CORRESPONDENCE

To the Editor of the *Mathematical Gazette*

DEAR SIR,

May I draw the attention of your readers to a matter of mutual interest to teachers and training college lecturers?

The increase in the length of the course of training for non-graduate teachers from two to three years is necessitating an increase in staffs of training colleges and this is likely still further to denude secondary schools of their mathematics teachers and so aggravate an already serious situation. The committee of the Mathematics Section of the Association of Teachers in Colleges and Departments of Education has given some thought to the problem and some colleges have been able to solve their staffing difficulties by appointing retired teachers or lecturers on a part-time basis. It is felt that more use could be made of this potential source of supply if there existed some central coordinating body for the collecting and collating of information. The Mathematics Section is willing to act in this capacity for the time being and any teacher or lecturer who is contemplating retirement and who would be willing to undertake part time work in a training college is invited to communicate with the undersigned at Redland College, The Promenade, Bristol, 8.

Yours etc., KATHLEEN SOWDEN

To the Editor of the *Mathematical Gazette*

DEAR SIR,

May I add a note to the interesting article by Mr R. F Wheeler on Force, Power and Gravitational Units printed in the December 1959 issue of the Mathematical Gazette. The title is

Pound Weight and Pound Mass

The confusion between these two terms arises partly from history and partly from the changes that have occurred in thought.

In Kaye and Laby s Tables of Physical and Chemical Constants (Longmans Green and Co. 1911) First edition under the heading of British Imperial Standards on p. 4 we find this statement "According to the Weights and Measures Act, 1878 the pound is the weight in vacuo of a platinum cylinder called the imperial standard pound." Note that the pound is *a weight* (or force) and *not a mass.*

The eleventh edition (1956) of the same tables on p. 3 changes the above to "The Imperial Standard Pound, defined by the Weights and Measures Act, 1878, is a cylinder of platinum of diameter slightly less than its height etc. The Standard Pound defines the avoirdupois pound" There is no mention here of weight or mass, the pound *is* the cylinder of platinum, although the related paragraphs are under a central heading MASS.

In 1928 the British National Physical Laboratory published a report on The Units and Standards of Measurement employed at the N.P.L. It quoted (p. 23) the same Act and says "The Imperial Standard Pound is defined as follows: The imperial standard for determining the weight

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of the imperial standard pound *is of* platinum the form being that of a cylinder etc.

In March 1946 the N.P.L. published another report "A Discussion on Units and Standards" which was reprinted from Proc. Roy Soc, A Vol. 186, 1946. Under the heading "The Standards of Mass" it says "The fundamental standard in the British system of units is the Imperial Standard Pound (Weights and Measures Act, 1878). This was constructed in 1844 in the form of a cylindrical piece of platinum.... It is to be noted that this standard is the avoirdupois pound containing 7000 gr." Again no mention of weight, but one notices a very gradual introduction of mass through the title of the section.

The section goes on to say "In the metric system it is of interest to refer to the original conception underlying the definition of the unit of mass. The kilogram was originally defined by reference to a "natural" standard, i.e. the mass of the cubic decimetre of water. The material representation of this standard was a simple cylindrical piece of platinum. ...

Here the word mass is employed with no mention of weight, and it would be of interest to know which of these two terms was employed in those days in France when the Metric system was invented.

In 1955 the United States Department of Commerce (National Bureau of Standards, Miscellaneous Publications 214) published a report on Units of Weight and Measure. Note the use of the singular Here both the kilogram and the pound are defined as units of mass, indeed the pound is defined in terms of the Kilogram. Weight is not mentioned except in the title of the report.

In 1956 the Government of Canada passed an Act respecting Weights and Measures (15, George VI) and says on p. 194 of the printed report "The pound is the only unit or standard measure of weight from which all other Canadian weights and measure having reference to weight are derived" So it looks as if the word weight will remain in popular usage while mass will be oftener used in scientific writings. It would be interesting to know if the term "unit of mass" has legal sanction in England.

If, as physicists say, the pound is the unit of mass in the British system the unit of force must be the weight of one g'th of the weight of this pound i.e. the weight of about half an ounce. It is called the *poundal*.

If, as engineers say, the pound is the unit of force in the British system the unit of mass must be the mass of a lump of matter whose mass is g-times the mass of this particular pound. It is called the *slug*.

The engineer who deals with statical problems only is safe with his definition of the pound as a force, e.g. in the use of the abbreviation "psi" for a pressure. But when the engineer is engaged in a dynamical problem he has to be careful to insert the g where necessary If a mass M pounds has a velocity V ft per second the physicist says it has Kinetic energy $\frac{1}{2}MV^2$ foot poundals. The engineer referring to a weight of W pounds having a velocity V ft per second has to say the Kinetic Energy $-\frac{1}{2}\frac{W}{g}V^2$ foot pounds. The physicist would measure pressure

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in *pounds-weight* per square inch, or more likely in dynes cm^{-2} Many of us remember our school boy days when, after working out a problem in Dynamics, we looked up the answers to see whether we had to divide by g to make our result agree with the answer.

Yours etc., JOHN SATTERLY

To the Editor of the *Mathematical Gazette*

DEAR SIR,

Mr. Whitfield in his review of my book on "Three-Dimensional Dynamics", which appeared in Vol. XLIII, No. 346, of the Mathematical Gazette, after congratulating me on giving a correct proof of the variational principles in impulse theory (Kelvin's and Robin's Theorems) goes on to say that my statement that "Bertrand's Theorem involves no stationary property" is false. This is a statement which I think needs clarifying.

Bertrand's Theorem states that the kinetic energy of any free system when set in motion by a set of impulses is greater than that of the same system when subject to frictionless constraints and set in motion by the same impulses. If frictionless constraints can be imposed on a system, the constraints being such that they can be so continuously varied that the resulting motion differs by as little as one pleases from the actual motion of the free system, then Bertrand's Theorem can certainly be associated with a stationary property, since in the result

$$\tfrac{1}{2}\Sigma m(\mathbf{v}^2 \quad \mathbf{v}^2) = \tfrac{1}{2}\Sigma m(\mathbf{v}_2{}' \quad \mathbf{v}_2{}') - \tfrac{1}{2}\Sigma m(\mathbf{v}_2 - \mathbf{v}_2{}') \quad (\mathbf{v}_2 - \mathbf{v}_2{}'),$$

where $\mathbf{v}_2$ corresponds to the free system and $\mathbf{v}_2{}'$ to the constrained system, we can replace $\mathbf{v}_2{}'$ by $\mathbf{v}_2 + \delta\mathbf{v}_2$ giving

$$\Sigma m(\mathbf{v}_2 \quad \delta\mathbf{v}_2) = 0,$$

i.e.

$$\delta\Sigma m(\mathbf{v}_2 \quad \mathbf{v}_2) = 0,$$

so that the actual motion corresponds to a stationary value of the kinetic energy The actual motion in this case corresponds to the constrained motion which has the maximum kinetic energy. Thus, for instance, suppose we consider a uniform rod AB of mass M and length $2a$ set in motion by an impulse J applied at A at right-angles to AB. Let the motion be defined in terms of $\mathbf{v}$, the velocity of G the centre of mass, together with $\boldsymbol{\omega}$, the angular velocity of the rod. The direction of $\mathbf{v}$ will clearly be at right-angles to AB in the direction of J Hence, taking $\boldsymbol{\omega}$ to have the appropriate direction, the equations to determine the motion are

$$Mv = J, \qquad I\omega = aJ,$$

I being the moment of inertia of the rod with respect to an axis through G perpendicular to the rod. We thus have

$$v = J/M, \quad \omega = aJ/I = 3J/Ma.$$

Now we can clearly apply a frictionless constraint to the system by fixing a point of the rod by means of a smooth pin, and the motion as

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