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On the Estimation of Atmospheric Aerosol Extinction Parameters from Measurements in the Johnson–Cousins Photometric System

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Abstract: A procedure to estimate the aerosol extinction parameters of Angstrom's formula using data acquired with photometric broadband standard filters, the Johnson–Cousins filters, is presented. The method is based on the transformation of the heterochromatic magnitudes to the equivalent mean wavelength monochromatic magnitudes by an iterative approach, correcting subsequently the effects of water-vapour extinction in the longer-wavelength filters. As an example, the procedure has been used in the determination of the extinction parameters at the location of our observatory using the filters B, V, R_c and I_c on the night of 2008 August 28.

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1 Introduction

The evaluation of the atmospheric extinction at a particular site is of great importance in determination of the quality of the astronomical observations, especially in photometric measurements. For this reason, many studies devoted to this subject have been carried out in recent decades (Hayes & Latham 1975; Gutiérrez-Moreno et al. 1982; Rufener 1986; Reimann et al. 1992; Sterken & Manfroid 1992; Burki et al. 1995; Schuster & Parrao 2001; Lombardi et al. 2008). Atmospheric extinction in the optical domain is usually described in terms of three main components: scattering by molecules (Rayleigh scattering), scattering by particles of dust or droplets (aerosol scattering) and molecular absorption (mainly due to ozone, water vapour and other gases). In this way, the total monochromatic extinction coefficient (at wavelength λ), $k(\lambda)$, can be written as

$$k(\lambda) = k_{\mathrm{R}}(\lambda) + k_{\mathrm{A}}(\lambda) + k_{\mathrm{mol}}(\lambda) \tag{1}$$

where $k_{\mathrm{R}}(\lambda)$ is the term due to Rayleigh scattering, $k_{\mathrm{A}}(\lambda)$ represents the contribution of the aerosols to the atmospheric extinction and $k_{\mathrm{mol}}(\lambda)$ is due to the molecular band's absorption.

Rayleigh scattering is dominant in the short-wavelength part of the optical spectrum, owing to its strong λ^{-4} dependence. In regard to this, various expressions have been developed to take into account the Rayleigh extinction coefficient $k_{\mathrm{R}}(\lambda)$. Among these formulae, one of the most employed is due to Hayes & Latham (1975), which can be expressed as

$$k_{\mathrm{R}}(\lambda) = 0.0094977\lambda^{-4}\left[0.23465 + \frac{107.6}{146 - \lambda^{-2}} + \frac{0.93161}{41 - \lambda^{-2}}\right]^2 \exp\left(-\frac{h}{7.996}\right) \tag{2}$$

where h is the height of the observation site in km (assuming a density scale height of 7.996 km and a normal atmospheric pressure at $h = 0$). Other proposed expressions are (Cox 2000)

$$k_{\mathrm{R}}(\lambda) = 0.008569\lambda^{-4}[1 + 0.00113\lambda^{-2} + 0.00013\lambda^{-4}]\frac{p}{p_o} \tag{3}$$

and (Leckner 1978; Iqbal 1983)

$$k_{\mathrm{R}}(\lambda) = 0.008735\lambda^{-4.08}\frac{p}{p_o}, \tag{4}$$

where p and p_o are, respectively, the pressure at the altitude of the observation site and the sea-level pressure (1013.25 mbar).[1]

Although scattering by droplets of water and dust particles can be treated separately (Iqbal 1983), the usual description of all-aerosols scattering involves the use of the well-known empirical Angstrom's turbidity formula

$$k_{\mathrm{A}}(\lambda) = \beta\lambda^{-\alpha}\exp(-h/H), \tag{5}$$

[1]Note that, in these two last cases, the coefficient that multiplies λ^{-4} and $\lambda^{-4.08}$ must be multiplied by 1.08574 in order to express the extinction coefficient in mag/airmass, as is usual in astronomical photometry.

where H is the scale height of aerosols (the usually adopted value is $H = 1.5$ km), α the wavelength exponent, related to the size distribution of the aerosol particles, and β the turbidity coefficient, which measures the amount of aerosols present in the atmosphere in the vertical direction (related to the extinction optical thickness). Note that, in the framework of the Mie theory, modeling the aerosols as homogeneous spheres with radius r and complex refractive index n', the aerosol extinction coefficient can be defined as (van de Hulst 1981)

$$k_{\mathrm{A}}(\lambda) = \int_0^{\infty} \pi r^2 Q_{\mathrm{ext}}(r, \lambda, n') N_c(r) dr, \qquad (6)$$

where Q_{ext} is the extinction efficiency factor from Mie theory and N_c is the aerosol number density in a vertical column (cross-section 1 cm^2) in the radius interval from r to $r + dr$. For aerosol particles in the interval from 0.06 to 10 μm, the function N_c can be approximated by the Junge distribution (Pakstiene 2001)

$$N_c(r) = ur^{-(v+1)}, \qquad (7)$$

where the multiplier u depends on the concentration of the particles. In this case, Equation 6 gives Angstrom's formula, with the Junge exponent, v, related to α by means of

$$\alpha = v - 2, \qquad (8)$$

and β being linear with u (Pakstiene 2001), and hence related to the concentration of particles. Parameter α is of the order of 1 for particles with size close to the wavelength of the observed light and 2 for particles smaller than this. For particles with diameters much larger than λ, $\alpha < 1$ and the aerosol extinction is sensibly independent of the wavelength (Gutiérrez-Moreno et al. 1982). In regard to the parameter β, note that in some cases it is expressed as an empirical function of the visibility in the horizontal direction, also called *meteorological range* (Iqbal 1983; Gueymard 2001).

Molecular absorption in the optical domain can be attributed mainly to three components: ozone, water vapour and O_2. Absorption by ozone contributes especially in the Huggins band (0.3–0.35 μm) and the Chappuis band (centred around 0.575 μm). This contribution is usually expressed (in mag/airmass) as

$$k_{\mathrm{oz}}(\lambda) = 1.08574 \sigma_{\mathrm{oz}}(\lambda) l, \qquad (9)$$

where $\sigma_{\mathrm{oz}}(\lambda)$ is the attenuation coefficient for ozone absorption and l is the amount of ozone in cm (NTP), that is, the height of gaseous ozone if all the ozone in a vertical column of unit area were brought to normal or standard temperature and surface pressure (NTP or STP). Because the ozone is mainly concentrated at altitudes between 10 and 35 km, we can consider that its contribution to the extinction does not depend on the altitude of the observatory. However, the amount of ozone is strongly dependent on the latitude and season (Iqbal 1983; King & Byrne 1976), ranging between 0.22 and 0.46 cm (NTP). This amount can be estimated using the data of the Total Ozone Mapping Spectrometer (TOMS) for any location in the world.[2] Ozone attenuation coefficients are tabulated (Iqbal 1983) for different wavelengths.[3]

Whereas for $\lambda \leq 0.60$ μm the atmospheric extinction is due basically to Rayleigh scattering, aerosol scattering and ozone absorption (usually being expressed in terms of the total extinction as the sum of these three contributions), at higher wavelengths O_2 presents three weak absorption bands centred at 0.63, 0.69 and 0.76 μm and water vapour shows bands of importance at 0.72, 0.82 and 0.94 μm. These two contributions can be modelled in a form not suitable for use with the usual Bouguer's law (Iqbal 1983). In regard to this, the spectral transmittance of the water-vapour component for the extinction in the I_c filter can be expressed as (Iqbal 1983)

$$\tau_{\mathrm{wa}}(\lambda) = \exp\left[\frac{-0.2385\sigma_{\mathrm{wa}}(\lambda)wX}{(1 + 20.07\sigma_{\mathrm{wa}}(\lambda)wX)^{0.45}}\right], \qquad (10)$$

$\sigma_{\mathrm{wa}}(\lambda)$ and w being, respectively, the attenuation coefficient for the water-vapour absorption and the amount of precipitable water. This is the total amount of water vapour in the zenith direction, between the surface of the Earth and the top of the atmosphere, often described as the thickness of the liquid water that would be formed if all the vapour in the zenith direction were condensed at the surface of unit area. According to Leckner (1978), w (in cm) can be estimated using

$$w = 0.493(\phi_r/100) \exp\left(26.23 - \frac{5416}{T}\right) T^{-1}, \qquad (11)$$

where ϕ_r is the relative humidity (in %) and T is the ambient temperature in K.

Due to its importance in obtaining accurate ground-based photometric data, the characterization of the atmospheric extinction is of great interest in astronomy. Note that it depends on many factors, including the geographical location and altitude of the observing site. However, whereas the Rayleigh scattering is mainly a function of the altitude above sea level — for a particular wavelength — and the ozone absorption is closely dependent on the latitude, with only small seasonal variations (and thus relatively stable for a given observing site), the atmospheric aerosols above an observatory site are the most sensitive contributors to the commonly observed variation in the atmospheric extinction behaviour over the years at the site. In regard to this, the aerosol extinction is strongly variable with the season and with the amount of material ejected by volcano eruptions

[2] `http://toms.gsfc.nasa.gov/teacher/ozone_overhead.html`

[3] Bessell (1990) has presented an expression that is sometimes used to estimate these coefficients.

(Gutiérrez-Moreno et al. 1982; Rufener 1986). Note, in this sense, that the degree of constancy of the atmospheric extinction coefficient — especially its aerosol component — indicates the photometric quality of the observing site. In this way, monitoring the aerosol extinction is interesting in order to detect variations that could point toward degrading or improving observing conditions. Owing to the widespread use and validity of Angstrom's formula for characterizing the aerosol extinction, and due to the relation of the parameters α and β to the concentration, nature and size of the aerosol particles, the measure of these quantities and their changes with time is of great importance to determine the photometric properties of a particular observatory site.

Using suitable monochromatic filters in the optical region $\lambda \leq 0.60\ \mu$m, the determination of the parameters α and β is straightforward from the knowledge of the Rayleigh scattering and ozone absorption (although small variability of the ozone content over timescales as short as a few hours implies that Equation 9 is valid only for calculating mean coefficients). Note that, in this case, the total extinction coefficient can be calculated, for a particular wavelength, using Bouguer's law

$$m(\lambda) = m_o(\lambda) + k(\lambda)X, \tag{12}$$

where $m(\lambda)$ is the instrumental magnitude of a star measured at the wavelength λ, $m_o(\lambda)$ the corresponding instrumental magnitude outside the atmosphere and X the relative (to the zenith) airmass.

However, in most cases the filters used in the extinction measurements have finite bandwidths (as in the Johnson–Cousins or Strömgren systems). In this circumstance, Bouguer's law is not strictly valid and it must modified to take into account the effect of the finite bandwidth of the filter, the energy distribution of the stellar source and the variation of the extinction with the wavelength. The theory of heterochromatic extinction has been developed by Strömgren (1937) and King (1952) and examined carefully by Young (1992). In this theory, the measured light is

$$L = \int_0^\infty R(\lambda)I(\lambda)\tau(\lambda)d\lambda, \tag{13}$$

where $R(\lambda)$ is the instrument spectral response, $I(\lambda)$, the stellar spectral irradiance and $\tau(\lambda)$, the atmospheric transmission function, related with the extinction coefficient (expressed in magnitudes/airmass) by means of

$$\tau(\lambda) = \exp[-0.921k(\lambda)X]. \tag{14}$$

Defining the heterochromatic magnitude as $m_{\text{band}} = -2.5\log L + C$, and expanding the product of the stellar irradiance and the atmospheric transmission function in a Taylor series around the mean wavelength of the filter, λ_o,

$$\lambda_o = \frac{\int_0^\infty \lambda R(\lambda)d\lambda}{\int_0^\infty R(\lambda)d\lambda}, \tag{15}$$

we obtain, after some manipulation,

$$m_{\text{band}} = m(\lambda_o) - 2.5\log\left(1 + \frac{1}{2}\mu_2^2[I(\lambda)\tau(\lambda)]''_{\lambda=\lambda_o} + \frac{1}{6}\mu_3^3[I(\lambda)\tau(\lambda)]'''_{\lambda=\lambda_o} + \ldots\right), \tag{16}$$

where

$$m(\lambda_o) = -2.5\log I(\lambda_o)\tau(\lambda_o) + C \tag{17}$$

is the monochromatic magnitude at $\lambda = \lambda_o$,

$$\mu_k^k = \frac{\int_0^\infty R(\lambda)(\lambda-\lambda_o)^k d\lambda}{\int_0^\infty R(\lambda)d\lambda}, \tag{18}$$

and $[I(\lambda)\tau(\lambda)]^n_{\lambda=\lambda_o}$ is the nth derivative at $\lambda = \lambda_o$ of the product of the stellar irradiance and the atmospheric transmission function normalized at the value at $\lambda = \lambda_o$ (that can be expanded in terms of the derivatives of the stellar irradiance and the atmospheric transmission function using Tartaglia's triangle).

Equation 16 expresses the link between heterochromatic and monochromatic magnitudes through the series expansion of the product of the stellar irradiance and atmospheric transmission function. Note that, in most cases, the measured magnitudes are heterochromatic, and the monochromatic magnitudes only can be obtained by using the above series expansion, which depends on the stellar spectral irradiance (that can be approximated by the Planck black body equation) and the atmospheric transmission function. In the last case, note that this function depends on the atmospheric extinction coefficients at $\lambda = \lambda_o$, $k(\lambda_o)$, that can only be strictly determined using the monochromatic magnitudes.

The estimation of the aerosol atmospheric extinction parameters α and β requires the use of a model including all contributions to atmospheric extinction (Rayleigh, molecular bands, etc.). In many cases (Rufener 1986; Reinmann et al. 1992; Kumar et al. 2000; Schuster & Parrao 2001; Stalin et al. 2008), the general procedure involves the subtraction of the Rayleigh and molecular band components (mainly ozone) from the measured extinction coefficients, assuming that they are monochromatic or reducing them by some method to monochromatic values (Mohan et al. 1999; Kumar et al. 2000). Usually, this implies the assumption of a particular model describing the Rayleigh extinction coefficients (as in the equations included at the beginning of this section) and the ozone extinction coefficients (assuming the amount of ozone, in some cases using a standard value of $l = 0.3$ cm). In some cases, a reasonable mean value taken from the bibliography is used for one of the aerosol

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parameters (α or β), the other being determined from the observations.

Various photometric systems have been used in the estimation of the aerosol contribution to the atmospheric extinction. In particular, the Geneva system, constituted by seven filters (U, B_1, B, B_2, V_1, V, G) with λ_o between 0.34 and 0.58 μm, has been used in studies of the atmospheric extinction at La Silla Observatory (Rufener 1986; Burki et al. 1995). Moreover, the Vilnius system (Zdanavicius 1996), with seven filters characterized by λ_o below 0.70 μm, has also been used in aerosol extinction measurements (Forbes et al. 1996; Pakstiene 2001). In other cases, the Strömgren system has been successfully employed to determine α and β (Sterken & Manfroid 1992; Schuster & Parrao 2001), sometimes in combination with other photometric filters (Reimann et al. 1992). Note that these filters have $\lambda \leq 0.60\,\mu$m, being particularly well suited for studying the aerosol extinction for this reason. In some particular cases (Gutiérrez-Moreno et al. 1982), monochromatic extinction values have been obtained in order to measure the different components of the atmospheric extinction.

As an example of these measurements, Burki et al. (1995) obtained a value of $\alpha = 1.39$, relatively stable, and $\beta = 0.0116$ for the meteorological aerosols — removing the effects from the volcanoes — at the ESO La Silla Observatory, using data from 1975 to 1994, indicating the very good quality of the site. However, notable changes in the values of α were measured for the aerosols due to the volcanoes El Chinchón (1982, in Mexico) and Pinatubo (1991, the Philippines). Thus, in the former case, values of α ranging from -1.2 to 0.85 were obtained over a period of 800–900 days. Values of $\alpha = 1.3$ and $\beta = 0.006$ were adopted by Rufener (1986) based on data taken in the same observatory in the period of lowest extinction before the eruption of El Chinchón. At the Cerro Tololo Inter-American Observatory (CTIO, $h = 2.2$ km), Gutiérrez-Moreno et al. (1982), using data from 1964 to 1980, measured β and α values ranging from 0.002 to 0.081 and 0.2 to 2.6, respectively — with mean values of $\beta = 0.021$ and $\alpha = 1.2$. Higher values of β were related to the presence of volcanic ashes during 1964–1966, whereas in the following years a decreasing trend of β was observed.

Broadband filters, such as the standard Johnson–Cousins filters, have been extensively used in the determination of the atmospheric extinction coefficients owing to their availability and widespread use in photometry. Nevertheless, the use of the $UBVR_cI_c$ system in the estimation of the aerosol extinction parameters has been sparse, probably due to: (i) its greater departure from monochromaticity, owing to the broadband character; (ii) the presence of the I_c filter, that can be affected by water vapour extinction (not included in the usual approach because of its negligible effect at shorter wavelengths). In regard to this, note that some authors (Mohan et al. 1999; Kumar et al. 2000) have used the following expression, due to Golay (1974), to obtain approximate values of the monochromatic extinction coefficients:

$$k_{\text{band}} = k(\lambda_o)\left[1 - \left(\frac{\delta}{T\lambda_o^2} - \frac{5}{\lambda_o}\right)\frac{\mu_2^2 n}{\lambda_o} + \frac{n(n+1)\mu_2^2}{2\lambda_o^2}\right], \tag{19}$$

where $\delta = 0.014388$ mK, T is the temperature of the star and n is the exponent of the power-law function describing the behaviour of the extinction coefficient. The above expression can be easily obtained from Equation 16 retaining the second-order term, making the approximation $\log(1+x) \approx 0.43429x$ and approximating the first derivative of the stellar irradiance function for $\delta \gg \lambda_o T$. Because the exponent n is *a priori* unknown, it is usual to take for it the value corresponding to the Rayleigh extinction dependence ($n = 4$).

However, by the extensive use of the Johnson–Cousins system in photometric work (in contrast to the aforementioned passbands), it can be considered as a good candidate to evaluate aerosol extinction parameters in many observatories, especially as part of long-term studies on extinction properties. In this way, we propose a general procedure to obtain the aerosol extinction parameters using Johnson–Cousins filters and correcting by their heterochromaticity. This procedure is based on the iterative determination of the terms in the series expansion (up to third order, in this study) included in the logarithmic part of Equation 16, assuming that the total extinction coefficient can be approximated as a power-law function. The subsequent steps imply the usual determination of the extinction aerosol coefficient by subtracting the other main components (Rayleigh, ozone and water-vapour, in this case for the I filter) from the monochromatic extinction coefficients and, from that, the estimation of the aerosol extinction parameters. In regard to this, in the following section we describe the method. Subsequently, an example of application is analysed. Next, the obtained results are discussed, and finally, conclusions are presented.

2 Procedure

As indicated above, Equation 16 shows the relation between heterochromatic and monochromatic magnitudes for a particular instrument response (through the moments μ_k^k), using the stellar irradiance model corresponding to the observed star and the atmospheric extinction function for the site and time of the observations. Note that the instrument function can be calculated from the transmission curves of the filters and spectral response of the detector, whereas the stellar irradiance can be approximated by the black body function for the temperature of the star. Nevertheless, the atmospheric extinction function is *a priori* unknown. In fact, in the usual monochromatic approach of Bouguer's law, the extinction coefficients are determined from the measured (uncorrected by the heterochromaticity of the filters) instrumental magnitudes. A more rigorous approach requires the knowledge, at least, of the approximate form of the atmospheric extinction function along the optical range. In this way, we have adopted here the well-known power-law function (King 1952; Budding 1993)

$$k(\lambda) = a\lambda^{-n}, \qquad (20)$$

that leads to

$$\tau(\lambda) = \exp[-0.921 a\lambda^{-n} X]. \qquad (21)$$

The values of a and n depend on the particular model of the extinction that, as indicated above, is *a priori* unknown. In this way, according to Equation 16, the monochromatic magnitude can be expressed as

$$m(\lambda_o) = m_{\text{band}} + 2.5 \log \phi(T, \lambda_o, a, n, X), \qquad (22)$$

where

$$\phi(T,\lambda_o,a,n,X) = 1 + \frac{1}{2}\mu_2^2[I(T,\lambda)\tau(a,n,X,\lambda)]''_{\lambda=\lambda_o} + \frac{1}{6}\mu_3^3[I(T,\lambda)\tau(a,n,X,\lambda)]'''_{\lambda=\lambda_o} + \ldots \qquad (23)$$

In this way, we propose here an iterative procedure to evaluate the equivalent monochromatic magnitudes in order to determine the corresponding monochromatic extinction coefficients and, from them, the aerosol extinction parameters. This procedure is based on the following steps:

1. First, the determination of the extinction coefficients for the mean wavelengths of each Johnson–Cousins filter using Bouguer's law, as is usual, using the observed heterochromatic magnitudes. In this way, these approximate extinction coefficients are evaluated by means of

$$m_{\text{band}} = m_1(\lambda_o) = m_{o,1}(\lambda_o) + k_1(\lambda_o)X. \qquad (24)$$

2. The fit of the power-law model given by Equation 20 to the coefficients previously obtained. Thus, an initial estimate of the parameters a and n is obtained.
3. Determination of the derivatives included in the definition of ϕ up to the desired order assuming the a and n values obtained in the preceding step.
4. Estimate of the approximate monochromatic magnitudes, $m_2(\lambda_o)$, for the mean wavelengths using Equation 22 for each observed heterochromatic magnitude (at the corresponding relative airmass).
5. Using the estimated monochromatic magnitudes previously determined, repetition of steps 1–4 until the difference between the monochromatic magnitudes corresponding to the two last iterations be smaller than a previously fixed value ($|m_{i-1}(\lambda_o) - m_i(\lambda_o)| \leq \epsilon$).
6. Subtraction of the contribution of the water-vapour extinction to the monochromatic magnitude in the mean wavelength of the I_c filter using the model of Iqbal (1983), interpolating the attenuation coefficient for that wavelength.
7. Determination of the extinction coefficients, $k_i(\lambda_o)$, for the monochromatic magnitudes obtained in the two last steps, using Bouguer's law.
8. Subtraction of the Rayleigh and ozone extinction coefficient components, modelled using some of the functions included in the preceding section, from $k_i(\lambda_o)$, in order to get the aerosol extinction coefficient.
9. And finally, as is usual, the fit of Angstrom's equation (Equation 5) to the aerosol extinction component obtained in the former step.

3 Observations and Analysis

In order to test the procedure outlined in the preceding section, we have analysed the atmospheric extinction in our observatory ($\phi = 38°52'58.5''$ N, $\lambda = 7°0'39.9''$ W, $h = 169$ m) on the night of 2008 August 28. In this way, we have used a refractor telescope of $D = 8$ cm and $f = 910$ mm mounted over the 0.4-m main telescope. An SXV-H9 CCD camera (Starlight Xpress Ltd, UK; 1392×1040 pixels, 33.9×25.3 arcmin^2, $1.46''$/pixel) was used to take images of the star HR 655 ($V = 5.25$, $B - V = -0.01$, spectral type A0V, $T = 9790$ K) at different zenith angles. These images (23 for each filter) were taken in the B, V, R_c and I_c bands by using a motorized SupaSlim filter wheel (True Technology Ltd., UK). Calibration was done using bias, darks and flats frames, as is usual. The subsequent analysis to determine the heterochromatic instrumental magnitudes was carried out by means of the program AIP4WIN (Berry & Burnell 2007). The relative airmass for each observation was calculated using Bemporad's formula (Budding 1993) by means of a FORTRAN code elaborated by the authors.

The general procedure described in the former section was applied with the aim of obtaining the extinction aerosol parameters. In this regard, the following details were kept in mind:

- The function ϕ was expanded up to third order, using the moments μ_2^2 and μ_3^3 calculated from the transmission curves of the filters, as provided by the manufacturer (Optec Inc., USA). However, the evaluation of the monochromatic instrumental magnitudes was also carried out using the second-order approximation and the non-logarithmic approximation

$$m(\lambda_o) = m_{\text{band}} + 0.54287\mu_2^2\,[I(T,\lambda)\tau(a,n,X,\lambda)]''_{\lambda=\lambda_o}, \qquad (25)$$

with comparative purposes.
- We have adopted the Rayleigh extinction model of Leckner (1978), due to its simplicity. Note, nevertheless, that the three models described in the introduction

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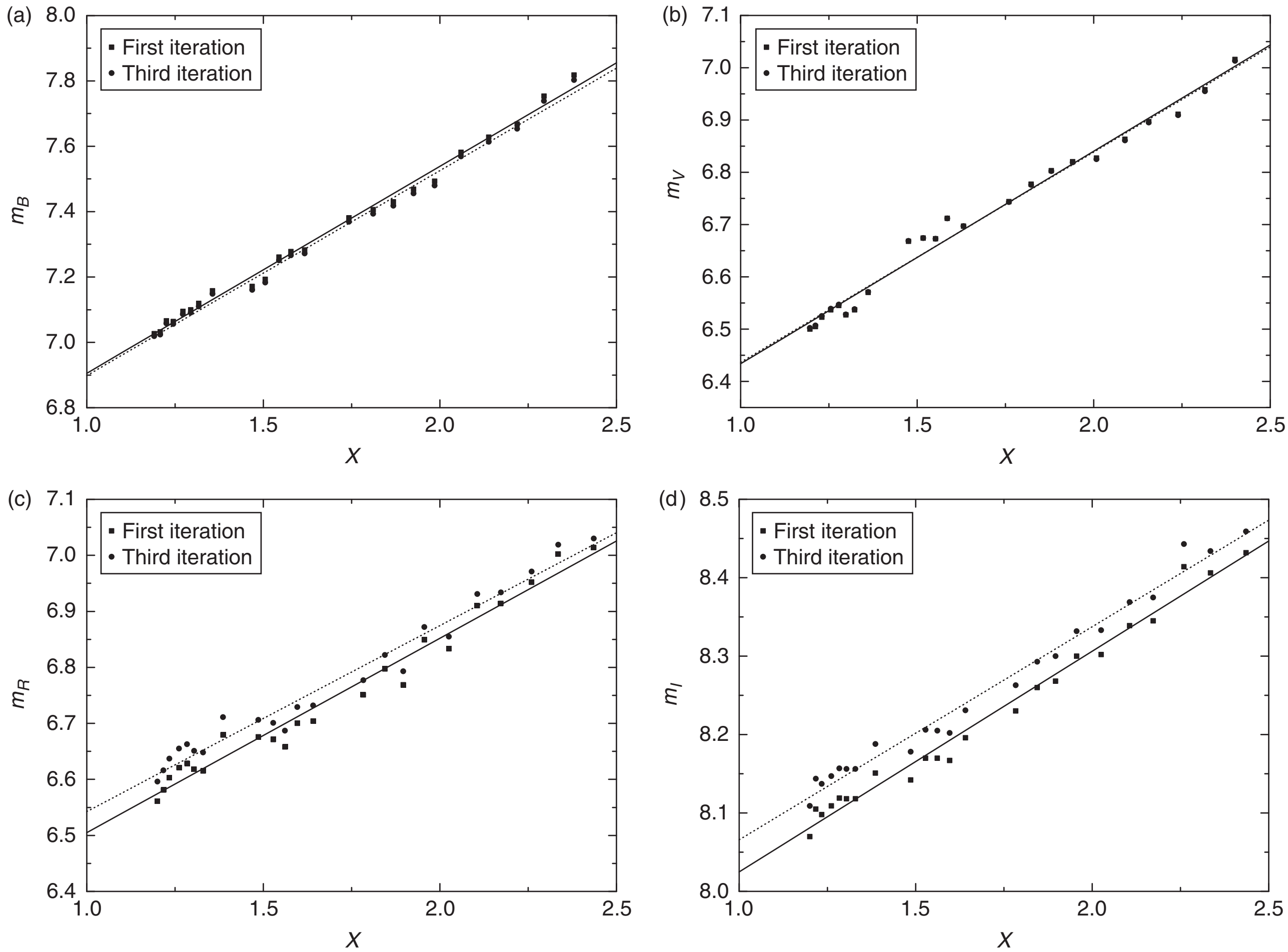

Figure 1 Instrumental magnitudes against the relative airmass for the filters B (a), V (b), R_c (c), and I_c (d). Squares represent the original heterochromatic magnitudes (first iteration), and circles the monochromatic magnitudes obtained at the third iteration. Solid and dashed lines are the best-fit linear models for both series of data, respectively.

are very similar and that some differences only arise for $\lambda \leq 0.50\,\mu$m.

- We have adopted the ozone extinction model of Iqbal (1983), with the attenuation coefficients — obtained by Vigroux (1953) — interpolated for the mean wavelength of each filter. Instead of the traditional approach that fixes the amount of ozone, l, at a mean reasonable value of 0.3 cm (or other), we have used the TOMS value for the location of the observatory interpolated for the date of the observations. In this case, however, this value is equal to 0.3 cm.
- The effect of the water-vapour extinction in the magnitudes of the filter I_c was modeled using Equation 10, employing the attenuation coefficients of Leckner (1978). This correction can be expressed as

$$\Delta m = -1.08574\frac{0.2385\sigma_{\rm wa}(\lambda)wX}{(1+20.07\sigma_{\rm wa}(\lambda)wX)^{0.45}}. \qquad (26)$$

The value of the amount of precipitable water in our observatory was estimated by using the measures of T and ϕ_r obtained with a WM918 electronic weather station (Huger Electronics GmbH, Germany) at the time of the observations. Mean values of 63% for ϕ_r and 299.2 K for T were derived from these measurements. In this way, Equation 11 provided a mean value of $w = 3.5$ cm.

4 Results and Discussion

4.1 Determination of the Monochromatic Magnitudes

Figure 1 shows the instrumental magnitudes (heterochromatic and monochromatic, in this last case obtained at the third iteration) against the relative airmass for the different filters used in this work. Note that the fourth iteration provided the same results as the third. Moreover, no significant differences were observed between the model up to third order and the lower-order approximations. As is expected for the spectral type of the star, the main differences between the heterochromatic and monochromatic magnitudes are show for the filters R_c and I_c.

Table 1 displays the corresponding extinction coefficients for the data presented in Figure 1, besides the power-law function parameters a and n derived from them. Note that a slight decrease in the extinction coefficients is observed when using the data of the monochromatic magnitudes, resulting in a slight increase of the n value. In this way, the selection of an A0 star, with a

Table 1. Extinction coefficients in mag/airmass derived from the fits in Figure 1 and parameters *a* and *n* of the corresponding power-law function

	Heterochromatic magnitude	Monochromatic magnitude[a]
B	0.63(1)	0.63(1)
V	0.41(1)	0.40(1)
R_c	0.35(1)	0.33(1)
I_c	0.277(8)	0.268(8)
a	0.233(7)	0.223(7)
n	1.14(2)	1.18(2)

[a]Obtained at the third iteration.

Table 2. Monochromatic extinction coefficients in mag/airmass derived from Equation 19 using *n* = 1.18 and *n* = 4

	$k(\lambda_o)$ $n = 1.18$	$k(\lambda_o)$ $n = 4$
B	0.62	0.60
V	0.40	0.38
R_c	0.32	0.26
I_c	0.26	0.23

negligible colour index, explains the small differences between the extinction coefficients obtained using heterochromatic and monochromatic magnitudes, because in this case the extinction coefficient is not very much affected by the heterochromaticity of the filters. In this regard, the $n = 1.18$ value obtained for the monochromatic magnitudes lies within the range of expected values (1 to 4) and it is very different from the value corresponding to pure Rayleigh scattering that is usually employed in Equation 19, as was indicated in Section 1. In regard to this, we have calculated the monochromatic extinction coefficients for the different filters using Equation 19 with $n = 1.18$ and $n = 4$. They are presented in Table 2. Note that the approximation for $n = 1.18$ is fairly good, whereas the use of $n = 4$ provides values lower than those obtained with the method presented here, especially for the R_c and I_c filters. This is an expected result because the crude Rayleigh approximation fails for longer wavelengths due to the presence of the other sources of extinction. In relation to the aerosol extinction, this is especially true for low-altitude observatories such as ours.

4.2 Determination of the Aerosol Extinction Parameters

We have estimated the aerosol extinction parameters by means of the extinction coefficients derived from the monochromatic magnitudes determined above (previously, the effects of the water-vapour extinction on the I_c filter were removed using Equation 26). In this way, the usual approach was chosen based on the subtraction of the Rayleigh and ozone components (using the models indicated in Section 3) from the total extinction coefficient

$$k_A(\lambda) = k(\lambda) - k_R(\lambda) - k_{oz}(\lambda). \qquad (27)$$

From the derived aerosol extinction coefficients, the parameters α and β of Angstrom's formula were

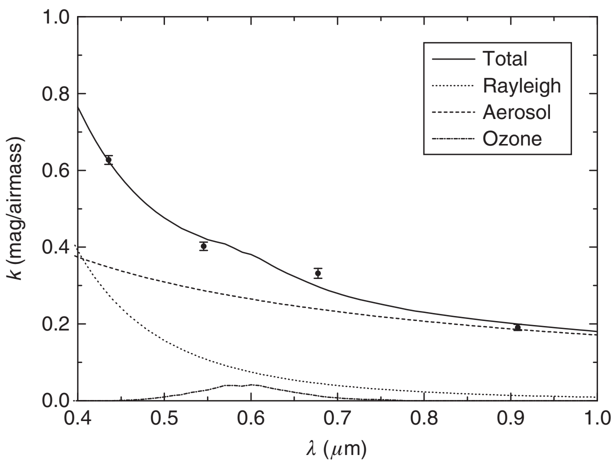

Figure 2 Total and Rayleigh, aerosol and ozone extinction coefficients as function of the wavelength. Data points corresponding to the mean wavelengths of the filters used in this work are included.

determined. Thus, the obtained values were $\alpha = 0.85(7)$ and $\beta = 0.158(7)$. Figure 2 displays the extinction model (including Rayleigh, aerosol and ozone extinction) and the corresponding extinction coefficient for each filter.

The estimated aerosol extinction coefficients are well placed in the expected usual ranges for these parameters. Thus, β values vary from 0.0 to 0.5 in most cases (Alnaser & Awadalla 1995; Gueymard 1998; Kaskaoutis & Kambezidis 2006). In this regard, the value obtained here is representative of moderately turbid skies (Iqbal 1983). In respect to the parameter α, usual values are between 0.5 and 2.5 (Iqbal 1983; Mecherikunnel et al. 1983; de La Casinière et al. 1997). Note that in many cases a value of 1.3 is adopted for this parameter, which can be considered a reasonably good average for most natural atmospheres — for example, an annual mean value of 1.27 ± 0.53 was obtained by Chaiwiwatworakul & Chirarattananon (2004) in a location near Bangkok. Our α value is low, indicating particles of large size (Iqbal 1983; Brine & Iqbal 1983). These results are in agreement with the summer atmospheric conditions in the location of our observatory (Badajoz, Spain), characterized by the general absence of rainfall and the influence of air masses from desert lands in North Africa (that causes very turbid skies in the effect named as *calima*). Note the high values of the extinction coefficients presented in the former subsection, which can be representative of observatories at low altitude in these summer conditions. In fact, they are comparable to previous measurements performed in the same location (Jurado Vargas et al. 2002).

Note that, if the contribution of the water-vapour extinction in the I_c filter is ignored, the corresponding values of the aerosol extinction parameters are $\alpha = 0.39(6)$ and $\beta = 0.219(7)$. Whereas the β value is still well placed in its usual range, the α is relatively low. In this regard, the I_c monochromatic extinction coefficient corrected by the contribution of water-vapour is about 0.19 mag/airmass, whereas the uncorrected value is

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0.27 mag/airmass (Table 1). This significant difference, that can be explained by the low altitude of our observatory — about one-half of the precipitable water is concentrated in the first 2 km above sea level (Iqbal 1983) — indicates the need to estimate the effect of the water-vapour extinction in the bands affected by it. In regard to this, note that the estimated amount of precipitable water used in this work, 3.5 cm, is close to values obtained in summer conditions in locations at similar latitudes (Iqbal 1983; Halthore et al. 1997; Cachorro et al. 1998).

5 Conclusions

The determination of the aerosol extinction parameters can be performed in a simple way by using usual broadband photometric standard filters (widely used in many observatories, both professional and amateur) in different bands of the optical spectrum. The procedure is based on the estimation of the monochromatic instrumental magnitudes in the mean wavelength of each filter from the heterochromatic magnitudes measured against the airmass. The method assumes that the monochromatic extinction coefficient can be approximated by the well-known power-law function in order to transform the heterochromatic magnitudes to the monochromatic magnitudes by calculating iteratively the term 2.5 log ϕ (T, λ_o, a, n, X) (Equation 22) up to the desired order of approximation. Note that this procedure avoids the need to assume an *a priori* value of the exponent n (as it is also iteratively determined) in the power-law extinction function in order to correct the heterochromatic magnitudes. The monochromatic instrumental magnitudes thus obtained can be used in the usual manner to determine the extinction coefficients (correcting them previously by the non-Bouguer extinction components, as the water-vapour in the case of the I_c filter). Subsequently, the aerosol extinction coefficients (that are needed to estimate the α and β parameters) are calculated subtracting the Rayleigh and ozone extinction contributions for the mean wavelength of each filter.

In this work, an application example has been carried out using the photometric standard Johnson–Cousins filters available in our observatory (B, V, R_c and I_c). However, the procedure can be performed with any photometric system or combination of filters of different broadband or intermediate band systems (such as the Strömgren system), avoiding the use of near-monochromatic filters. Besides this, the contribution to the extinction of the water vapour in the longer-wavelength filters (such as the I_c) can be estimated and subtracted from the monochromatic magnitudes. Note that the use of monochromatic magnitudes instead of the heterochromatic ones allows us to loosen the usual criterion of choosing stars of spectral type close to A0 in the measurements of the extinction coefficients, extending the range of stars useful for them. In this regard, the method is suitable for standard equipment in low-level observatories and, for this reason, adequate in long-term observations aiming to study the evolution of aerosol extinction employing small telescopes and the usual photometric filters.

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References

Alnaser, W. E. & Awadalla, N. S., 1995, EM&P, 70, 61
Berry, R. & Burnell, J., 2007, AIP4Win: Astronomical Image Processing for Windows, version 2.2.0
Bessell, M. S., 1990, PASP, 102, 1181
Brine, D. T. & Iqbal, M., 1983, SoEn, 30, 447
Budding, E., 1993, An Introduction to Astronomical Photometry (Cambridge: Cambridge University Press)
Burki, G., Rufener, F., Burnet, M., Richard, C., Blecha, A. & Bratschi, P., 1995, AASS, 112, 383
Cachorro, V. E., Utrillas, P., Vergaz, R., Durán, P., de Frutos, A. M. & Martínez-Lozano, J. A., 1998, ApOpt, 37, 4678
Chaiwiwatworakul, P. & Chirarattananon, S., 2004, Energy and Buildings, 36, 650
Cox, A. N. (ed.), 2000, Allen's Astrophysical Quantities (4th ed.; New York: Springer-Verlag)
de La Casinière, A., Bokoye, A. I. & Cabot, T., 1997, JApMe, 36, 509
Forbes, M. C., Dodd, R. J. & Sullivan, D. J., 1996, BaltA, 5, 281
Golay, M., 1974, Introduction to Astronomical Photometry (Boston: Dordrecht-Holland)
Gueymard, C. A., 1998, JApMe, 37, 414
Gueymard, C. A., 2001, SoEn, 71, 325
Gutiérrez-Moreno, A., Moreno, H. & Cortés, G., 1982, PASP, 94, 722
Halthore, R. N., Eck, T. F., Holben, B. N. & Markham, B. L., 1997, JGR, 102, 4343
Hayes, D. S. & Latham, D. W., 1975, ApJ, 197, 593
Iqbal, M., 1983, An Introduction to Solar Radiation (New York: Academic Press, Inc.)
Jurado Vargas, M., Merchán Benítez, P. & Sánchez Bajo, F., 2000, EJPh, 21, 245
Kaskaoutis, D. G. & Kambezidis, H. D., 2006, AtmRe, 79, 67
King, I., 1952, AJ, 57, 253
King, M. D. & Byrne, D. M., 1976, JAtS, 33, 2242
Kumar, B., Sagar, R., Rautela, B. S., Srivastava, J. B. & Srivastava, R. K., 2000, BASI, 28, 675
Leckner, B., 1978, SoEn, 20, 143
Lombardi, G., Zitelli, V., Ortolani, S., Pedani, M. & Ghedina, A., 2008, A&A, 483, 651
Mecherikunnel, A. T., Gatlin, J. A. & Richmond, J. C., 1983, ApOpt, 22, 1354
Mohan, V., Uddin, W., Sagar, R. & Gupta, S. K., 1999, BASI, 27, 601
Pakstiene, E., 2001, BaltA, 10, 651
Reimann, H. G., Ossenkopf, V. & Beyersdorfer, S., 1992, A&A, 265, 360
Rufener, F., 1986, A&A, 165, 275
Schuster, W. J. & Parrao, L., 2001, RMxAA, 37, 187
Stalin, C. S, Hegde, M., Sahu, D. K., Parihar, P. S., Anupama, G. C., Bhatt, B. C. & Prabhu, T. P., 2008, BASI, 36, 111
Sterken, C. & Manfroid, J., 1992, A&A, 266, 619
Strömgren, B., 1937, Handbuch der Experimentalphysik, Band 26, Astrophysik (Leipzig: Akademische Verlagsgesellschaft)
Van de Hulst, H. C., 1981, Light Scattering by Small Particles (New York: Dover)
Vigroux, E., 1953, AnPhy, 8, 709
Young, A. T., 1992, A&A, 257, 366
Zdanavicius, K., 1996, BaltA, 5, 549

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