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COSMIC-RAY TRANSPORT AND ACCELERATION

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ABSTRACT

We review the transport and acceleration of cosmic rays concentrating on the origin of galactic cosmic rays. Quasi-linear theory for the acceleration rates and propagation parameters of charged test particles combined with the plasma wave viewpoint of modeling weak cosmic electromagnetic turbulence provides a qualitatively and quantitatively correct description of key observations. Incorporating finite frequency effects, dispersion, and damping of the plasma waves are essential in overcoming classical discrepancies with observations as the $\kappa_{\rm fit}$-$\kappa_{\rm ql}$ discrepancy of solar particle events. We show that the diffusion-convection transport equation in its general form contains spatial convection and diffusion terms as well as momentum convection and diffusion terms. In particular, the latter momentum diffusion term plays a decisive role in the acceleration of cosmic rays at super-Alfvénic supernova shock fronts, and in the acceleration of ultra-high-energy cosmic rays by distributed acceleration in our own galaxy.

Subject headings: acceleration of particles — convection — cosmic rays — diffusion — shock waves

1. INTRODUCTION

In this review I am going to concentrate on the transport and acceleration of Galactic cosmic rays. Using the measured energy spectra of Galactic cosmic rays above 1 GeV nucleon^{-1} in the neighborhood of the solar system one estimates the energy densities (e.g., Wolfendale 1983) $w_N \simeq 0.5$ eV cm^{-3} in cosmic-ray nucleons and $w_e \simeq 0.006$ eV cm^{-3} in cosmic-ray electrons and positrons. Note that due to spectral index values larger than 2 these energy densities are determined by particles with energies of 1–30 GeV nucleon^{-1} in case of nucleons and by electrons with energies of 400 MeV to 20 GeV so that the effects of solar modulation cannot drastically change these estimates. Below 400 MeV the electron spectrum will flatten due to the onset of Coulomb and ionization losses (e.g., Pohl 1993). The very high energy ($E_{\rm kin} > 10^{15}$ eV) cosmic rays do not influence the estimates of the energy densities. The analysis of the diffuse radio background and Galactic γ-ray emission has shown that these local values are fairly representative for the whole Galaxy of volume V = 350 kpc^3 = 10^{67} cm^3. The measured abundance of radioactive cosmic-ray clocks indicates a mean residence time of ~ 1 GeV nucleon^{-1} cosmic-ray particles of about $\tau \simeq 10^7$ yr. We then obtain for the cosmic-ray power in our Galaxy $Q_c = (w_N + w_e)V/\tau \simeq 3 \times 10^{40}$ ergs s^{-1} which is distributed nearly uniformly over the Galaxy. The time history of cosmogenic nuclei established that within a factor of 2 the flux of extrasolar cosmic rays has been constant over the past 10^9 yr, indicating that the cosmic-ray power has had this value steadily at least over the last 10^9 yr. Since the mean residence time of cosmic rays in the Galaxy is two orders of magnitude shorter, possible cosmic-ray sources have to inject steadily at least a source power equal to Q_c to account for this situation.

Besides the many observed elemental and isotopic subtleties a successful model for the origin of Galactic cosmic rays therefore has to explain the following key ingredients:

1. An over 10^9 yr constant cosmic-ray power of $\sim 10^{40}$ ergs s^{-1};
2. A nearly uniform and isotropic distribution of cosmic-ray nucleons and electrons with energies below 10^{15} eV over the Galaxy;
3. Elemental and isotopic composition similar to solar flare particles;
4. Electron/nucleon ratio in relativistic cosmic rays at the same energy of about 0.01;
5. The formation of power-law energy spectra for all species of cosmic rays over large energy ranges accounting for the systematic differences in the spectral index values of primary and secondary cosmic-ray nucleons and cosmic-ray electrons.

The general problem of the origin of cosmic rays can be divided into two parts. The first part concerns the actual origin or injection of the cosmic rays into the Galaxy by sources which keep up the power Q_c over a long time, while the second part concerns the subsequent behavior of the cosmic rays, their motion, transport, and confinement in the Galaxy. We consider both in turn.

1.1. *Global Cosmic-Ray Source Energetics*

Due to the large steady cosmic-ray power, only four types of Galactic objects can serve as potential cosmic-ray source candidates, since their estimated total power output into the Galaxy Q is larger than the minimum requirement Q_c:

1. *Supernova explosions:* numerical simulations of the collapse of a star of 10 $M_\odot$ ($M_\odot$: solar mass) indicate that the resulting supernova explosion sets free an energy of $\sim 10^{51}$ ergs. And with a supernova rate of 1 per 30 years in our Galaxy this yields a power input into the Galaxy of $Q \simeq 10^{42}$ ergs s^{-1}.
2. *Neutron stars:* estimating the rate of rotational energy loss from known radio pulsar's periods P and their derivative $\dot{P}$ as $\dot{E}_{\rm rot} = -4\pi I\dot{P}/P^3$, where $I \simeq 10^{45}$ g cm^2 denotes the neutron

star's moment of inertia, summing over all observed pulsars, and applying corrections for the fraction of neutron stars not seen due to their beamed radiation, one estimates a total power input into the Galaxy of $Q \simeq 10^{41}$ ergs s^{-1}. This neutron star origin is supported by the detection of three pulsars as MeV–GeV γ-ray point sources.

3. *Stellar winds from young hot O/B stars:* hot stars of spectral type O, B, and Wolf-Rayet stars drive powerful winds by their strong radiation pressure with mass-loss rates ($\dot{M} \simeq 10^{-7} - 10^{-5}\ M_\odot\ yr^{-1}$) that exceed the Sun's mass-loss rate by some nine orders of magnitude. Although such stars only live $\sim 10^7$ years on the main sequence, the large mass-loss rate means that a substantial fraction ($\sim$50%) can be lost during its main-sequence lifetime, implying an important contribution to the energy balance of the interstellar medium. Summing over their spatial distribution function a steady power input of $Q \simeq 10^{41}$ ergs s^{-1} into the interstellar medium has been estimated (Casse & Paul 1982).

4. *Flare stars of spectral class K-M:* we know that our Sun produces cosmic rays with energies up to several tens of GeV during large flares. It has been observed that many late-type stars of type M and K also flare in the optical and radio frequency band. Lovell (1974) has estimated the total contribution of Galactic flare stars to the interstellar energy flux in cosmic rays. He estimates a power input of $Q \simeq 3 \times 10^{40}$ ergs s^{-1} from these objects which would mainly go into cosmic rays of energies below 300 MeV nucleon^{-1}. In this model cosmic rays with higher energy result from further acceleration in the interstellar medium. This hypothesis is certainly supported by the similar isotopic and elemental composition of solar flare and Galactic cosmic rays.

Note that in the case of the first three source candidates the power input estimates refer to the total power input into the Galaxy. They require a very efficient acceleration process for cosmic rays, that ultimately converts a large percentage of the total power (1%–10%) into cosmic rays. Also note that probably all four source candidates contribute to the cosmic-ray power and that these object classes are not independent from another: supernova explosions result from the collapse of young massive O/B stars and lead to the formation of neutron stars at the center of the explosion. A large star formation rate and the presence of many young hot stars will also lead to enhanced supernova explosions and an enhanced birth rate of neutron stars.

As a result we note that the global energetics requirements of cosmic rays can be met if an efficient acceleration process for cosmic rays can be found.

1.2. *Cosmic-Ray Scattering, Confinement, and Isotropy*

It is generally recognized that due to their small Larmor radii, for cosmic-ray nucleons (of momentum p) $R_L = (pc/\mathrm{eV})/300(B/\mathrm{Gauss})$ cm, as compared to Galactic dimensions, the majority of cosmic rays with energies below $\sim 10^{15}$ eV propagate along the Galactic magnetic field. Because of the observed isotropy and age of cosmic rays, it seems clear that the cosmic rays cannot propagate freely along the lines of force but must be continually scattered. If they would propagate freely with the speed of light, they would leave the Galaxy within 10^4 to 4×10^5 yr, as the dimensions of our Galaxy suggest. But from the measured abundance of the cosmogenic cosmic-ray clocks we know that their average lifetime in the Galaxy is $\sim 10^7$ yr. Moreover, if there would be no scatterings, we would expect a strong anisotropy toward the direction of the Galactic center due to the peculiar location of the solar system, since there should be more sources of the type discussed in § 1.1 toward the inner Galaxy. Yet we do not see this anisotropy for cosmic rays with energies less than 10^{15} eV which apparently is washed out due to multiple scatterings of the cosmic rays on their path from the sources to us.

The scattering cannot be by particles (Coulomb scattering) since the energies of cosmic rays are much higher than nuclear binding energies and such collisions would destroy all nuclear species heavier than protons in the instellar medium which is not the case (Kulsrud & Pearce 1969). Moreover, the mean free path for Coulomb collision of relativistic nucleons in the dilute interstellar medium of order $\sim 10^{23}\gamma/n_{\rm H}(\mathrm{cm}^{-3})$ cm is far too long. Thus the most likely scattering mechanism is off *plasma waves,* that is, fluctuating electromagnetic fields in the interstellar medium. How these plasma waves are produced and how they affect the dynamics of cosmic-ray particles will be the topic of this paper.

2. THEORY OF COSMIC-RAY TRANSPORT AND ACCELERATION

The dynamics of cosmic-ray particles in cosmic plasmas is determined by their interaction with ambient electromagnetic, photon, and matter fields. Among these by far quickest is the particle-wave interaction with electromagnetic fields which very often can be separated into a leading field structure F_0 and superposed fluctuating fields δF. Theoretical descriptions of the transport and acceleration of cosmic rays in interplanetary and interstellar plasmas are usually based on two transport equations which both are derived from the collisionless Boltzmann-Vlasov equation into which the electromagnetic fields of the interplanetary and interstellar medium enter by the Lorentz force term. The first of the equations, the *Fokker-Planck equation,* results from applying the quasi-linear approximation (Vedenov, Velikhov, & Sagdeev 1962; Kennel & Engelmann 1966; Lerche 1968; Hasselmann & Wibberenz 1968). The quasi-linear approach to wave-particle interaction is a first-order approach in the ratio $q_L \equiv (\delta F/F_0)^2$ and requires smallness of this ratio with respect to unity. In most cosmic plasmas this is well satisfied as has been either established by direct in situ electromagnetic turbulence measurements in interplanetary plasmas, or by saturation effects in the growth of fluctuating fields. Nonlinear wave-wave interaction rates and/or nonlinear Landau damping set in only at appreciable levels of $(\delta F)^2$ and thus limit the value of $q_L \leq 1$. In any investigation it is decisive to single out the leading force field F_0.

Of particular interest to the astrophysical community are magnetized plasmas of high conductivity so that any large-scale steady electric fields are absent. One considers the behavior of energetic charged particles in a uniform magnetic field, $\boldsymbol{B}_0 = B_0\boldsymbol{e}_z$, with superposed small-amplitude plasma turbulence ($\delta\boldsymbol{E}$, $\delta\boldsymbol{B}$) in the rest frame of the plasma turbulence supporting fluid (e.g., the solar wind). The plasma turbulence is represented by its Fourier transform in space coordinates, so that the total electromagnetic field is

$$B_T = B_0 + \delta B = B_0 e_z + \int d^3k B(k, t) \exp(ik \cdot x) ,$$
$$E_T = \delta E = \int d^3k E(k, t) \exp(ik \cdot x) . \tag{1}$$

In the quasi-linear approach we study the effect of the plasma turbulence on the particles by calculating first-order corrections to the particle's orbit in the uniform magnetic field B_0, and ensemble-averaging over the statistical properties of the plasma turbulence (Jokipii 1966). In the *mixed comoving coordinate system,* in which the *space coordinates* are measured in the laboratory system and the *particle's momentum coordinates* are measured in the rest frame of the background plasma, that supports the plasma turbulence and in which the turbulence is homogeneous in space and time, the gyrophase-averaged phase space density $f(z, p, \mu, t)$ evolves according to the Fokker-Planck equation (Kirk, Schlickeiser, & Schneider 1988)

$$\frac{\partial}{\partial \mu}\left(D_{\mu\mu}\frac{\partial f}{\partial \mu} + D_{\mu p}\frac{\partial f}{\partial p}\right) + \frac{1}{p^2}\frac{\partial}{\partial p}p^2\left(D_{\mu p}\frac{\partial f}{\partial \mu} + D_{pp}\frac{\partial f}{\partial p}\right)$$
$$= -S + \Gamma\left(1 + \frac{uv\mu}{c^2}\right)\left(\frac{\partial f}{\partial t} - \frac{1}{c}\frac{\partial u}{\partial t}\Gamma^2 E\frac{\partial f}{\partial p_z}\right)$$
$$+ \Gamma(u + v\mu)\left(\frac{\partial f}{\partial z} - \frac{1}{c}\frac{\partial u}{\partial z}\Gamma^2 E\frac{\partial f}{\partial p_z}\right), \tag{2}$$

where $u(z, t)$ denotes the bulk speed of the background plasma (e.g., the solar wind speed in the case of the interplanetary medium). Here $\mu = p_z/p$ is the cosine of the particle's pitch angle, S denotes the source term of particles, $\Gamma = (1 - u^2/c^2)^{-1/2}$, and $E = (p^2c^2 + m^2c^4)^{1/2}$ is the total particle energy. Equation (2) is derived assuming that the flow velocity $u = u(z, t)e_z$ is parallel to the background magnetic field. The three Fokker-Planck coefficients

$$D_{\mu\mu} \equiv \lim_{t\to\infty}\frac{1}{2t}\langle \Delta\mu(t)\Delta\mu^*(t + \tau)\rangle$$
$$= \mathscr{R}\int_0^\infty d\tau\langle \dot{\mu}(t)\dot{\mu}^*(t + \tau)\rangle , \tag{3a}$$

$$D_{\mu p} \equiv \lim_{t\to\infty}\frac{1}{2t}\langle \Delta\mu(t)\Delta p^*(t + \tau)\rangle$$
$$= \mathscr{R}\int_0^\infty d\tau\langle \dot{\mu}(t)\dot{p}^*(t + \tau)\rangle , \tag{3b}$$

$$D_{pp} \equiv \lim_{t\to\infty}\frac{1}{2t}\langle \Delta p(t)\Delta p^*(t + \tau)\rangle$$
$$= \mathscr{R}\int_0^\infty d\tau\langle \dot{p}(t)\dot{p}^*(t + \tau)\rangle \tag{3c}$$

have to be calculated (Hall & Sturrock 1967; Krommes 1984; Achatz, Steinacker, & Schlickeiser 1991) from the ensemble-averaged first-order particle orbit.

In the presence of low-frequency magnetohydrodynamic turbulence such as Alfvén waves, whose magnetic field component is much larger than their electric field component ($|\delta B| = (c/V_A)|\delta E|$, Alfvén velocity $V_A \ll c$), the particle's distribution function $f(z, p, \mu, t)$ adjust very rapidly to quasi-equilibrium through pitch-angle diffusion, which is close to the isotropic distribution. In this case a second cosmic-ray transport equation can be derived from the Fokker-Planck equation (2) by a well-known approximation scheme (Jokipii 1966; Hasselmann & Wibberenz 1968; Schlickeiser 1989a) which is commonly referred to as the *diffusion-convection equation* for the pitch-angle-averaged phase space density $F(z, p, t)$ and which for nonrelativistic bulk speed $u \ll c$ reads

$$\frac{\partial F}{\partial t} - S_0 = \frac{\partial}{\partial z}\left(\kappa\frac{\partial F}{\partial z}\right) - \left[u + \frac{1}{4p^2}\frac{\partial}{\partial p}(p^2 v a_1)\right]\frac{\partial F}{\partial z}$$
$$+ \left(\frac{p}{3}\frac{\partial u}{\partial z} + \frac{v}{4}\frac{\partial a_1}{\partial z}\right)\frac{\partial F}{\partial p} + \frac{1}{p^2}\frac{\partial}{\partial p}\left(p^2 a_2\frac{\partial F}{\partial p}\right), \tag{4}$$

where the spatial diffusion coefficient κ, the rate of adiabatic deceleration a_1, and the momentum diffusion coefficient a_2 are determined by pitch-angle averages of the three Fokker-Planck coefficients (3) as

$$\kappa = \frac{v^2}{8}\int_{-1}^{1} d\mu\,\frac{(1 - \mu^2)^2}{D_{\mu\mu}} ,$$
$$a_1 = \int_{-1}^{1} d\mu(1 - \mu^2)\frac{D_{\mu p}}{D_{\mu\mu}} , \tag{5}$$
$$a_2 = \frac{1}{2}\int_{-1}^{1} d\mu\left(D_{pp} - \frac{D_{\mu p}^2}{D_{\mu\mu}}\right) .$$

S_0 is $(\frac{1}{2})\int_{-1}^{1} d\mu S$ and v is the cosmic-ray particle velocity. Since equations (2) and (4) are derived in the mixed comoving coordinate system, the rate of adiabatic deceleration a_1 does not include $\partial u/\partial z$. In its most general form the diffusion-convection equation contains spatial diffusion and convection terms ("transport of cosmic rays") as well as momentum diffusion and convection terms ("acceleration of cosmic rays"). Since the pioneering papers of Fermi (1949, 1954) it has become customary to refer to the latter two as Fermi acceleration of second and first order, respectively. Whether in any specific situation all four terms arise depends solely on the nature and the statistical properties of the plasma turbulence and the background medium.

For example, if one because of their low-frequency neglects all electric field effects, and regards the turbulence as purely magnetic ("magnetostatic approximation") in the turbulence supporting medium, $D_{\mu\mu}$ is the only nonvanishing Fokker-Planck coefficient, and the diffusion-convection equation contains no second-order Fermi acceleration term ($a_2 = 0$). First-order Fermi acceleration may arise if the bulk speed $U(z)$ of the turbulence-carrying background medium undergoes compression, that is, $\partial U/\partial z < 0$. Despite its frequent use, its applicability is rather limited, and it has led to fundamental inconsistencies in the past.

As emphasized, fundamental to all deductions of cosmic-ray transport equations and their parameters is the knowledge of the particle's Fokker-Planck coefficients (3). Inserting the equations of motion, making the usual assumption that the turbulence's Fourier components at different wavevectors are uncorrelated, and averaging over the initial phase ϕ_0 yields the three Fokker-Planck coefficients (3), and with these the three transport parameters (5) of the diffusion-convection equation after the time dependence of power spectra of the plasma turbulence have been specified.

3. MODELING WEAK COSMIC ELECTROMAGNETIC TURBULENCE

As a more reliable (than the magnetostatic approximation) representation of weak turbulence the Fourier transforms of the magnetic and electric field fluctuations are represented as superposition of individual plasma modes ("wave viewpoint") of frequency $\omega = \omega_j(\boldsymbol{k}) = \omega_{r,j}(\boldsymbol{k}) - \iota\Gamma_j(\boldsymbol{k}), j = 1, \ldots, N$, which can have both real and imaginary parts, so that

$$[\boldsymbol{B}(\boldsymbol{k}, t), \boldsymbol{E}(\boldsymbol{k}, t)] = \sum_{j=1}^{N} [\boldsymbol{B}^j(\boldsymbol{k}), \boldsymbol{E}^j(\boldsymbol{k})] \exp(-\iota\omega_j t) . \quad (6)$$

Maxwell's induction law relates $\boldsymbol{B}^j(\boldsymbol{k}) = (c/\omega_j)\boldsymbol{k} \times \boldsymbol{E}^j(\boldsymbol{k})$. As a consequence, the fluctuation's correlation tensor becomes

$$P_{\alpha\beta}(\boldsymbol{k}, \tau) = \sum_{j=1}^{N} P^j_{\alpha\beta}(\boldsymbol{k}) \exp[+\iota\omega^j_{r,\beta}(\boldsymbol{k})\tau - \Gamma^j_\beta(\boldsymbol{k})\tau] , \quad (7a)$$

where

$$P^j_{\alpha\beta}(\boldsymbol{k}) = \langle B^j_\alpha(\boldsymbol{k}) B^{j*}_\beta(\boldsymbol{k}_s)\rangle \delta(\boldsymbol{k} - \boldsymbol{k}_s) . \quad (7b)$$

With this approach (eq. [7]) Jaekel & Schlickeiser (1992) and Schlickeiser & Achatz (1993a, b) have calculated the quasi-linear Fokker-Planck coefficients (3) for general electromagnetic plasma modes in the weak turbulence approximation. Their results can be employed once the relevant plasma modes in a given physical system are identified either from in situ measurements (as in the interplanetary medium) or from calculations of the growth rates of individual modes.

To identify the relevant low-frequency plasma modes in a given physical system, that is not accessible by direct in situ measurements of the electromagnetic fluctuations, the generation and damping of plasma waves has to be considered. A useful start is provided by the plasma wave dispersion relation of a cold magnetized electron-proton plasma. For parallel (to $\boldsymbol{B}_0$) propagating waves at frequencies much smaller than the electron gyrofrequency $|\omega_r| \ll |\Omega_{e0}|$ the solution of the dispersion relation is (Achatz, Steinacker, & Schlickeiser 1991)

$$\omega \equiv \omega^{f,b}_{r,L} = -\frac{V_A^2 k_\parallel^2}{2\Omega_{p0}} + \sigma_{f,b} V_A k_\parallel \sqrt{\frac{V_A^2 k_\parallel^2}{4\Omega_{p0}^2} + 1} , \quad (8)$$

where $\sigma_{f,b} = \pm 1$ refers to forward (i.e., positive phase speed $\partial\omega/\partial k_\parallel > 0$) and backward (i.e., negative phase speed) waves. Here we use the convention that solutions with positive frequency $\omega > 0$ are physically left-handed polarized, while solutions with negative frequency $\omega < 0$ are physically right-handed polarized, implying the symmetries

$$\omega^j_{r,R}(k_\parallel) = -\omega^j_{r,L}(-k_\parallel); \quad \Gamma^j_R(k_\parallel) = \Gamma^j_L(-k_\parallel) . \quad (9)$$

The solutions (8) comprise several well-known plasma modes with different asymptotic dispersion relation:

1. At small wavenumbers $|k_\parallel| \ll 2k_c$ where $k_c \equiv \Omega_{p0}/V_A$ and frequencies $|\omega| \ll \Omega_{p0}$ equation (8) reduces to $\omega^2 \simeq V_A^2 k_\parallel^2$ which describes nondispersive ($V_{ph} = \omega/k_\parallel = \text{const}$) forward ($V_{ph} > 0$) and backward ($V_{ph} < 0$) moving *Alfvén waves* which are either right-handed ($\omega < 0$) or left-handed ($\omega > 0$) circularly polarized.
2. At large wavenumbers $|k_\parallel| \gg 2k_c$ the left-handed Alfvén branch develops into the left-handed *ion cyclotron wave* branch, which can propagate both forward and backward and which at all values of $k_\parallel$ has nearly the same frequency $\omega \simeq \Omega_{p0}$.
3. At large wavenumbers $|k_\parallel| \gg 2k_c$ and frequencies between $\Omega_{e0} < \omega < -\Omega_{p0}$ the right-handed Alfvén branch develops into the right-handed *Whistler wave* branch, which is dispersive ($V_{ph} \neq \text{const}$), because of the quadratic wave number dependence of its dispersion relation $\omega \simeq -\Omega_{p0}(k_\parallel/k_c)^2$, and which can propagate both forward and backward.
4. At very large wavenumbers $|k_\parallel| \gg 43k_c$ the right-handed Whistler wave branch develops into the right-handed *electron cyclotron wave* branch $\omega \simeq \Omega_{e,0}$, which again can propagate both forward and backward.

The modifications by including finite temperature effects due to a warm Maxwellian background plasma distribution on the cold-plasma results are mainly fourfold:

1. Oblique propagating low-frequency magnetohydrodynamic waves are quickly damped depending on the plasma beta of the background plasma (Barnes 1969; Foote & Kulsrud 1979; Achterberg 1981). Because of this modification the theory of resonant interactions of cosmic-ray particles with plasma waves has concentrated almost exclusively on parallel propagating waves.
2. The right-handed polarized Alfvén-Whistler branch is almost unaffected by the background plasma near $-\Omega_{p0}$,

$$\Gamma_R(\sigma_j k_\parallel \geq 0) = \Gamma_L(\sigma_j k_\parallel \leq 0) \simeq 0 , \quad (10)$$

where we have used symmetry (9b).

3. The parallel propagating left-handed polarized Alfvén-ion-cyclotron branch near Ω_{p0} and the right-handed polarized electron cyclotron waves near Ω_{e0} are strongly cyclotron damped by the thermal protons and electrons, respectively (Davila & Scott 1984; Achatz et al. 1993). In case of damping the wave frequencies ω^j have real (ω^j_r) and imaginary ($\Gamma^j > 0$) parts. For the left-handed waves Achatz et al. (1993) have found that

$$\Gamma_L(\sigma_j k_\parallel > 0) \simeq |k_\parallel| v_{th,p} \exp\left(-\frac{k_c}{\beta^{1/6}|k_\parallel|}\right) , \quad (11)$$

where $v_{th,p}$ refers to the thermal background proton velocity and $\beta \equiv (v_{th,p}/v_A)^2$ denotes the plasma beta. The value of the threshold, where damping becomes strong, of $k_\parallel \simeq \beta^{-1/6} k_c$ strictly holds only for $\beta \leq 1$.

4. Also the real part of the frequency of the left-handed polarized waves is slightly modified at large $k_\parallel$ (Achatz et al. 1993). This is important for resonant calculations of the mean free path. Since the convergence $\omega \to \Omega_{p0}$ for $|k_\parallel| \to \infty$ remains, this modification is of minor importance when resonance broadening is taken into account (see below).

4. COSMIC-RAY TRANSPORT PARAMETERS

At large wavenumbers dispersive and damping effects come into play and the simple linear Alfvénic frequency-wavenumber relation $\omega \propto k_\parallel$ no longer holds. Long-wavenumber plasma waves are particular important for the wave-particle interaction of nonrelativistic ions and mildly relativistic electrons and positrons. Damping of waves enters twofold into the calculation of the Fokker-Planck coefficients and the transport parameters of the convection-diffusion equation: first, due to the strong damping of the ion-cyclotron waves in the warm interplanetary plasma the intensity of these waves is very low; second, wave damping also modifies the resonance function in the Fokker-Planck coefficients from sharp δ-function resonance function to broadened Breit-Wigner resonance functions (Krommes 1984, ch. 5.5.3). Achatz et al. (1993) have shown that if one would consider only the first modification he would find that the quasi-linear mean free path of cosmic-ray particles is drastically larger than the measured one, in obvious contradiction to the observational evidence. For a consistent picture he therefore has to account correctly also for the second modification of wave damping and include the resonance broadening from wave damping, with the aim to arrive at a consistent explanation of the measurements. This has been considered recently by Schlickeiser & Achatz (1993a, b). For low-frequency ($\omega_r \ll |\Omega_{e0}|$) and parallel-moving plasma waves they derive for the Fokker-Planck coefficient (3a)

$$D_{\mu\mu} = \frac{\Omega^2(1-\mu^2)}{B_0^2} \sum_{j=f,b} \int_{-\infty}^{\infty} dk_\parallel R_j(k_\parallel) \times \left(1 + \sigma_j \Sigma^j(|k_\parallel|) \frac{k_\parallel}{|k_\parallel|}\right) S^j(|k_\parallel|) , \quad (12a)$$

where the resonance function R_j is given in Breit-Wigner form

$$R_j(k_\parallel) = \frac{\Gamma_L^j(k_\parallel)}{\Gamma_L^j(k_\parallel)^2 + \xi_j^2(k_\parallel)} \quad \text{with} \quad \xi_j(k_\parallel) \equiv v\mu k_\parallel + \Omega - \omega_{r,L}^j . \quad (12b)$$

S^j and $\Sigma^j(|k_\parallel|)$ denote the total wave intensity and the magnetic helicity of the forward- and backward-moving plasma mode, respectively. In case of negligible damping, $\Gamma \to 0$, use of $\lim_{\Gamma\to 0}[\Gamma/(\Gamma^2+\xi^2)] = \pi\delta(\xi)$ readily reduces equation (12) to the resonant-diffusion limit used by Achatz et al. (1991), but as we have emphasized this limit strictly holds only in case of negligible wave damping. We mention here also the expressions for the other two Fokker-Planck coefficients (3b) and (3c) under the same assumption of the slab geometry that has led to equations (12). Low-frequency waves have a much larger magnetic field than electric field component, $|\delta B| \simeq (c/|V_{\rm ph}|)|\delta E|$. We therefore may expand $D_{\mu p}$ (eq. [3b]) to first order in the small quantity $V_{\rm ph}/c$, whereas D_{pp} is of second order in $V_{\rm ph}/c$. Performing the same manipulations as above with $D_{\mu\mu}$ Schlickeiser & Achatz (1993a) have obtained

$$D_{\mu p} = \frac{\Omega^2(1-\mu^2)p}{vB_0^2} \sum_{j=f,b} \int_{-\infty}^{\infty} dk_\parallel S^j(|k_\parallel|)k_\parallel^{-1} \times \left(1 + \sigma_j \Sigma^j(|k_\parallel|) \frac{k_\parallel}{|k_\parallel|}\right) R_j(k_\parallel)(\xi_L - \omega_{r,L,j}) , \quad (13)$$

and

$$D_{pp} = \frac{\Omega^2 p^2(1-\mu^2)}{v^2 B_0^2} \sum_{j=f,b} \int_{-\infty}^{\infty} dk_\parallel S^j(|k_\parallel|) \times \frac{\omega_{r,L,j}^2 + \Gamma^2}{k_\parallel^2} \left(1 + \sigma_j \Sigma^j(|k_\parallel|) \frac{k_\parallel}{|k_\parallel|}\right) R_j(k_\parallel) . \quad (14)$$

The most important modification of earlier studies is the implied resonance broadening of the wave-particle interaction. The hitherto relevant sharp δ-function resonances have to be replaced by Breit-Wigner type resonance functions, into which the plasma beta enters. All Fokker-Planck coefficients (12)–(14) can readily be calculated if the plasma turbulence intensity, helicity, and other plasma parameters are specified.

5. TRANSPORT OF COSMIC RAYS

For parallel propagating waves (referred to as "slab model") the calculation of quasilinear Fokker-Planck coefficients and transport parameters of the diffusion-convection equation is well developed. At large particle energies the influence of particle interactions with damped ion-cyclotron and electron-cyclotron waves is negligibly small, and the Fokker-Planck coefficients can well be calculated in the resonant diffusion limit. Schlickeiser (1992) has reviewed the status of cosmic-ray transport parameter calculations in this limit. The most noteworthy results are that (1) the famous discrepancy between observed ($\kappa_{\rm fit}$) and quasi-linear ($\kappa_{\rm ql}$) spatial diffusion coefficients for solar flare particles can be resolved once the magnetostatic approximation is discarded and the influence of cross- (i.e., the fraction of forward- and backward-moving waves) and magnetic helicities are properly taken into account, (2) dissipation range spectra yield finite values of $\kappa_{\rm ql}$, and (3) momentum diffusion is unavoidable ($a_2 \neq 0$) for nondegenerate ($|H_c| \neq 1$) values of the cross-helicity of nondispersive waves.

At small, especially nonrelativistic, particle energies the situation is more complex. On the one side, damping of the cyclotron waves reduces the intensity of waves necessary to scatter nonrelativistic particles through pitch angles of 90°. Smith, Bieber, & Matthaeus (1990) and Achatz et al. (1993) have pointed out that this wave dissipation would lead to unreasonably large spatial diffusion coefficients of these particles if calculated in the resonant diffusion limit. On the other hand, the analysis of Schlickeiser & Achatz (1993a) has shown that as a consequence of wave damping resonance broadening of the wave-particle interaction sets in, so that the pitch-angle Fokker-Planck coefficient has to be calculated with the full Breit-Wigner resonance function (see eq. [12]). Figure 1 shows the resulting Fokker-Planck coefficient $D_{\mu\mu}$ of a nonrelativistic ($E_{\rm kin}$ = 100 MeV) and a relativistic ($E_{\rm kin}$ = 3 GeV)

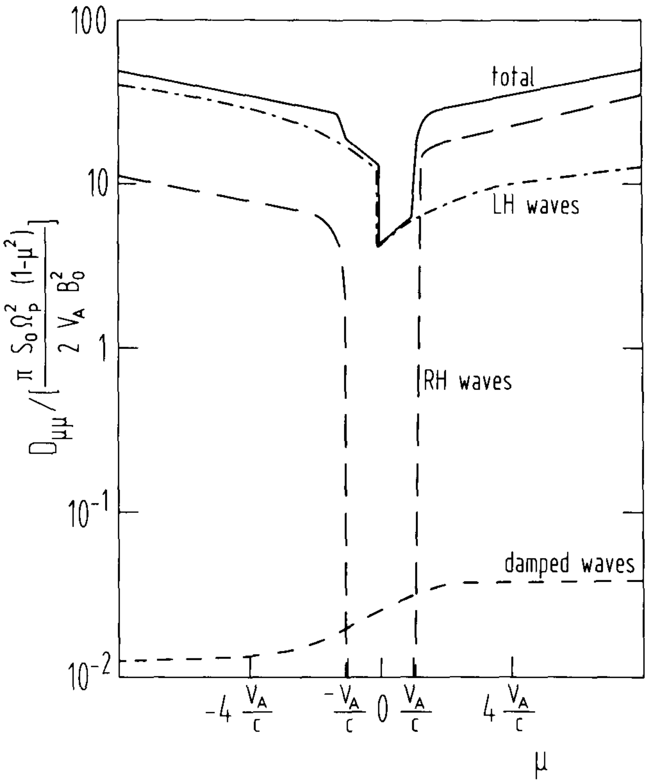

FIG. 1*a*

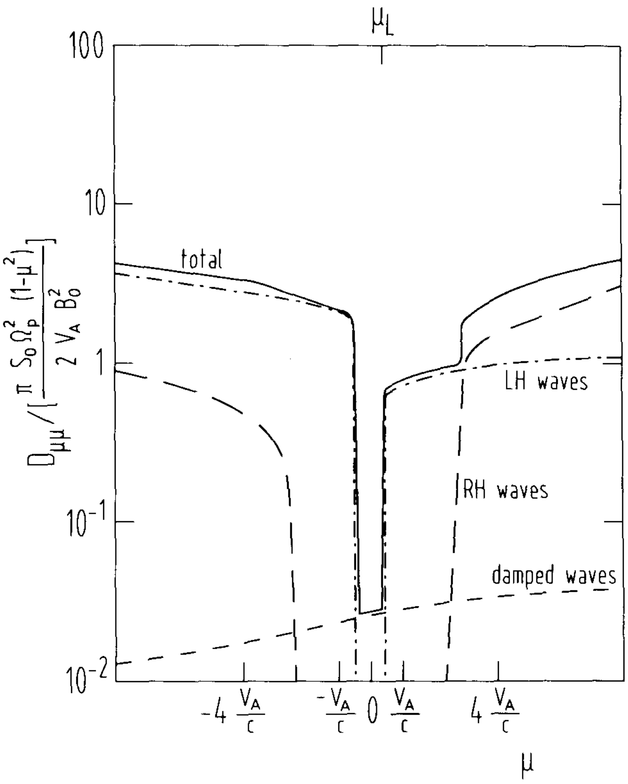

FIG. 1*b*

FIG. 1.—Pitch-angle Fokker-Planck coefficients of a 3 GeV (*a*) and a 100 MeV (*b*) cosmic-ray proton calculated for a nonwhistler-dissipative slab turbulence power spectrum with a spectral index $q = 1.5$ and cross helicity $H_c = 0.5$ in a background medium with plasma beta $\beta = 5$. Besides the total Fokker-Planck coefficient the individual contributions from the undamped left-handed polarized (LH), undamped right-handed polarized (RH), and the damped left-handed waves are shown.

cosmic-ray proton calculated by Schlickeiser & Achatz (1993b) for a nonwhistler-dissipative turbulence spectrum where the left-handed polarized waves are exponentially reduced above a critical wavenumber $k_\parallel \geq \Omega_{p0}/V_A$. Besides the total Fokker-Planck coefficient we also show the individual contributions to scattering from the undamped left-handed polarized (LH), undamped right-handed polarized (RH), and the damped left-handed waves. At all pitch angles the damped left-handed waves provide small but nonnegligible scattering of particles. Whereas at relativistic energies (Fig. 1*a*) at all pitch angles the resonant interaction with undamped waves controls the scattering, at nonrelativistic energies (Fig. 1*b*) there exists a small pitch-angle interval $|\mu| \leq V_A/c$ where the undamped RH and LH waves do not contribute to the pitch-angle scattering. Here the value of Fokker-Planck coefficient is due to the small contribution from the damped left-handed polarized waves. Since according to equation (5a) the spatial diffusion coefficient κ and the associated mean free path of particles $\lambda \equiv 3\kappa/v$ are sensitively determined by the minimum value of $D_{\mu\mu}$ we find a quite different behavior of the mean free path at small and large energies. The resulting mean free path shown in Figure 2 obviously consists of two terms: the first describes the resonant scattering from undamped waves and exhibits the well-known power-law behavior in particle rigidity if the turbulence spectrum is of Kolmogorov type in the inertial range; the second term describes the contribution from scattering by the damped waves and only occurs below some limiting kinetic energy E_H above which particles at all pitch angles interact with undamped waves. Toward small energies this contribution approaches a constant independent of particle kinetic energy which dominates the resonant scattering contribution and for interplanetary plasma parameters agrees remarkably well with Palmer's (1982) consensus value.

6. DIFFUSIVE SHOCK ACCELERATION OF COSMIC RAYS

Following Fermi's (1949, 1954) two classical papers on first- and second-order Fermi acceleration there was an ongoing debate on which of the two processes is responsible for the energization of charged particles in cosmic plasmas. This issue seemed to be settled in favor of first-order Fermi acceleration when the important role of shock wave acceleration was recognized by Axford, Leer, & Skadron (1977), Krymsky (1977), Bell (1978), Blandford & Ostriker (1978). Cosmic shock waves are a direct consequence of violent dynamical phenomena in the universe, as high-velocity stellar winds and supernova envelopes impinging on interplanetary and interstellar plasmas with small signal speeds. As a consequence fast super-Alfvénic shock waves form. There is common agreement that the Galactic cosmic rays at energies below $\lesssim 10^{14}$ eV nucleon^{-1}

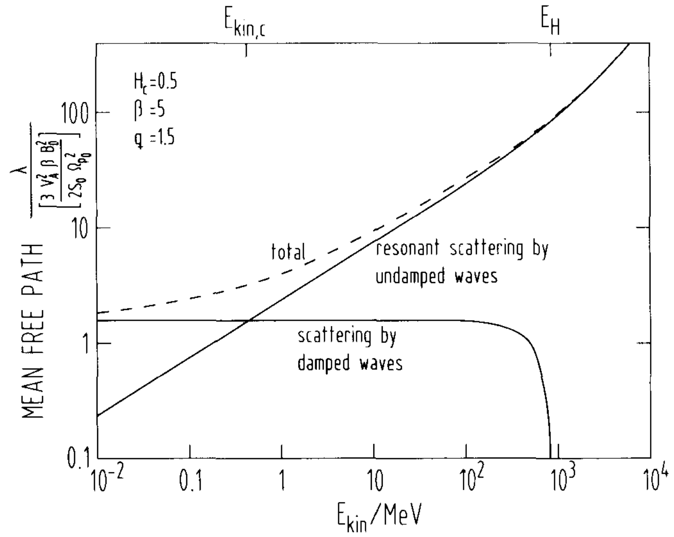

FIG. 2.—Mean free path of cosmic-ray protons as a function of kinetic energy calculated for a nonwhistler-dissipative slab turbulence power spectrum with a spectral index $q = 1.5$ and cross-helicity $H_c = 0.5$ in a background medium with plasma beta $\beta = 5$. At small energies the contribution from the interaction with the damped waves provides a constant mean free path, while at large energies the well-known power-law dependence in kinetic energy per nucleon results from the resonant interaction with the undamped waves.

are accelerated near supernova shock fronts. In the classical treatment of diffusive shock wave acceleration pitch-angle scattering of charged particles with purely magnetic Alfvén plasma waves is used to confine the particles near the shock, and the particles gain energy by multiple crossings of the shock. The diffusion-convection transport equation then to be solved in the test particle limit is of first order in momentum yielding simple power-law particle spectra with the spectral index being solely determined by the shock wave gas compression ratio, and being almost insensitive to the actual microscopic physical conditions in the acceleration region, such as the turbulence level and the power spectrum of the Alfvénic turbulence. The recent work of Campeanu & Schlickeiser (1992) has shown that such a simplified treatment of diffusive shock wave acceleration is not justified.

Following earlier work by McKenzie & Westphal (1969) and Scholer & Belcher (1971), Campeanu & Schlickeiser (1992) have investigated the interaction of backward upstream Alfvén waves amplified by the upstream precursor particle distribution (Bell 1978) with a plane, parallel, super-Alfvénic but nonrelativistic shock. From the Rankine-Hugoniot relations and Alfvén wave properties they rigorously demonstrated that the shock amplifies the incoming upstream backward waves and generates forward waves downstream, implying a nondegenerate ($|H_c| \neq 1$) downstream cross-helicity state that makes second-order Fermi acceleration unavoidable. Comparing the respective timescales they established that the acceleration by downstream momentum diffusion is generally more quicker than acceleration by multiple shock crossings. For a wide range of plasma parameter values this finding leads to a quite different view than hitherto on how energetic particles become accelerated at parallel shocks: the acceleration is primarily due to the cyclotron damping of the electric fields of the Alfvén waves by the particles, whereas the energy gain by multiple shock crossings is a minor effect. In their analysis Campeanu & Schlickeiser (1992) also succeeded in relating the parameters, that determine momentum diffusion of particles, to properties of the shock wave.

Schlickeiser, Campeanu, & Lerche (1993) have recently presented the first exact analytical solutions to the full cosmic-ray transport equation describing the diffusive acceleration of cosmic-ray protons at parallel nonrelativistic shock waves including the unavoidable momentum diffusion term in the downstream region of the shock, and using the correct relation of the momentum diffusion coefficient to the shock wave property. Mainly for mathematical convenience they restricted their analysis to the case of energy-independent spatial diffusion coefficients, although the generalization to power-law energy dependence is well under way. The downstream region was chosen to be of finite extent L to avoid the unphysical situation of downstream waves acting as an infinite source of energy for the particles. The downstream solution is given as an infinite sum of power laws whose spectral indices follow from a transcendental eigenvalue equation. Each individual power-law component is weighted by expansion coefficients that depend on the actual downstream position. Both the eigenvalues and the expansion coefficients are controlled by three parameters:

The scattering center compression ratio, R, which depends on the gas compression ratio and the upstream plasma beta. R regulates the strength of first-order Fermi acceleration of particles at the shock.

The Peclet number, $N = U_2 L/\kappa_2$, which is a direct measure of the extent of the downstream acceleration region.

A function Φ, which measures the strength of momentum diffusion and which is fixed by the transmission and reflection coefficients of the incoming upstream Alfvén waves and the upstream plasma beta.

The following main results were obtained:

1. Since in the most general case of finite N and Φ the downstream solution is a superposition of an infinite sum of power laws, the shape of the downstream distribution function is concavely curved. At large momenta far away from the injection momentum the largest negative eigenvalue dominates the spectrum, in contrast to momenta close to injection where many eigenvalues contribute to the total spectrum.
2. In the formal limit of vanishing momentum diffusion $\Phi \rightarrow 0$ and infinite extent of the downstream region $N \rightarrow \infty$ our general solution approaches the classical result of Axford et al. (1977), Krymsky (1977), Bell (1978), and Blandford & Ostriker (1978), $F(p) \propto p^{-3R/(R-1)}$.
3. For small values of the Peclet number $N < \min(1, \Phi^{-1})$ diffusive shock wave acceleration becomes very inefficient. At large momenta the power-law spectral index is proportional to N^{-1}, in agreement with the finding of Schlickeiser (1989b).
4. For efficient momentum diffusion of particles $\Phi \gg \max(1, N^{-1})$ the acceleration is dominated by the second-order Fermi mechanism. At large momenta the spectral index of the resulting power law of the particle's phase space density

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F approaches the limiting value -3 below, indicating a much more efficient acceleration in this limit than in the limit (2) where the limiting spectral index is -4 for $R \to 4$. In case of the efficient momentum diffusion of particles in the downstream region of the shock, the particle spectra become flatter than in the original treatment of diffusive shock acceleration. This removes the legendary (Lerche 1980; Dröge, Lerche, & Schlickeiser 1987) discrepancy of the original theory with the explanation of flat particle spectra in some shell-type supernova remnants, and with the independence of spectral indices from evolutionary effects in these sources. It has been shown before by Dröge et al. (1987) and Schlickeiser & Fürst (1989) on a less rigorous level how the inclusion of efficient downstream momentum diffusion (i.e., large values of $\Phi \gg \max[1, N^{-1}]$) can account for flat synchrotron radio spectra ($S \propto \nu^{-\alpha}$, $\alpha < 0.5$) of shell-type supernova remnants, and the results of Schlickeiser et al. (1993) indeed support this earlier conjecture. The observed dispersion in spectral index values below $\alpha = 0.5$ is attributed to a distribution of large Φ-values in different remnants, whereas the dispersion in large spectral indices $\alpha > 0.5$ results from a distribution of compression ratios in different remnants.

7. SUMMARY AND CONCLUSIONS

We have reviewed the transport and acceleration of cosmic rays concentrating on the origin of Galactic cosmic rays. We demonstrated that quasi-linear theory for the acceleration rates and propagation parameters of charged test particles combined with the plasma wave viewpoint of modeling weak cosmic electromagnetic turbulence leads to a qualitatively and quantitatively correct description of key observations. Incorporating finite frequency effects, dispersion, and damping of the plasma waves has been essential in overcoming classical discrepancies with observations as the $\kappa_{\rm fit}$-$\kappa_{\rm ql}$ discrepancy of solar particle events. We pointed out that the diffusion-convection transport equation in its general form contains spatial convection and diffusion terms as well as momentum convection and diffusion terms. In particular, the latter momentum diffusion term plays a decisive role in the acceleration of cosmic rays at super-Alfvénic supernova shock fronts.

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