Appendix A

Conventions, spinors, and currents

A.1 Conventions

The space-time coordinates $(t, x, y, z) = (t, \vec{x})$ are denoted by a contravariant four-vector (c and $\hbar$ are set equal to 1):

$$x^{\mu} = (x^0, x^1, x^2, x^3) = (t, x, y, z). \tag{A.1}$$

The metric tensor is

$$g_{\mu\nu} = g^{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \tag{A.2}$$

$$p^{\mu} = (p_0, \vec{p}), \qquad p_{\mu} = g_{\mu\nu} p^{\nu} = (p_0, -\vec{p}\,). \tag{A.3}$$

Momentum four-vectors are similarly defined,

$$p^{\mu} = (E, p_x, p_y, p_z), \tag{A.4}$$

and the inner product

$$p_1 \cdot p_2 = p_{1\mu} p_2^{\mu} = (E_1 E_2 - \vec{p}_1 \vec{p}_2). \tag{A.5}$$

We frequently meet products of the totally antisymmetric tensor $\varepsilon_{\alpha\beta\gamma\mu}$ (note $g_{\mu}^{\nu} = \delta_{\mu}^{\nu}$)

$$\varepsilon_{\alpha\beta\gamma\mu}\varepsilon^{\alpha\beta\gamma\nu} = -6\delta_{\mu}{}^{\nu}, \tag{A.6}$$

$$\varepsilon_{\alpha\beta\mu\nu}\varepsilon^{\alpha\beta\rho\sigma} = -2\begin{vmatrix} \delta_{\mu}^{\rho} & \delta_{\nu}^{\rho} \\ \delta_{\mu}^{\sigma} & \delta_{\nu}^{\sigma} \end{vmatrix}, \tag{A.7}$$

$$\varepsilon_{\alpha\mu\nu\sigma}\varepsilon^{\alpha\lambda\rho\tau} = \begin{vmatrix} \delta_{\mu}^{\lambda} & \delta_{\nu}^{\lambda} & \delta_{\sigma}^{\lambda} \\ \delta_{\mu}^{\rho} & \delta_{\nu}^{\rho} & \delta_{\sigma}^{\rho} \\ \delta_{\mu}^{\tau} & \delta_{\nu}^{\tau} & \delta_{\sigma}^{\tau} \end{vmatrix}. \tag{A.8}$$

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A.2 Dirac matrices and spinors

Anticommutation of γ-matrices:

$$\{\gamma^\mu, \gamma^\nu\} = \gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu}, \tag{A.9}$$

$$\gamma^5 \equiv \mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3, \quad \{\gamma^\mu, \gamma^5\} = 0. \tag{A.10}$$

The σ-matrix:

$$\sigma^{\mu\nu} = \frac{\mathrm{i}}{2}[\gamma^\mu, \gamma^\nu]. \tag{A.11}$$

Reduction of the product of three γ-matrices:

$$\gamma^\mu\gamma^\rho\gamma^\nu = S^{\mu\rho\nu} + \mathrm{i}\epsilon_\lambda^{\mu\nu\rho}\gamma^\lambda\gamma_5, \tag{A.12}$$

with

$$S^{\mu\rho\nu} = g^{\mu\rho}\gamma^\nu + g^{\rho\nu}\gamma^\mu - g^{\mu\nu}\gamma^\rho. \tag{A.13}$$

A familiar representation of γ-matrices is

$$\gamma^0 = \begin{bmatrix} \mathbf{1} & 0 \\ 0 & -\mathbf{1} \end{bmatrix}, \tag{A.14}$$

$$\{\gamma^i\} = \boldsymbol{\gamma} = \begin{bmatrix} 0 & \boldsymbol{\sigma} \\ -\boldsymbol{\sigma} & 0 \end{bmatrix}, \quad \gamma_5 = \gamma^5 = \begin{bmatrix} 0 & \mathbf{1} \\ \mathbf{1} & 0 \end{bmatrix}, \tag{A.15}$$

where

$$\boldsymbol{\sigma}^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{\sigma}^2 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \quad \boldsymbol{\sigma}^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \tag{A.16}$$

are the familiar Pauli matrices and

$$\mathbf{1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

is the 2×2 unit matrix.

The spinors u and v satisfy the Dirac equation,

$$(\not{p} - m)u(p, s) = 0, \tag{A.17}$$

$$(\not{p} + m)v(p, s) = 0. \tag{A.18}$$

The normalization of spinors is

$$\bar{u}(p, s)u(p, s) = 2m, \tag{A.19}$$

$$\bar{v}(p, s)v(p, s) = -2m, \tag{A.20}$$

and the completeness relation is

$$\sum_s u(p, s)\bar{u}(p, s) = \not{p} + m, \tag{A.21}$$

$$\sum_s v(p, s)\bar{v}(p, s) = \not{p} - m. \tag{A.22}$$

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A.3 Currents

Vector:

$$J_\mu(x) = \bar{\Psi}(x)\gamma_\mu\Psi(x) = \Psi(x)^+\gamma_0\gamma_\mu\Psi(x). \tag{A.23}$$

Axial:

$$J_{\mu5}(x) = \bar{\Psi}(x)\gamma_\mu\gamma_5\Psi(x). \tag{A.24}$$

Decompositions of the currents or products of them are very useful. Let $\ell_\mu = p_\mu + p'_\mu$ and $q_\mu = p'_\mu - p_\mu$, then

$$\bar{u}(p')\gamma^\mu u(p) = \frac{1}{2m}\bar{u}(p')(\ell^\mu + \mathrm{i}\sigma^{\mu\nu}q_\nu)u(p), \tag{A.25}$$

$$\bar{u}(p')\gamma^\mu\gamma_5 u(p) = \frac{1}{2m}\bar{u}(p')(\gamma_5 q^\mu + \mathrm{i}\gamma_5\sigma^{\mu\nu}\ell_\nu)u(p), \tag{A.26}$$

$$\bar{u}(p')\mathrm{i}\sigma^{\mu\nu}\ell_\nu u(p) = -\bar{u}(p')q^\mu u(p), \tag{A.27}$$

$$\bar{u}(p')\mathrm{i}\sigma^{\mu\nu}q_\nu u(p) = \bar{u}(p')(2m\gamma^\mu - \ell^\mu)u(p). \tag{A.28}$$

Additional identities can be found in Appendix A of the article by M. Nowakowski, E. Paschos and J. M. Rodriguez (*Eur. J. Phys*. **26**, 545–560, 2005) and in Appendix C of this book.

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