PHYSICAL INTERACTIONS BETWEEN STARS

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ABSTRACT. Physical arguments are presented to show that two-body, tidal-capture binaries should form in abundance ($N_b \sim 10^3$) during the evolution of globular clusters by the time that core collapse begins. Interactions amongst these binaries and with core single stars will cause ejections from the cluster which pump energy into the system producing a bounce and re-expansion. Detailed numerical Fokker-Planck evolutionary calculations presented here confirm this scenario and indicate that this process is likely to be the dominant energy input for most clusters. During the re-expansion phase $r_{core} \propto t^{1/3}$, $r_{half} \propto t^{2/3}$ with the core containing several hundred very close binary star systems.

1. INTRODUCTION

The mathematical treatment of globular cluster evolution has almost always idealized the interacting stars, treating them as point masses. Most of the work presented at this conference has also adopted this usually excellent approximation. The purpose of this paper is to show that, at critical stages in the life of a cluster, finite size effects will have an important, even dominant, influence on its subsequent evolution. This possibility has been noted by several authors, including: Lightman and Fall (1978), Milgrom and Shapiro (1978), Alexander and Budding (1979), Ozernoy and Dokuchaev (1982a,b), Inagaki (1984a,b), Krolik (1983) and Stodolkiewicz (1985). Here I will review the physical situation and then present the first detailed model evolution (through and past core collapse) for a stellar system which is forming and ejecting binaries produced by the two-body tidal capture process. It appears from this work that such physical interactions are the primary mechanism for arresting and reversing core collapse and also for driving the late asymptotic expansion phase.

To develop the physical framework required to understand the problem, we will consider a simple one-component system and begin by comparing three time scales in the central regions of the cluster. The inverse relaxation time ($\nu_r \equiv t_r^{-1}$) is conventionally

$$\nu_r \equiv 15.4\ G^2 m_*^2 n \ell n \Lambda / v^3 \quad , \tag{1}$$

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where v is the three-dimensional local velocity dispersion, (m_*, n) the stellar mass and local number density and Λ typically approximated by 0.4 N. All quantities, including N, may be thought of as locally defined; e.g., N ≡ N(r) the number of particles interior to r. Close two-body encounters locally occur at a rate $\nu_2 = n\sigma v$ where the cross section can be written in terms of minimum orbital separation R_{min} as

$$\sigma = \pi\left(R_{min}^2 + R_{min}\frac{2Gm_*}{u^2}\right) \approx \pi R_{min}R_*\left(\frac{v^*}{u}\right)^2 \quad , \tag{2}$$

where v_* is the escape velocity from a star ($v_*^2 \equiv 2Gm_*/R_*$) and u the relative velocity. For physical (including tidal) encounters we will find that, as expected, R_{min} cannot be much larger than R_*, so we write $R_{min} \equiv aR_*$ and note the weak dependence of a on v. Thus we have

$$\sigma = \pi a(v)R_*^2(v_*/v)^2, \text{ where } \qquad a(v) \equiv 0.54(v_*/v)^{0.2}; \tag{3}$$

giving

$$\nu_2 = 16.1\ naR_*^2\left(\frac{v_*}{v}\right)^2 v \quad , \tag{4}$$

where integration over an assumed local Maxwellian has been performed ($2\langle u^2\rangle = v^2$) and, for a(v) in equation (3), we have used the calculations of Press and Teukolsky (1977) for tidal capture of n = 3 polytropes. Finally, we adopt (from Hut, 1985) for the rate of production of three-body binaries hard enough to survive

$$\nu_3 = 126\ n^2G^5m_*^5/v^9 \quad . \tag{5}$$

Let us first compare the rates of relaxation and two-body capture from equations (1) and (4):

$$\frac{\nu_r}{\nu_2} = 0.24\,\frac{\ln\Lambda}{a}\left(\frac{v_*}{v}\right)^2 \quad . \tag{6}$$

Since (v_*/v) is large in normal globular clusters and the other numerical factors together are close to unity, we would not normally expect two-body processes to be important. As an example, if we adopt the parameters (v_* = 600 km/s, v = 20 km/s, $N_0 = 10^{5.0}$) we find that (ν_r/ν_2) = 1,200 and two-body processes should be utterly negligible in importance. However, as core collapse proceeds, the central velocity dispersion increases, $v^2 \propto \rho^{0.10}$ (Cohn, 1980), the number of stars in the core decreases, $N_c \propto \rho^{-0.35}$ leading to a decrease in $\ln\Lambda$ and, most important, the evolution becomes slower and slower per relaxation time as isothermality becomes better in the central regions with, at late times, $\nu_{r,eff} \equiv (\dot{\rho}/\rho) = 10^{-2.5}\ \nu_r$ (Cohn, 1980). Thus, by the time the central density has increased by six orders of magnitude as core collapse approaches, the velocity dispersion has increased by a factor of 2, and the number of stars in the core decreased to 800, giving ($\nu_{r,eff}/\nu_2$) = 1. Well beyond this point the two-body processes will be more important than the heat flow induced by relaxation, so energy input from binaries should be able to reverse the collapse. This is close to the point in our numerical computation (cf. Fig. 1) when re-expansion does in fact, begin.

Second, we may compare the two-body and three-body processes using equations (4) and (5):

$$\frac{\nu_3}{\nu_2} = \frac{0.4}{aN^2}\left(\frac{v_*}{v}\right)^2 \quad . \tag{7}$$

This shows that the three-body processes will not be able to compete with tidal capture, until N_c is quite small ($N_c < 0.7 \times (v_*/v)^{0.9}$, and the core has reached a correspondingly large central density. Another way to express this point is to note that $(\nu_3/\nu_r) = (3.7/N^2 \ln\Lambda)$ so that very few stars (perhaps only a few binaries) will be left in the system when three-body processes become significant regardless of the internal structure of the stars. As a result, the total number of binaries formed by the three-body process will be much smaller than the number of two-body binaries already present. Of course, if most of the stars were "hard" enough such as degenerate dwarfs or neutron stars then, even when the core was reduced to one binary, finite size effects would not have become important. But for normal stars, and stellar systems such as globular clusters or galactic nuclei it appears that two-body processes will almost always be more important than three-body collisions among single stars. We will make this argument more quantitative in the next section.

After tidally captured binaries form, they will affect the system in two ways. First they may interact with singles or other binaries in close encounters. In these an exchange of stars often occurs, and then typically the reaction products leave the encounter at velocities of order the orbital velocity ($\Delta v \approx 1/4\ v_* \approx 10^2$ km/s) which will cause ejection from the cluster. To evaluate the effect on the cluster one need only consider the reverse process, bringing mass Δm in from infinity to the cluster center where it is given a velocity $v_0{}^2$. The energy release is obviously $-(+\phi_0 + 1/2\ v_0{}^2)\Delta m$. The energy input to the cluster caused by mass Δm leaving is, of course, the negative of this and it is distributed according to the integral, $G\Delta m\int dm/r$, which puts equal amounts of energy in equal logarithmic intervals of radius for an isothermal cluster. As a consequence, this form of "heating" is not centrally concentrated but deposited typically far from the core. In addition, more distant encounters of stars with binaries will deposit a comparable amount of energy directly into the central regions by the processes investigated by Spitzer and Mathieu (1980). In the detailed integrations to be reported on shortly, we have neglected this process and we therefore underestimate the importance of the two-body binaries. Finally we note that, ultimately, we can expect an equilibrium density of binaries to be established since, if the density is low, then the ejection rate by the dominant binary-binary interactions is slow but the formation rate by single-single interactions is rapid so the binary number density will increase (and vice versa). We can expect the post-collapse expansion, if one exists, to be similar to that found by Goodman (1984) with the half mass radius increasing as $t^{2/3}$, since as noted in that work and by others (Henon, 1961; Lynden-Bell and Eggleton, 1980), the overall evolution is insensitive to the details of the energy generation process in the post-collapse era.

2. AN IDEALIZED EVOLUTION UNDER THE INFLUENCE OF TIDAL CAPTURE BINARIES

The calculations were performed by T. Statler who has extensively modified the code developed by H. Cohn (1980) for orbit averaged Fokker-Planck evolution to include the various binary rates. Details of the work will be published elsewhere (Statler, Ostriker and Cohn, 1984). The only significant modification made to Cohn's treatment of collisions was to take $\Lambda = 0.4\ N_{core}$ rather than keeping it constant throughout time. For binary creation we use the orbit averaged cross sections calculated by Press and Teukolsky (1977). All binaries are taken to be at the average separation $\langle R_{sep}\rangle = 2.5\ R_*$ and no allowance is made for mergers. The inelasticity of the sticking collisions is specifically included. Energy release during single-binary encounters is calculated via a modified form of the Hut (1984) cross sections.

$$\frac{d\sigma}{d\Delta} = \frac{135}{8}\left(\frac{v_*}{u}\right)^2 \pi R_*^2 (1 + \Delta)^{-4}\Delta^{-0.5} \quad , \tag{8}$$

where $\Delta \equiv |\Delta E/E_{binary}|$. We note how much larger the numerical factor is in this cross section than in the binary formation cross section given by equation (4). In binary-binary interactions Mikkola (1983) is adopted for close encounters ($\Delta > 1$), and a further modified version of the Hut cross sections taken for the distant collisions. After each encounter we check the probability of ejection of any of the components entering into the interaction (sharing the energy released in accordance with momentum conservation). If a star, single or binary, can escape, it is removed from the distribution function but if it cannot, then its energy is not changed (i.e., we ignore heating) and the interaction is ignored. This is done in the "Fokker-Planck step" of the integration. Then, keeping the particle density conserved (in the appropriate action-action space), Poisson's equation is solved iteratively until self-consistency is reached with the new distribution function. In this step the potential change due to the mass lost in the F-P step by ejected stars causes all exterior stars to lose energy and their orbits to expand.

The temporal evolution is presented in the accompanying figures where the dimensionless units are for $[R] = r_0$, $[t] = t_{rh,0}$, $[\rho] = (M/r_0^3)_0$, $[v^2] = (GM/r_0)_0$ where M and r_0 are the mass and scale radius of the initial Plummer model. Physical units displayed assume the initial model to have $r_0 = 1.13$ pc and to contain 3×10^5 stars, each of mass $m_* = 0.7\ M_\odot$ and radius $R_* = 0.57\ R_\odot$. Table I summarizes the physical quantities at various epochs (labeled by arrows in Figure 1). The core radius is defined in the conventional way for isothermal spheres $[4\pi G\rho(0) \equiv 3v^2(0)/r_c^2]$, the core radius used in photometric discussions being three times larger; r_h is the half mass radius, $\sigma_s(0)$ the central one-component velocity dispersion of singles. N_b is the current number of binaries, T_b their mean age at that time and ν_{bf} the current rate of binary formation.

In Figure 1 we see that the central density follows the pre-core collapse evolution nearly identical to that without binaries reaching very high values briefly at $t = 3.3 \times 10^9$ years. The time to peak den-

Table I

Model Cluster at Four Epochs

Epoch:	Initial	Early	Peak	Late
$t/t_{rh,0}$	0	7.84	14.68	76.4
t(yr)	0	1.76(9)	3.303(9)	1.72(10)
r_c(pc)	.799	.295	2.03(-3)	1.98(-2)
r_h(pc)	1.47	1.56	2.16	8.38
$\sigma_s(0)$(km/s)	11.6	11.6	17.3	5.8
N_b	0	335	1462	370
T_b(yr)	0	7.36(8)	8.12(8)	1.19(9)
ν_{bf}(binaries/yr)	1.07(-7)	3.05(-7)	9.29(-6)	2.91(-7)

sity in Cohn's (1980) work was found to be t_c = 15.8 $t_{rh,0}$; binaries accelerate the collapse at late times and peak density is reached at t_c = 14.7 $t_{rh,0}$ in the present calculations. Subsequently the central density decays as the 4/3 power of the time. The central velocity (Fig. 2) is nearly constant until nearly the peak when it climbs briefly to 18 km/s (from an initial value of 12 km/s) and subsequently decays as $t^{-1/3}$. The core and half mass radii behave (Fig. 3) in expected ways prior to maximum concentration and then expands as $t^{1/3}$ and $t^{2/3}$, respectively, in the post-collapse phase. Thus the post-collapse phase is not truly homologous and, although all radii expand, the cluster becomes more and more centrally concentrated.

The post-collapse density distributions (4a,b) show the buildup of an extensive and expanding nearly isothermal structure with, of course, $\rho_b = \rho_{bo}(\rho_s/\rho_{so})^2$; the binaries remaining primarily in the center even as some diffuse into the cluster halo.

We have treated the cluster as an isolated system. Had we taken a tidal cutoff at R_t corresponding to a fixed mean density, then we would have found the mass decreasing linearly (cf. Stodolkiewicz, 1985) at late times as core energy input pushed the expanding cluster beyond the tidal limit.

Figure 5 shows the rates of formation and ejection of stars via various processes. Note that the primary process, binary formation increases only gradually in the precollapse phase and then decreases approximately as $t^{-1.2}$. During and after core collapse the binary-binary ejection process is most important and the three-body binary creation rate (dot dash curve) is never competitively important. The total number of binaries formed during the run is approximately 20,000 (20% of mass) of which 2,000 (2% of mass) had formed prior to the moment of maximum density. However, the lifetime of binaries once made is relatively short, so the number currently in the cluster declines from 2,000 at core collapse to 200 at the end of the numerical evolution. The self-similar nature of the central regions in the post-collapse epoch is evident in the central ratio of binaries to singles which is constant at (2/1) by mass throughout the late phases.

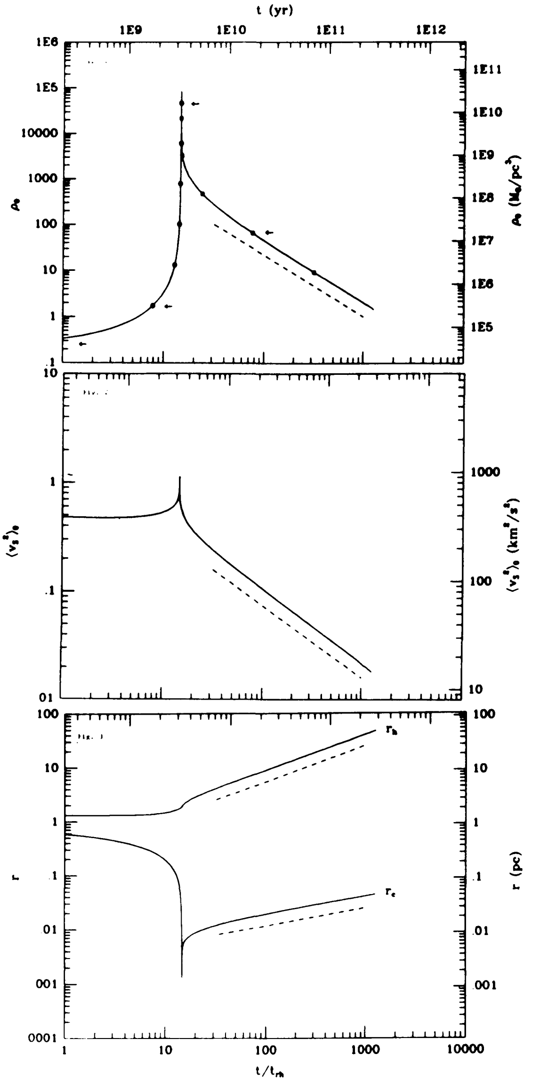
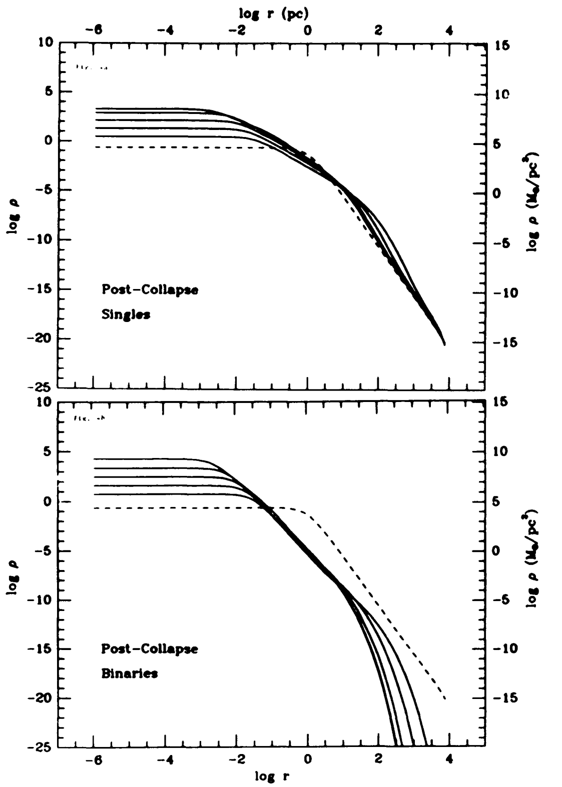


Figure 1. Cluster central density vs time in physical and dimensionless units (cf. §2 for definitions). Arrows mark "early" peak and "late" epochs referred to in Table I. Dashed line shows $\rho_0 \propto t^{-4/3}$ found from analytical treatment eq. (15a).

Figure 2. Central velocity dispersion (squared) of single stars vs time. To high accuracy the binaries always have $\langle v_{b0}{}^2 \rangle = 1/2 \langle v_{s0}{}^2 \rangle$. Dashed line shows $v^2 \propto t^{-2/3}$ dependence of equation (14a).

Figure 3. Half mass radius r_h and core radius r_c vs time with analytical estimates, ($r_h \propto t^{2/3}$, $r_c \propto t^{1/3}$) from equations (14b) and (15b) shown as dashed lines.

Although the calculation is not intended to be realistic in that the cluster tidal radius, a spectrum of masses and other important features have been knowingly neglected; it is of interest to check on the mean age of the binaries as shown in Fig. 6. There we see that up to a time of t = 1.5 × 10^{10} yrs the mean age of the binaries has not exceeded 9 × 10^8 years. Thus gravitational radiation, which would cause the stars to merge into singles, would probably not have been effective. If, however, as seems not unlikely, other angular momentum loss processes are much more efficient than gravitational radiation, then at late stages the evolution may be significantly different than what we have found. However, during the important bounce and re-expansion phases the binaries are typically quite young so our treatment should be valid. Other calculations not presented here indicate that neither angular momentum loss nor other causes of stellar fusion are likely to significantly alter the qualitative features of the numerical evolution shown in Figures 1-6 for an isolated cluster.



Figures 4a,b. Post-collapse density distributions of singles (a) and binaries (b) shown at various times with the five solid lines corresponding to the five square symbols in Fig. 1. The initial Plummer model is shown as a dashed line.

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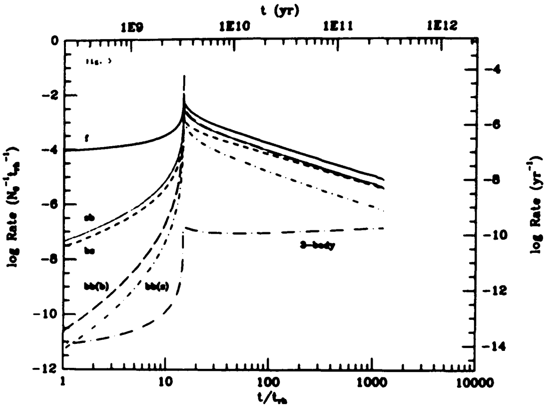

Figure 5. Rates (integrated over the whole cluster) for various processes. Here f designates two-body tidal-capture binary formation rate; sb and bs are the collision rates between binaries and singles that eject singles and binaries, respectively, and bb(b), bb(s) the collision rates between binaries that eject binaries or singles. The rate of formation of binaries by three-body interactions among single stars is always relatively unimportant.

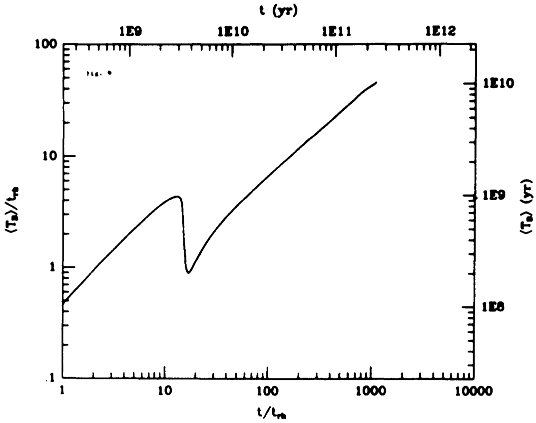

Figure 6. The mean age of the binaries in the cluster at any time vs the time itself. For comparison the timescale for gravitational radiation to bring the stars together would be about $10^{9.3}$ years.

3. ANALYTICAL ANALYSIS OF CLUSTER EVOLUTION

The number of binaries increases only gradually, approximately giving

$$N_b = 10^{1.5} \left(\frac{v_0}{v_*}\right)^{1.8} \frac{N_0}{\ell n \Lambda_0} \left[1 - \left(\frac{\Delta t}{t_c}\right)^{0.29}\right] , \quad (9)$$

as collapse is approached $\Delta t \equiv t_c - t$). During the collapse there is a significant overshoot and, although Figure 5 shows that three-body encounters among the single stars are never important, physical collisions and three-body encounters among binaries would be important--and would presumably lead to core bounce at a somewhat lower density than the one calculated. Now we can check more quantitatively the circumstances under which three-body binaries would be more important than capture binaries. The number of three-body binaries formed in the absence of the two-body processes up to the instant of bounce is approximately $10^{1.5}$ (Hut, Goodman and Cohn, 1984). This implies that if $(N/\ln 0.4N) > (v_*/v_0)^{1.8}$ or, approximately, $N > 1.6 \times 10^4 \, v_{0,10}^{-1.8}$, then two-body processes will dominate even at maximum core density. This criterion is satisfied for most clusters.

After core bounce the inner and outer cluster regions are increasingly decoupled. The core evolves on the two-body collisional time scale which implies that

$$\frac{1}{t} = \text{const} \times \nu_2 \approx \text{const} \times n_0 R_*^2 v_*^2 v^{-1} \, . \qquad (10)$$

In addition, equilibrium within the core requires that

$$G m_* n_0 = \text{const} \times v_0^2 r_c^{-2} \, . \qquad (11)$$

Now let us consider the outer parts. At the half-mass radius the conductive luminosity must balance energy input

$$\text{const} \times v \ln\Lambda \frac{dm_* v^2}{dr} = \frac{d}{dt}\left(Mv^2\right) \, , \qquad (12)$$

so that the evolutionary time scale in the outer parts is set by the requirement

$$\frac{1}{t} = \text{const} \times \frac{v m_* \ln\Lambda}{M R_h} = \text{const} \times \frac{m_* v^3 \ln\Lambda}{GM^2} \, . \qquad (13)$$

Thus, if we can treat $(t\dot{M}/M)$ as a small quantity (a point we shall come back to), then equation (13) tells us that

$$v = K_1 \left(\frac{G m_*}{\ln\Lambda}\right)^{1/3} N^{2/3} \, t^{-1/3} \quad ; \qquad (14a)$$

and

$$R_h = K_2 \, [G m_* (\ln\Lambda)^2]^{1/3} N^{-1/3} t^{2/3} \quad . \qquad (14b)$$

Where K_1, K_2 are numerical constants to be determined and we have used the virial theorem $(G m_* N/R_h) \simeq 2.5 \, v^2$. Then, with $v(t)$ known, and assuming that (v_0/v) = const, we have, from equations (10) and (11)

$$n_0 = K_3 \, v_*^{+2} [N^2/(G^5 m_*^5 \ln\Lambda)]^{1/3} \, t^{-4/3} \; ; \qquad (15a)$$

and

$$r_c = K_4 \, v_*^{-1} (\ln\Lambda)^{-1/6} (G^2 m_*^2 N)^{1/3} \, t^{1/3} \quad . \qquad (15b)$$

As shown by the dashed lines in Figures 1-3, the power law dependencies derived here fit our numerical results. A rough quantitative fit is obtained if K_i = (0.74, 2.0, 0.22, 0.52). Then, given these dependencies and the requirement that the mass loss from the center times the central potential equal the energy input into the cluster (const × Mv^2/t), one can show (cf. Statler et al.,1984) that ϕ_0 will be close to that in a truncated isothermal cluster and that $M = \text{const} \times [1 + \alpha \ln (t/t_0)]^{-\beta}$ where α and β positive are constants of order unity, the exact value of which depends on ϕ_0/v_0^2). Thus $(t\dot{M}/M) << 1$ as required.

4. DISCUSSION AND OBSERVATIONAL CONSEQUENCES

Many scientists over the years have argued that a significant number of globular clusters must have reached and passed through the stage of "core collapse". From the recently developed theoretical arguments referred to earlier in this paper, these clusters should look like very concentrated isothermal spheres, and there is some evidence (Djorgovski and King, 1984; Djorgovski, 1984) that such objects have indeed been found. The calculations presented here indicate that hundreds of close two-body binaries should be found in the inner parts of such systems concentrated roughly as the square of the surface light intensity). These binaries should be divided into comparable numbers of normal-normal binaries (W Ursa Majoris stars) and normal-degenerate binaries (cataclysmic variables). These last should be seen as soft X-ray sources (cf. Grindlay, 1984) and, of course, since neutron star remnants will be $\sim 10^{-2}$ degenerate remnants, thus of order one X-ray binary per cluster might be anticipated. This last remark of course cannot be taken as a "prediction" since it was the overabundance of X-ray binaries in globular clusters which led Fabian, Pringle and Rees (1975) originally to propose the mechanism of tidal capture in its modern guise.

Finally we note that, if one drops the limitation of isolation, and instead assumes a tidal limit such that $\rho = 0$ outside the radius at which $[3M(R)/4\pi R^3] = \rho_{tidal}$ = const, then we expect the cluster mass to go linearly to zero on the half-mass relaxation time scale. The investigation of this process, which would lead to the total destruction of the cluster, is left to subsequent calculations (Statler et al., 1984).

5. ACKNOWLEDGEMENTS

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REFERENCES

Alexander, M. E. and Budding, E. 1979, Astron. & Astroph., 73, 227.
Cohn, H. 1980, Ap. J. 242, 765.
Djorgovski, S. 1985 (this conference).
Djorgovski, S. and King, I. R. 1984, Ap. J. (Lett.), 277, L49.
Dokuchaev, V. I. and Ozernoy, L. M. 1982, Astron. & Astroph., 111, 16.
Fabian, A. C., Pringle, J. E. and Rees, M. 1975, Monthly Notices, Roy. Astron. Soc., 172, 15P.
Goodman, J. 1984, Ap. J., 280, 298.
Grindlay, J. 1985 (this conference).
Henon, M. 1961, Ann. Astrophy., 24, 369.
Hut, P. 1984, Ap. J. (Suppl.), 55, 301.
_______ 1985 (this conference).
Hut, P., Goodman, J. J., and Cohn, H. 1984, in preparation.
Inagaki, S. 1984a, M.N.R.A.S., in press.
Inagaki, S. 1984b, preprint.
Krolik, J. H. 1983, Nature, 305, 506.
Lightman, A. P. and Fall, S. M. 1978, Ap. J. 221, 507.
Lynden-Bell, D. L. and Eggleton, P. 1980, Monthly Notices, Roy. Astron. Soc., 191, 483.
Mikkola, S. 1983, Monthly Notices, Roy. Astron. Soc., 203, 1107.
Milgrom, M. and Shapiro, S. L. 1978, Ap. J. 223, 991.
Ozernoy, L. M. and Dokuchaev, V. I. 1982, Astron. & Astroph., 111, 1.
Press, W. H. and Teukolsky, S. A. 1977, Ap. J. 213, 183.
Spitzer, L. and Mathieu, R. D. 1980, Ap. J. 241, 618.
Statler, T. S., Ostriker, J. P., and Cohn, H. 1984, in preparation.
Stodolkiewicz, J. S. 1985 (this conference).

DISCUSSION

HUT: I am very glad to see the first detailed Fokker-Planck treatment of the global effects of dissipative encounters on the evolution of globular clusters. I have two remarks. First, I would recommend to include wide encounters with single stars in your calculations. Although these events change the binding energy of the binary only by a relatively small amount, this change can nevertheless be significant with respect to the much smaller field star energies. Weak encounters with hard binaries can give off ∿10 kT, which could turn out to be significant. Second, a treatment of close encounters will be very difficult, since resonant scattering will result in near-central collisions (my numerical orbit calculations show that in at least half of the resonant scattering cases, the centers of two of the three stars approach each other to within ten percent of their radii). Therefore, I expect that close encounters will produce heavy stars by merging (possibly repeatedly); such a heavy star will evolve quickly, and eventually shed its mass which can escape out of the shallow potential well of a globular cluster.

OSTRIKER: I agree with your remarks and since any process which leads to mass loss from the cluster has the same effect on the overall evolution as any other a striking collision perhaps can be neglected.

COHN: I have a question addressed to the observers - what limits do we have on the presence of binaries in the *cores* of globular clusters?

PRYOR: There is no evidence in the cusps, where the stars are unresolved. Our and Griffin & Gunn's observations of the radial velocities of ∿100 M3 giants show very little evidence for binaries. If the distribution of periods of the binaries when their components were on the main sequence was that of Abt & Levy, <20% of the stars were in binaries then. Because only giants are observed, there is no direct evidence on the abundance of binaries with separations less than ∿0.3AU.

SHARA: White dwarf - main sequence star physical collisions set up very strong shock waves in the non-degenerate star, heating much of that star to $\sim 10^8$K. CNO cycle hydrogen burning then produces ∿10 times the main sequence star binding energy in one main sequence star expansion timescale; thus the main sequence star must be totally disrupted. The energy liberated into gas motions (and light?) is supernova-like ($\sim 10^{50}$ erg) and should very effectively sweep globular clusters clear of accumulated gas. Details can be found in Shara & Shaviv, MNRAS 183, 687 (1978).

LARSON: What is the role of red giants and horizontal-branch stars in close interactions? Although they are much less numerous than main-sequence stars, they are also much bigger, and my simple arithmetic suggests that they should be at least as important.

OSTRIKER: If one adopted a simple scaling of the cross sections you would certainly be correct. However, red giant (and to a less extent horizontal branch stars) have relatively weakly bound envelopes which could not absorb sufficient energy to catch stars in a dense globular cluster. We, conservatively, neglect the capture by evolved stars altogether.

SPITZER: In principle the ejection of hard tidal-capture binaries could be averted if the two stars approached each other sufficiently rapidly. Could you comment on the rates of the different processes that will bring the two stars closer together, particularly gravitational radiation and effects of a common gaseous envelope? How do these rates compare with the rate of ejection of hard binaries?

OSTRIKER: Detailed curves in the accompanying paper show that in some cases gravitational radiation can be an important process.

GRINDLAY: Let me comment, first, that I agree with your assertions that two body tidal capture, not exchange collisions, is the dominant mode of formation of the observed X-ray binaries in globular clusters and, second, that the production of tidal-capture binaries in globulars *is* important for the cluster evolution (as evidenced by the observed central cusps and your work, which suggests these may be dominated by binaries). My question is: if the central cusps are indeed dominated by tidal-capture binaries and these are largely main sequence-main sequence or main sequence-giant systems, would you expect them to be bluer than the surrounding cluster core due to the formation of "blue stragglers?" We (Hertz and Grindlay 1984, *Ap.J.*, in preparation) have shown that the central cusp in NGC6624 is in fact bluer than the surrounding core and have speculated that this process may be responsible.

KING: (1) Are these two-body-capture binaries important for the late evolution of the cluster, or do they kick themselves out immediately? (2) In making X-ray sources from them, do you require an exchange encounter with a neutron star, in order to make an ordinary pair into something capable of making X-rays?

OSTRIKER: (1) The two-body binaries are important *because* they are kicked out fairly promptly. This puts energy into the remaining stars and causes re-expansion. (2) Exchange reactions can occur but neutron stars can capture normal stars as well.

LIGHTMAN: Isn't it true that once a small number of physical collisions and coalescences have occurred in a core of two-body-formed binaries, there will be a tendency for a single massive star to grow (by coalescence) very rapidly at the expense of the others - both because of its increased size (increased cross section) and its increased mass (increased gravitational focussing)? Because close encounters with two-body-formed binaries have impact parameters of order the stellar radius, the energy released in encounters may be dissipated through the coalescence process rather than put into recoil that can eject the binary. Thus, we may have to face the issue of a growing massive star in the core, involving a number of uncertainties.

OSTRIKER: I agree but massive stars will have a relatively short lifetime before exploding. Thus the net outcome of mass loss from the cluster is the same and the evolution should be the same. We are doing other calculations to test this point.

WEBBINK: In the picture you have outlined, it is clearly the evaporation of hard binaries from the cluster core in energetic encounters with a third star which is the stabilizing mechanism against cluster core collapse. The question I have (for you or Piet Hut) is whether these energetic encounters necessarily require the third star passing within the binary separation; and wouldn't one then expect three-body collisions (given that the separation is only $\sim 3R_*$) rather than exothermic (hardening) encounters in most cases?

OSTRIKER: Yes, the typical collision is close and involves binaries and singles (binary-binary encounters are also quite important). In any case after a complicated interaction stars are ejected from the tightly interacting subsystem with velocities comparable to the initial binary orbital velocities. These first reaction products will tend to be ejected from the cluster.

KING: Regarding seeing binaries in a central cusp, I think that a post-collapse cusp must be so small that it would be quite hopeless to see into it.

OSTRIKER: Yes, I agree but if it contains special stars, say cataclysmic variables in abundance, then these might leave a signature on the observed spectrum of the central parts.