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ANGULAR-MOMEMTUM TRANSFER BY NONRADIAL OSCILLATIONS IN MASSIVE MAIN-SEQUENCE STARS

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Angular mementum distribution is one of the most important factors for stellar structutre and evolution. Among other mechanisms, angular momentum is transfered by non-axisymmetric oscillations (nonradial oscillations). In this mechanism the angular momentum is carried mainly by the Reynolds stress, which is proportional to the product between radial and azimuthal components of oscillation velocity; i.e., $v_r'v_\phi'$ (ϕ direction is the direction of rotation velocity). In the linear oscillation analysis, the phase difference between v_r' and v_ϕ' is $\pi/2+\delta$ with $|\delta| << 1$. A finite value of δ, which arises from excitation or damping of the oscillation, makes the time average of $v_r'v_\phi'$ finite. Positive angular momentum is transfered from the driving zone to the damping zone by a prograde mode (Osaki 1986).

Lee & Saio (1986, 1987) and Lee (1988) have shown that low-frequency oscillations are excited in rotating massive main-sequence stars, and proposed that the oscillations thus excited are responsible for the light and line-profile variations observed in OB (Be) stars. In the present paper, we discuss the efficiency of angular momentum transfer by such low-frequency oscillations. The oscillation is generated by oscillatory convection (inertial oscillation) with negative energy in the convective core resonantly coupled with a g mode in the envelope (Lee & Saio 1990). They are prograde, although the phase velocity is very low, with respect to the core rotation. The oscillation energy flows from the core to the envelope, and hence positive angular momentum is carried from the core to the envelope, if the angular velocity of rotation of the core is equal to or faster than that of the envelope.

Let us begin with the angular momentum conservation equation given by

$$\frac{\partial(\rho\varpi v_\phi)}{\partial t}+\nabla\cdot(\rho\varpi v_\phi \mathbf{v})=-\frac{\partial p}{\partial\phi}-\rho\frac{\partial\Phi}{\partial\phi}, \tag{1}$$

where $\varpi=r\sin\theta$ and Φ is the gravitational potential. When we decompose each physical quantity of a rotating star into the axisymmetric and non-axisymmetric parts, the velocity and the density, for example, are written as

$$(v_r,v_\theta,v_\phi)=(0,0,\Omega r\sin\theta)+(v_r',v_\theta',v_\phi') \quad \text{and} \quad \rho=\bar\rho+\rho', \tag{2}$$

where the bar ($\bar{\ }$) denotes the (zonally averaged) axisymmetric part and the prime ($'$) the non-axisymmetric part ($\overline{f'}=0$). Then, taking the zonal average of equation (1) and integrating over the solid angle, we obtain

$$\frac{\partial j(r,t)}{\partial t}=-\frac{1}{r^2}\frac{\partial(r^2\Psi)}{\partial r}-\left\langle\rho'\frac{\partial\Phi'}{\partial\phi}\right\rangle-\frac{\partial\langle\varpi\overline{\rho'\upsilon'_\phi}\rangle}{\partial t}, \tag{3}$$

where the angular bracket means the integration of the enclosed quantity over the whole solid angle, and

$$j(r,t)\equiv\langle\omega\bar{\rho}\bar{\upsilon}_\phi\rangle, \quad \text{and} \quad \Psi\equiv\langle\varpi(\bar{\rho}\overline{\upsilon'_\phi\upsilon'_r}+\bar{\upsilon}_\phi\overline{\rho'\upsilon'_r})\rangle. \tag{4}$$

Usually, the second and third terms in the right hand side of equation (3) have only small contributions if the tidal effects play no important role. Neglecting these terms, we have

$$\dot{J}(r,t)\equiv\int_0^r 4\pi r^2\frac{\partial j(r,t)}{\partial t}dr\cong-4\pi r^2\Psi, \tag{5}$$

which represents the change in the equilibrium angular momentum within the sphere of the radius r. We assume that the non-axisymmetric parts of the physical quantities are caused by non-radial oscillations with $m\neq 0$. Then, equation (3) describe the angular momentum transfer by non-axisymmetric waves. We also assume that the functional form of the amplitude of non-axisymmetric oscillations can be represented by eigenfunctions for linear wave equations. Then, knowing that the Eulerian velocity perturbation generated by a non-radial oscillation is related to the displacement vector $\boldsymbol{\xi}(r,\theta,\phi,t)$ as

$$\boldsymbol{\upsilon}'=i(\sigma+m\Omega)\boldsymbol{\xi}-r\sin\theta(\boldsymbol{\xi}\cdot\nabla\Omega)\mathbf{e}_\phi, \tag{6}$$

we can evaluate the right-hand-side of equation (3) by using the eigenfunctions of the non-radial oscillation. As in a previous study (e.g., Lee & Saio 1986), we employ the series expansion in terms of the spherical harmonic functions to represent the non-radial oscillation of rotating stars. For example, the radial component of the displacement vector is given as

$$\frac{\xi_r}{r}=\sum_{\ell\geq|m|}^{\infty}S_\ell(r)Y_\ell^m(\theta,\phi)e^{i\sigma t}, \tag{7}$$

where σ is the oscillation frequency in an inertial frame, $\ell=|m|+2j-2$ for even modes and $\ell=|m|+2j-1$ for odd modes and $j=1,2,3,\cdots$.

The equilibrium model we employ is the $10M_\odot$ zero-age main-sequence model, for which $\log(L/L_\odot)=3.73$, $\log T_{\rm eff}=4.41$ and $R=3.71R_\odot$. The model has a convective core, in which $\nabla-\nabla_{\rm ad}=10^{-3}$ is assumed. The outer boundary of the convective core is located at $x\equiv r/R=0.235$. We assume the angular frequency of rotation depends only on the coordinate r, and that

$$\bar{\Omega}=\bar{\Omega}_s\left\{1+\frac{b-1}{1+\exp[a(x-x_c)]}\right\}, \tag{8}$$

where $\bar{\Omega}_s$ is the angular velocity of rotation at the surface, $a=20$, $x_c=0.235$, and a parameter $b(\geq 1)$ is to specify the differential rotation. (The frequencies $\bar{\sigma}$ and $\bar{\Omega}$ are dimensionless ones normalized by $\sqrt{GM/R^3}=2.78\times10^{-4}\,\mathrm{s}^{-1}$.) We show an example of angular momentum transfer by the B_1 mode of $m=-2$ (see Lee & Saio 1986), which is

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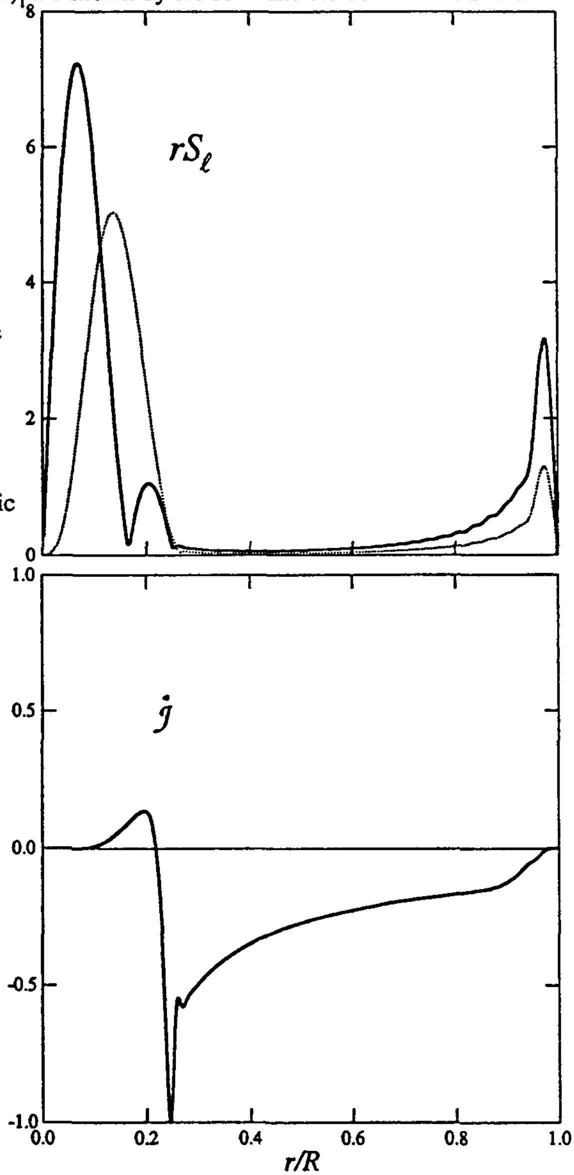
an even prograde wave excited by a resonance coupling between an oscillatory convective mode in the core and a gravity mode in the envelope. The oscillatory convective mode has negative energy of oscillation, while the gravity mode has positive energy of oscillation (Lee & Saio 1990). For $\overline{\Omega}_s = 0.162$ and $b = 1.12$, we calculate the B_1 mode by taking accout of the non-adiabatic effects. Its frequency is $(\overline{\sigma}_R, \overline{\sigma}_I) = (0.383, -2.97\times10^{-4})$. The eigenfunctions of $|rS_2(r)|$ and $|rS_4(r)|$ are shown by the solid and the dotted lines in the upper panel, and the qunatity $\dot{\jmath}(r,t)$ is given as a function of r/R in the lower panel, where $\dot{\jmath}(r,t)$ is normalized by its maximum value. As shown by the figure, $\dot{\jmath}(r,t)$ change its sign at the outer boundary of the core and is negative throughout the envelope, which means there is a net angular momentum flux from the core to the envelope. The rapid increase in $\dot{\jmath}(r,t)$ near the surface means that a large amount of angular momentum deposition to the stellar rotation occurs because of large non-adiabatic effects (radiative dissipation) there. Since the angular momentum density is low, the rotation velocity near the surface would be changed on a short time scale. The rotation law can be modified until the phase velocity becomes equal to the rotation velocity.



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