness, drawing up emotions and affects that unfold in the act of trading ("he could now feel it in his guts"): the "anxiety" of delta hedging, the "inexorability" of time decay, and "the pains and joys" of convexity. It's in the instantaneous volatility of making a trade to fix a new price that the quantitative dimension of volatility meets its qualitative counterpart.

The move to implied volatility and the real time of the market cross an ontological divide between the formal representation of possibility and probability (martingales and binomial tree diagrams) to the indexical contingency of trading, thereby connecting quantitative volatility to its qualitative counterpart. Watching implied volatilities stream by on the Bloomberg machine (the prices of options are represented by implied volatilities, not dollars and cents), opening a position by buying or selling an asset, determining the size of the corresponding hedge, and then establishing it to neutralize any directionality, are part of the market maker's "dance" that exposes him to the volatility of the market. As prices change, the hedge has to be reset to a risk-free, zero-volatility position, thereby constantly maintaining a risk-free position in the facing of changing volatility; it is this combination of dynamic delta hedging, volatility, time decay, and implied volatility that Ayache describes as in the "guts" of the market maker and makes the act of trading volatility analogous to surfing or dance.

Delta hedging, implied volatility, and dynamic replication transform Black-Scholes from an engineering device into a semiotic grammar of prices. Implied volatility moves Black-Scholes from the abstract calendar time of its mathematics into the real time of trading and market making. The price of the option is no longer the result of a calculation made within an abstract model; it is the result of trading in real time. Implied volatility is the market's estimate of the forthcoming volatility of the stock, and it's the basis for delta hedging and dynamic replication as real-time processes. Instead of being a theoretical price based upon a historical volatility, the market price of the option is a moving "price image" that will be put into play by implied volatility, delta hedging, and dynamic replication. Pricing becomes a real-time semiotic process built around the "now" of the market price in which the present forward-looking "volatility image" is already becoming a past upon which future action is based.

With implied volatility, the gap between model and reality begins to close. The starting input is what the theoretical use of Black-Scholes was supposed to calculate: the market prices of options. With the inversion of Black-Scholes to calculate implied volatilities, the model is used to calculate something it is now part of—the flow of market prices—rather than stand outside the process as a model of it. The implied volatility is not a historical average of past volatilities that approximates the instantaneous randomness of the stock's volatility that is supposed to be inputted into the formula. Instead, it is the model dependent representation of the stock's volatility needed to make the theoretical price match the market price, which is constantly updated by the process of delta hedging and dynamic replication. The model is now part of the process it purports to represent by producing a "present" representation of a forthcoming volatility that becomes its own past as soon as it is enunciated in a potentially never ending of creative destruction. The present price image is a slice into this process, which contains an image of itself as it unfolds until its expiration.

Implied volatility, delta hedging, and dynamic replication turn a model of pricing into a semiotic process built around the indexical moment of trading. Before implied volatility, Black-Scholes was treated like any other partial differential equation in physics; it was solved by combining various partial derivatives, specifying their boundary conditions (the initial and terminal states of the derivative contract), and solving the resulting combination by setting it equal to zero. However, once the model is inverted and the market price is inserted as the variable input, and then delta-hedged and dynamically replicated as time unfolds, the process becomes self-reflexive—implied volatility is the value you need to make the model produce a theoretical volatility for the stock that results in the present market price of the option; it's the model's representation of the future volatility of the stock at expiration that is discounted to get the market price of the option. It is this self-reflexivity that turns Black-Scholes into a complex sign in an unending process of semiosis, a process captured in Peirce's famous definition of a sign: "A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same relation to the same object for some interpretant" (*W* 1:541).

A sign is not a dyadic relation between sign and object, to be encoded by a sender and decoded by a receiver. Instead, a sign_1 determines another sign_2 to stand in the same relation to the object as it_1 does, ad infinitum in a potentially unending chain of self-reflexive sign generation. Delta hedging sets the risk free rate as the relation that is dynamically replicated by the movement of prices; each successive price is recalibrated to the risk-free rate, which determines the next price sign to stand in the same risk-free relation as it does, ad infinitum or to the expiration date of the option (which then can be "rolled over") perdures. Delta hedging and dynamic replication make the movement of prices a simulacrum of the semiotic process.

As a differential equation, Black-Scholes works in the realm of logic and the ratio-scaled calendar time. However, the calculation of implied volatility ties Black-Scholes to the indexical time of trading and the market; the starting and end points of the pricing process are the market prices of options, which are performative contracts among counterparties to buy or sell an option at a fixed market price. These are fed into an inverted use of Black-Scholes, which is constantly hedged and dynamically replicated as prices change. Implied volatility takes the "logic" of the Black-Scholes formula and the grammar of its pricing of volatility and in combination with delta hedging and dynamic replication creates a semiotic flow of prices. The "driver" of the option-pricing process is the volatility of the underlying stock. Delta hedging and dynamic replication initiate a complicated semiotic process in which the price sign of the option is a translation of the volatility of the underlier; via delta hedging, this sign determines another price sign to stand in a similar (risk-free) relationship to its object as it does, which is dynamically replicated over time until expiration.

Calculating implied volatility involved inputting the market price of the option into an inverted Black-Scholes formula; instead of using the historical volatility of the underlying stock to calculate the theoretical price of the option, you used the actual market price to calculate the "implied volatility" of the stock. The implied volatility was the value needed to make the theoretical option price match its market price. For options on the same stock with different strike prices, the resulting implied volatilities varied; this violates one of the assumptions of Black-Scholes—that the volatility of the underlying stock was constant. When graphed, the distribution looked like a "smile" and has been called that since its first appearance after the stock market crash on October 19, 1987 (Black Monday). Implied volatility has become the dominant measure of volatility in derivative finance; the VIX is the implied volatility of the S&P 500.

Implied volatility makes the reality of the market—the process of trading and market making—part of the model. Emanuel Derman (2002) has shown that there are at least two times of temporality at play in the market. The first is "calendar time," which is the infinitely divisible "empty, homogenous" time presupposed by the mathematics of Black-Scholes and in which all events are supposed to take place. The other time is the "instrinsic time" of trading, the indexical real time of the market, which is measured by the frequency and intensity of trading. Implied volatility and the trading practices of delta hedging and dynamic replication occur in "instrinsic time" even as they rely on values calculated in the calendar time of the pricing models that traders use. From a temporal standpoint, trading is a multilevel, mixed mode that brings together multiple durations and volatilities.

Implied volatility, delta hedging, and dynamic replication keep the trader and market maker inside the volatility of the market, in its temporal unfolding in "tick-time." If a market maker buys an option based on his assessment of its implied volatility (option prices are given in implied volatilities, not dollar amounts), he immediately delta hedges the position and repeats that as the price changes; he'll make money not from the directional movement of the stock price but from the rehedging of his position. If he initially bought a call option and didn't delta hedge his position, the call would be a directional bet on the stock going up in price; but by delta hedging the call (by shorting a "delta" amount of the stock), he removes all directional risk and exposes himself to the convexity of the option and bets on the volatility and not the directionality of price changes. He will make money whether the stock goes up or down, as long as the convexity is greater than the time decay.

Implied volatility, delta hedging, and dynamic replication "nail" the trader to the flow of prices—implied volatility is a kind of "price image" (playing off of Deleuze's "movement image") of price movement, and delta hedging and dynamic replication are forward projections of the future movement of prices even as the present price image slips into the past. Inside the process of dynamic replication, the implied volatility price image is a present that is becoming a past that determines a future (the present market price is used to calculate the implied volatility that is then used to calculate the stock price at expiration and then discounted to the present to calculate the present option price), which is a virtual reprise of Bergson's account of duration.

Reprising Performativity

I've gone through this rather exhausting reprise of Black-Scholes to highlight something that seems to me to be unprecedented in the use of mathematical models in finance. Mackenzie's research shows how the use of the Black-Scholes "model of options prices" as a "model for trading" produced some of the conditions necessary for the model to work. The success of the Black-Scholes created a self-referential feedback loop that pushes options prices in the direction predicted by the model and closes the spread between theoretical and market prices, which then reverses itself after 1987 with the arrival of the volatility smile. The story of the performativity of Black-Scholes ends with the rise of a "counter-performativity" inaugurated by the smile. Mackenzie and Callon do not look at the linguistic dimensions of performativity, and the story of the performativity of Black-Scholes seems to end with the creation of liquid options markets. However, I'd like to build upon their insights and push the issue of performativity into the very foundations of finance and suggest that calibration and dynamic replication bring out the performative and semiotic dimensions of derivative finance as a social practice.

One of Peirce's breakthroughs in his semiotic theory was to move beyond a dyadic consideration of signs to a triadic one, which would be the basis for his great classification of sign types. A sign is a mediation between itself, its object, and its interpretant, which itself is a sign in a potentially unending "discourse of signs." If we peer inside his definition of a sign, it contains a quasi-performative moment in the stipulation that a sign "determines its interpretant to stand in the same relation to the same object for some interpretant;" signs are self-reflexively self-similar in that they determine another sign, the interpretant, to stand in the same relation to its object that it does, as in his famous description of a sunflower as sign: "Thus, if a sunflower, in turning towards the sun, becomes by that very act fully capable, without further condition, of reproducing a sunflower which turns in precisely corresponding ways toward the sun, and of doing so with the same reproductive power, the sunflower would become a Representamen of the sun" (*CP* 2.274). Self-similarity is one of the defining features of fractals; a self-similar object is similar to a part of itself, such as a snowflake or crystal. One of the most interesting properties of fractals is that the recursive application of self-similarity to a part generates a whole that has the same formal properties as the part as in the way crystals or snowflakes form. Peirce's semiosis is a fractal process.

Michael's early work built upon Peirce's semiotic, with a special focus on indexical signs. His research on the noun-phrase hierarchy and ergative languages showed how a Saussurean analysis could include indexical categories. Saussurean proportionality defined the boundaries of the linguistic system by correlating systematic differences in sound with systematic differences in meaning—the phonological representation of Fregean senses. Grammatical categories include indexical categories, and building upon Jakobson (1985) and Benveniste (1971), Michael proposed that grammatical categories were themselves order by a principle of "metapragmatic recursion." In the noun-phrase hierarchy, grammatical marking relations would systematically order nominal categories in terms of their relative indexicality and meta-indexicality, which would also determine discourse relations such as coreference. At one extreme of the noun-phrase hierarchy would be the personal pronouns and deictics, which presupposed parameters of the ongoing speech event. At the other might be abstract nouns referring to entities presupposed to exist independent of any speech event, such as essences like "justice" or "beauty." A noun phrase's position in the hierarchy would determine its grammatical relations, including the case relations that would define co-reference in discourse.

Metapragmatic recursion gives a fractal structure to language; any utterance has an indexical and meta-indexical structure that is self-similar to the "totality" of language of which it is an instance. The uniqueness of language is that metapragmatic recursion takes indexicality as an object and builds a structure around it. In this respect, explicit performatives are a transparent objectification of the metapragmatic principles that structure language. This was first pointed out by Benveniste (1971), who noticed that explicit performatives were self-referential and their grammatical structure explicitly encoded the relation between indexicality and meta-indexicality.

Black-Scholes as a Leaky Grammar of Prices

What I would like to suggest is that implied volatility, delta hedging, dynamic replication, and the volatility smile make Black-Scholes into something like a "leaky grammar" for prices. It takes an equation that is formally equivalent to the heat diffusion equation in physics and transforms it into something that resembles a semiotic discourse grammar of prices.

The mathematics of Black-Scholes are formal extensional languages in which there is no indexicality or reference shift; they operate in calendar time, the "empty homogenous" and infinitely divisible time of classical physics. Frege tried to develop a logical language that would make all the semantic distinctions necessary for mathematics and science. By eliminating reference shift (the logical intensionality introduced by indexicals, intentionality, and modality), each semantic distinction would have a clear semiotic realization; the grammar of such a language would generate only semantically well-formed "utterances," without ambiguity or rhetorical flourish. It would be a language in which Saussurean proportionality would apply nonredundantly to all semantic categories. However much mathematics and science demanded such semantic clarity, it was a goal not realized in any natural language, as captured in this famous quote from Edward Sapir, which also indicates the limits of Saussurean proportionality: "The fact of grammar, a universal trait of language, is simply a generalized expression of the feeling that analogous concepts and relations are most conveniently symbolized in analogous forms. Were a language ever completely 'grammatical,' it would be a perfect engine of conceptual expression. Unfortunately, or luckily, no language is tyrannically consistent. All grammars leak" (1921, 39). Saussurean proportionality operated at the same level of abstraction as the Fregean notion of sense. It could be seen as a way of discovering how language systematically related sound to concepts ("senses") and their denotata. The problem was that indexicals such as deictics picked out different objects depending upon when and where they were uttered. There was no fixed denotation for an indexical, but simply a rule of use that in principle yielded a new referent each time it was uttered: the pronoun *I* refers to the person who uttered it. For a purified language of science, indexicals could be handled if they could be analyzed in term of nonindexical categories. The problem was that a prototypical singular term, a proper name, which picked out a fixed but unique referent, seemed to be recalcitrant to extensional analysis. Saul Kripke (1972) argued that a purely extensional analysis of proper names would not allow you to keep track of their referents across counterfactual situations ("possible worlds"); there was an irreducible indexical and meta-indexical component to naming and the ability of proper names to act as singular terms.

Proper names are used in discourse to pick out and track unique referents (usually potential participants of a speech or communicative situation). They seem to overlap with indexicals in picking out unique individuals in a speech situation but differ in that they are not shifters. The traditional theory of proper names is that they were disguised or abbreviated definite descriptions. Definite descriptions had no indexical elements because indexicality would introduce the possibility of reference shift and make the context logically intensional. So in the definite description model of proper names (used to preserve the extensionality of logic), a nonindexical or decontextualized description would uniquely pick out the referent (Frege's notion of sense was a candidate for such a definite description).

In his development of a semantics for modal logic, Kripke developed an alternative performative theory of proper names. Proper names secured their referents through an initial performative event of baptism, which was then historically maintained through co-reference. Instead of a proper name being a classificatory instance of a definite description (i.e., the unique object that satisfied the description), it was secured through co-presence between object and the utterance of the name and then maintained by "backward" reference to the original baptismal event. Although this "causal theory" of proper names was developed to secure the intelligibility of the logical intensionality of modality, similar treatments were developed for other intensional contexts, including those introduced by verbs of speaking, thinking, and decision making—performatives, mental state, and mental act verbs—the full range of subjectivity. From both linguistic and logical perspectives, there is no direct route from the nonindexicality of semantics to the indexicality of pragmatics; in the case of the logical intensionality of modality, a performative theory of naming backed up by a causally established discourse chain of co-reference allow proper names to "rigidly designate" (i.e., pick out the same person across possible worlds). The metapragmatic principle establishes not only the relation between name and denotation, but also replaces logic as the principle of discursive coherence.

Unlike physics whose equations are expected to hold for all time, financial pricing models often refer to a future parameter and then discount its value to the present. In the case of Black-Scholes, this future parameter is specified by the expiration date of the option. However, dynamic replication specifies that the delta hedge be reset back to the risk-free position each time the price changes, creating what is known as "continuous-time finance;" Robert Merton's introduction of continuous time mathematics in his proof of Black-Scholes would earn him the Nobel Prize. Dynamic replication makes Black-Scholes into a simulacrum of a semiotic process in which signs give birth to other signs; the addition of implied volatility would move the model from the abstract homogenous time of historical volatility to the indexical time of the market. Since implied volatility is forward looking—it's known as the "market's estimate" of the volatility of the stock at expiration—it effectively moves the semiotic dimension immanent in the mathematics of dynamic replication into real time. The inversion of the equation and the calculation of implied volatility of the stock meant that the volatility calculated was now tied to the real-time process of market prices, creating an "indexical grammar" calibrated to the real-time market price of the option, which was used to calculate the implied volatility of the stock at expiration.

The development of implied volatility, delta hedging, and dynamic replication calibrate Black-Scholes to the real time of trading and market making, what Emanuel Derman calls "intrinsic time" or "tick-time;" it makes the model indexical but at the price of its leaking (the volatility smile). Implied volatility "is the number needed to make the theoretical price match the market price." It's the number generated by the model (the inverted Black-Scholes) to make the theoretical model correspond to the indexical market price. The volatility smile indicates how the theoretical prices differ from the market price; the smile is represents a counterfactual performativity: it's the gap between the decontextualized model and its indexical instantiation that a performative model would have to close for the model to follow its assumptions; Mackenzie's performative thesis rests upon the observation that the spread between theoretical and market prices decreases because of the use of Black-Scholes. The smile is a measure of how much the model "leaks" or violates its presuppositions. Any particular calculation of implied volatility for a specific option gives us the value that aligns the theoretical price with the market price and indexes "performative success." If volatility was constant, then options at different strike prices should have the same implied volatility. But looking across options at the same strike price, the smile appears, indicating some cross-contextual performative failure.

Performatives are transparent objectifications of the metapragmatic principles that structure language. They create a "transparent" bridge between the indexical and nonindexical signs by making the former and instance of the latter, creating a fractal structure built up by what I've called "metapragmatic recursion." As Kripke's account of proper names and modality demonstrates, the gap between indexical and nonindexical reference is irreducible; there is no way that a nonindexical definite description can uniquely pick our a referent that is consistent with the logical treatment of modality (intensional logics of possibility and necessity) and intentionality. In ritual performativity, the gap is externalized: ritual time is that of myth and cosmology, while the ritual event is in real time, and if the ritual is successful, the real is turned into an instance of the cosmological, the mythical type is instantiated in the real time of social praxis by the ritual event. The gap between the time of myth and everyday social processes is where the volatilities of social life occur; the performative spread at the heart of ritual is the ritual model's representation of that volatility spread even as it tries to resolve it by creating a social order that has the "force" of an idealized social behind it.

Putting It All Together: Efficient Markets

All these issues come together in the model of an efficient market. The modern version was developed by Eugene Fama in the late 1960s at Chicago and published in 1970. The importance of this hypothesis should not be underestimated for both finance—it's a presupposition for many financial models, including Black-Scholes—and politics—it's at the core of the ideology of the free market in neoliberalism. Simply stated it says that in an efficient market the prices of financial assets such as stocks reflect all available information. Since information arrives randomly, stock prices should behavior randomly and it's impossible to "beat the market" on a risk-adjusted basis in the long run.

Fama's development of the hypothesis overlapped with that of the Black-Scholes model and became a working assumption for many of the pricing models developed during this period; it would later become a major motivation for the development of index funds—"you can't beat the market." Building upon the work of Bachelier, Fama recast the random walk of stock prices into a hypothesis about the market connecting information to randomness. Stock prices are supposed to follow Brownian motion, with the stock prices behaving like pollen particles only stock prices reflect the "bombardment" by information rather than water molecules. The information that market participants act upon arrives randomly, which is reflected in the stock price movement. But the total movement of prices in the market is the aggregation of these individual transactions, seen as the performative construction out of individual promises (contracts) to buy and sell. For any given company's stock, there is a range of possible prices. Individuals think about the information they are receiving about the stock and company, and then have to make a decision to buy or sell at a given price, and then issue a bid and if successful, make a promise and contract. Each stock price has a subjective probability attached to it, which determines whether they will buy or sell at a given price. The range of prices is of course given by the market, and the range of market prices are seen as counterfactual alternatives to the actual market price, that is, each possible price is an alternative possible world ("state prices") to the actual world. The movement of prices is often represented as a branching tree diagram, with each node representing a possible price and the probabilities associated with it, a lattice-like structure known as a "decision tree," indicating the alignment of market-level price movements and individual decision making.

The alternative prices to the actual market price of a financial asset are seen as counterfactual possibilities that could have been chosen by traders. Each possible price represents a possible world that is associated with a probability of occurrence. These possible states of the world form the backdrop for traders' decisions as Bloomberg machines stream implied volatilities across their screens. The model of trading activity is that as a trader is contemplating the flow of prices and information, he makes a decision to enter the market at a certain price with an associated probability of occurrence and then issues a performative promise to buy or sell at that price. The focus of his attention is the stream of actual prices that index a probability distribution of possible prices in minutely differing possible worlds. From this act of focal attention that starts with the modality of possible worlds and prices, the full armature of performatively mediated subjectively is implemented: modality > performative verbs of speaking > mental acts such as judging and deciding > mental states such as intending or believing—all of which take propositional complements as part of the cryptotypic structuring of modality and subjectivity in what Benjamin Lee Whorf would have called "Standard Average European" (1956, 138).

The flow of stock prices in the market is seen as the aggregation of individual acts of performative promises to buy and sell. In the efficient market hypothesis, the assumption of the fractal nature of Brownian motion for stock prices makes it possible to link the behavior of individuals with market behavior through the formal property of self-similarity: both the market and individuals are subject to the randomness of information at the heat of Brownian motion "homology" between pollen particles and information. The market has a probability distribution of possible prices as does the individual who chooses from those possible prices to arrive at his bid. Of course, what remains the most cross-contextual feature of the price is the model it comes out of: Black-Scholes, which also provides the information about implied volatilities that traders act on. So the price presupposes Black-Scholes or some equivalent (binomial trees), which everyone shares as part of their background, since the implied volatilities that appear on Bloomberg machines are calculated using Black-Scholes. The market price thus looks like an instance of a predetermined probabilistic interpretation of a world of possible prices; traders watch the prices (in the form of implied volatilities) stream by them as actual real-time instantiations of the probabilistically generated possible prices, and then enter into performative agreements to buy or sell at specific prices. The market isn't simply a processor of information to set prices, but a multilevel volatility processor that sets prices by communicating them. Every trade signals both a new price and also the continuity of the market—the one certainty in all the uncertainty produced by the volatility of the market is the market itself.

In Lieu of a Conclusion

Tracking the peculiar history of the "volatility of Black-Scholes" has also allowed me to revisit the formal-substantive divide of my earlier training. The performativity of Black-Scholes makes its discovery of volatility into the engine of contemporary capitalism, eventuating in the "politics of volatility" that we now enjoy (?). The inverted use of Black-Scholes to calculate implied volatilities, delta hedging, dynamic replication, and the volatility smile point to an "indexicalizing" of what started out as a purely theoretical model *of* option prices into a model *for* prices, in the mixed sense of "models of" and "models for" that Clifford Geertz used in his famous definition of religion; a performative is both a model of and a model for (or more precisely a model for what it is a model of). Geertz tied his "models of and for" to "long-lasting moods and motivations" (1973, 90), connecting performativity with other anthropological notions such as ethos, worldview, and ideology; neoliberalism as an ideology of free markets isn't far behind.

The efficient market is a "big-data" volatility processor, which in the case of the options market accesses and prices volatility through the elimination of directional risk. Harvey (1990) has suggested that postmodernism arose out of the volatility introduced by time-space compression in the early 1970s, which later morphs into the neoliberalism regnant during the Great Moderation. The pricing of volatility (perhaps like the pricing of abstract labor time or value in Marxist accounts) creates a new principle of financial innovation that makes derivative finance the cutting edge of derivative capitalism. Neoliberal policies of deregulation catalyze the rapid expansion of derivatives, which fuels the Great Moderation. At the core of this neoliberal ideology of free and unregulated markets is the notion of an efficient market whose origins are in Von Neumann and Morgenstern's axiomization of expected utility in their *Theory of Games and Economic Behavior* (1944), which became the foundation for portfolio theory and the efficient market hypothesis, which in turn became a presupposition of options pricing models. Indeed, the origins of what has become known as neoliberal ideology can also be traced back to the early 1970s and spring from similar game-theoretic considerations: John Rawls's *A Theory* *of Justice* (1971) and Robert Nozick's *Anarchy, State, and Utopia* (1974) form the bookends of the neoliberal debate between distributive justice and libertarianism that continues to this day.

The question is whether the historical timing is just coincidental or indicates a deeper connection between the rise of volatility and the development of neoliberalism. These themes reinforce each other in the Great Moderation that leads up to the financial crisis of 2007–8. During this period, there is rising inequality caused at least part in the rise of the financial sector, which almost doubles its share of corporate profits; there is a transvaluation among the young elite, as Ivy League schools report 40–50 percent of graduating seniors wanting to go into finance or investment banking. One way to interpret neoliberalism and derivative finance is that the former uses the latter to create wealth out of the volatility that is destabilizing the very people disenfranchised by the volatility introduced by the collapse of Fordist production in the late sixties and early seventies. One group gains from volatility while the other loses. The ties between ideology, worldview, Black Scholes, and the Great Moderation were revealed in Alan Greenspan's testimony in the US House of Representatives hearings after the financial crisis in which he admits his almost religious belief in the efficacy of Black-Scholes, "modern risk management," and efficient markets, leaving us to ponder the "politics of volatility" that have now resulted:

> I do have an ideology. My judgment is that free competitive markets are by far the unrivaled way to organize economies. We have tried regulation, none meaningfully worked. . . . In recent decades a vast risk management and pricing system has evolved combining the best insights of mathematicians and finance experts, supported by major advances in computer and communications technology. A Nobel Prize was awarded for the discovery of the pricing model that underpins much of the advance in derivatives markets. This modern risk management paradigm held sway for decades. The whole intellectual edifice, however, collapsed in the summer of last year. . . . I found . . . [a] flaw in the model that I perceived is the critical functioning structure that defines how the world works, so to speak.[2]

2. See https://youtu.be/CQ6WgiHq3CE.

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