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Theoretical modeling of convection II. Reynolds Stress Model

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Abstract. The Reynolds Stress Model (RSM) yields the dynamic equations for the second-order moments (e.g., heat fluxes) needed in the equations for the mean variables (e.g., mean temperature). The RSM equations are in general time dependent and non-local. We first discuss the "buoyancy only" case and the tests of the non-local model against a variety of data. We also "plumenize" the model in order to exhibit the up-down flows that characterize convection so as to show that a non-local RSM is fully equipped to account for the "plume aspect" of buoyant flows. Next, we extend the RSM to account for stable and/or unstable stratification and shear, a formalism that is needed to describe the overshooting region contributed by differentail rotation. We conclude by discussing the equation for the dissipation of turbulent kinetic energy which plays a key role in any RSM.

Keywords. Convection, turbulence, stars: interiors

1. Reynolds stress model for pure buoyancy

At a mathematical conference toward the end of the twenties, the Russian mathematician A.A. Friedmann (the same of the Friedmann universe) suggested that if the NSE (Navier-Stokes Equations) yield the dynamical equations for the mean components (see Eqs. 2.2, 2.3 of Canuto 2007, Part 1), they also yield the dynamic equations for the fluctuations' correlations such as the Reynolds stresses, the turbulent heat flux, etc. The Reynolds Stress Model, RSM, could have been born then, but that was not the case. One had to wait until 1940 when the Chinese physicist P.Y.Chou published the first dynamical equations for the momentum Reynolds stresses. He treated primarily shear flows and the engineering community has since then used the RSM. The successes of the RSM in that field are well documented and there is no need to dwell on them. Suffices it to say that closure problems still exist especially concerning the pressure correlations, but the work of many groups has considerably narrowed the uncertainties.

In Canuto (1992), a detailed derivation of the RSM equations was presented. For the *buoyancy only case,* the second order moments of interest to stellar structure studies are the turbulent kinetic energy K, the heat flux $J = \overline{w\theta}$, the temperature variance $\overline{\theta^2}$ and the kinetic energy in the z-direction $1/2\,\overline{w^2}$ whose dynamic equations are (Canuto 1992; Canuto and Dubovikov 1998; Canuto *et al.* 2001, 2002):

$$\frac{\partial K}{\partial t} + \underbrace{\frac{\partial F_{\rm KE}}{\partial z}}_{\text{non-locality}} = g\alpha J - \varepsilon \tag{1.1}$$

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$$\frac{\partial J}{\partial t} + \underbrace{\frac{\partial \overline{w^2\theta}}{\partial z}}_{\text{non-locality}} = -\overline{w^2}T_z + (1-\gamma_1)g\alpha\overline{\theta^2} - \tau^{-1}\pi_4^{-1}J \tag{1.2}$$

$$\frac{\partial \overline{\theta^2}}{\partial t} + \underbrace{\frac{\partial \overline{w\theta^2}}{\partial z}}_{\text{non-locality}} = -2JT_z - 2\overline{\theta^2}\pi_5^{-1}\tau^{-1} \tag{1.3}$$

$$\frac{\partial}{\partial t}\overline{w^2} + \underbrace{\frac{\partial \overline{w^3}}{\partial z}}_{\text{non-locality}} = \frac{2}{3}(1+2\beta_5)g\alpha J - \frac{2}{3}\varepsilon - 5\tau^{-1}(\overline{w^2} - \frac{2K}{3}) \tag{1.4}$$

where $\tau = 2K/\varepsilon$ is the dynamical time scale. The equation for the dissipation ε is:

$$\frac{\partial \varepsilon}{\partial t} + \underbrace{\frac{\partial \overline{w\varepsilon}}{\partial z}}_{\text{non-locality}} = c_1 g\alpha J\tau^{-1} - c_2\varepsilon\tau^{-1} \tag{1.5}$$

where c_1=2.88, c_2=3.8, $\overline{w\varepsilon} = 3/2\tau^{-1}F_{\rm KE}$ with $F_{\rm KE} = \overline{wK}$ (see Sect. 6). The suggested values of the constants are $\beta_5 = 1/2$, $\gamma_1 = 1/3$, $\pi_4 = 0.084$, $\pi_5 = 0.72$. The first consideration is that equations (1.1)–(1.5) are linked together. To solve the equation for J, one needs to know the temperature variance $\overline{\theta^2}$ and $\overline{w^2}$ which are given by two other equations. In the stationary and local limit, Eqs. (1.1)–(1.5) become algebraic and the solution has the MLT form (Canuto and Dubovikov 1998). To carry out the next step, consider (1.2) and neglect the temperature variance $\overline{\theta^2}$ since in an unstably stratified regime, potential energy, which is proportional to $\overline{\theta^2}$, transforms into kinetic energy. We have:

$$J = -\pi_4\tau\overline{w^2}\frac{\partial T}{\partial z} - \pi_4\tau\frac{\partial}{\partial z}\overline{w^2\theta} = J_{\rm L} + J_{\rm NL} \tag{1.6}$$

where:

$$J_{\rm L} = -\pi_4\tau\overline{w^2}\frac{\partial T}{\partial z} = -K_h\frac{\partial T}{\partial z}, \qquad J_{\rm NL} = -\pi_4\tau\frac{\partial}{\partial z}\overline{w^2\theta} \tag{1.7}$$

Eqs. (1.1)–(1.5) have been successfully used to study stellar convection by Kupka (1999), Kukpa and Montgomery (2002) and Montgomery and Kupka (2004). In the stellar case, the derivative $T_z = \partial T/\partial z$ is replaced by the super-adiabatic temperature gradient β (see also Sect. 2 in Canuto 2007).

2. Non-locality: third-order moments

Clearly, each of (1.1)–(1.5) entails a third-order moment (TOM) and models such as (2.15) of Canuto (2007) can only be a rough approximation. To obtain a more physical expression for the TOMs, one begins with the TOMs dynamical equations (Canuto 1992, Eqs. 55). For example, in the case of buoyancy forces only, the equation for $\overline{w^3}$ reads:

$$\frac{\partial}{\partial t}\overline{w^3} = -\frac{\partial}{\partial z}\underbrace{\overline{w^4}}_{\text{FOM}} + 3\overline{w^2}\frac{\partial \overline{w^2}}{\partial z} + 3g\alpha\overline{w^2\theta} - 2c_8\tau^{-1}\overline{w^3} \tag{2.1}$$

which shows that to proceed, we need to model the fourth-order moment (FOM), $\overline{w^4}$.

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2.1. *FOMs. Previous models*

Most previous FOM models (Tatsumi 1957; O'Brien and Francis 1962; Ogura 1962; Zeman and Lumley 1976; André *et al.* 1976, 1978; Bougeault 1981; Chen and Cotton 1983; Moeng and Randall 1984; Canuto 1992; Canuto *et al.* 1994) employed the quasi-normal approximation, QNA, whereby $\overline{abcd} = \overline{ab}\,\overline{cd} + \overline{ac}\,\overline{bd} + \overline{ad}\,\overline{bc}$. For example:

$$\overline{w^4} = 3\overline{w^2}^2, \quad \overline{w^3\theta} = 3\overline{w^2}\,\overline{w\theta}, \quad \overline{w^2\theta^2} = \overline{w^2}\,\overline{\theta^2} + 2\overline{w\theta}^2 \tag{2.2}$$

In the *convectively unstable* case, QNA is known to suffer from realizability problems, that is, the resulting TOMs contain denominators that become zero at some critical $\tau^2 N^2 \sim -20$, $N^2 = g\alpha\partial T/\partial z$, which easily attains in a convective planetary boundary layer (PBL). To prevent this from happening, Canuto *et al.* (2001) proposed an ad hoc procedure to limit the value of $\tau^2 N^2$ in the unstable case; as a result, the eddy sizes are chopped down and the transport is weakened. In the *stable case*, Moeng and Randall (1984) pointed out that Eq. (2.1) under QNA leads to a "wave equation":

$$\frac{\partial^2}{\partial t^2}\overline{w^3} = 3g\alpha|\partial T/\partial z|\overline{w^3} + \text{other terms} \tag{2.3}$$

with an oscillation frequency of $f = (3g\alpha|\partial T/\partial z|)^{1/2}$. Similar "wave equations" result from other TOMs equations. These oscillations are not observed in nature and are therefore spurious.

2.2. *FOMs. New model*

Since the QNA (with zero-cumulants) causes singular behaviors of the TOMs, a more physical FOM model with non-zero cumulants was proposed and tested by Cheng *et al.* (2005). In principle, to formulate a new FOM model, one could try to solve the dynamic equations of the FOMs, but this would require a new set of parameterizations for the pressure and dissipation terms, and most of all, the need to model the fifth-order moments. A new model was therefore proposed (Cheng *et al.* 2005) which we briefly sketch here. First, from the TOMs dynamic equations one subtracts the QNA part leaving behind the dynamic equations for the cumulants. For example, one has:

$$\frac{\partial}{\partial z}(\overline{w^4} - \overline{w^4}|_{\rm QNA}) = -2c_8\tau^{-1}\overline{w^3} + 3g\alpha\overline{w^2\theta} - 3\overline{w^2}\frac{\partial}{\partial z}\overline{w^2} \tag{2.4}$$

Next, it was assumed that the FOMs can be modeled by linear combinations of the TOMs, an assumption that assures that in the Gaussian limit, the TOMs vanish and the FOMs acquire the QNA form. For example, it was assumed that:

$$\frac{\partial}{\partial z}(\overline{w^4} - \overline{w^4}|_{\rm QNA}) = p_1\tau\overline{w^3} \tag{2.5}$$

The constants that appear in expressions like (2.5) were chosen so that that (2.4), (2.5) best match the full expressions (2.1) using as input the large-eddy simulation (LES) data for the TOMs and SOMs of Mironov *et al.* (2000). Most importantly, use was made of new aircraft data on the FOMs by Hartmann *et al.* (1999), to further determine these constants. The "best" values are listed in Table 1 of Cheng *et al.* (2005). The choice of such constants helps provide adequate damping that was lacking in previous models and effectively cancels the $\beta \sim \partial T/\partial z$ terms in the TOM equations, as first suggested to the author by Kupka (1999) on the basis of DNS data. The cancellation of the β terms not only greatly simplifies the TOM equations, but also avoids the singularities in the unstable case and eliminates the source of the spurious oscillations in the stable case.

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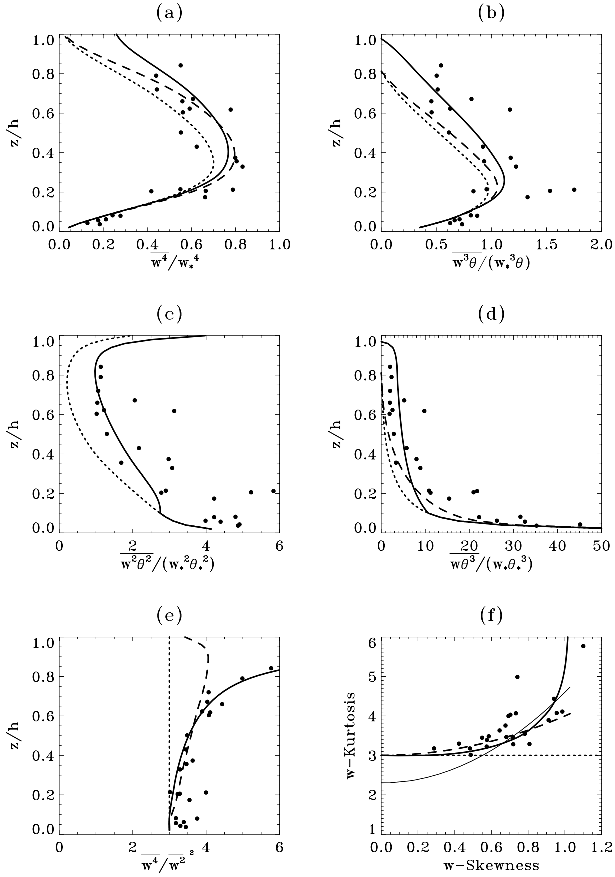

Figure 1. In (a-d), the normalized FOMs are plotted versus z/h in a convective PBL, using the LES data of Mironov *et al.* (2000) for lower order moments as input. The solid lines represent results from the present FOM model, the dashed lines represent results from the recent model of Gryanik and Hartman (2002) and the dotted lines represent the QNA. The filled circles are the aircraft data of Hartmann *et al.* (1999). In (e) the kurtosis of w is plotted versus z/h. The thick solid line represents the result from the new model, the dashed line represents the result from the model of Gryanik and Hartman (2002) and the dotted line represents the QNA, for comparison the aircraft data are the filled circles. In (f) the w-kurtosis K_w is plotted versus w-skewness S_w, using the new model (thick solid line), the model of Gryanik and Hartman (2002, dashed line) and the QNA (dotted line), for comparison the aircraft data are the filled circles, and the empirical formula $K_w = 2.3\,(S_w^2 + 1)$ is the thin solid line.

To assess their validity, the new FOMs were compared with measured data by plotting the modeled FOMs with the SOMs and TOMs from the LES data (Mironov *et al.* 2000) as input, versus z/h (h is the PBL height). In Fig. 1, the thick solid lines represent the new model results, the filled circles represent the aircraft data of Hartmann *et al.* (1999), the dashed and dotted lines represent the model results of Gryanik and Hartmann (2002) and QNA, respectively. The kurtosis of w from the models and from the aircraft data is plotted in Fig. 1e. To help assess the improvement shown in Fig. 1e, we refer the reader to the measurements of w-kurtosis by Lenschow *et al.* (1994, 2000) who stated that “The kurtosis increases with height from around 3 to about 5 near 0.9 z/z_i. Above it, the kurtosis increases sharply”. In Fig. 1f we plot the w-kurtosis K_w versus the skewness S_w from the new model (thick solid line) and from Gryanik and Hartmann (2002, dashed

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line) to be compared with the aircraft data (filled circles) and with the empirical formula (Alberghi *et al.* 2002, thin solid line)

$$K_w = 2.3\,(S_w^2 + 1). \tag{2.6}$$

Judging from the comparisons with these data, the new model exhibits significant improvements over the QNA and the Gryanik and Hartmann (2002) model.

2.3. *New TOM model with new FOMs*

Next, one employs the new FOMs into the TOM equations. The resulting equations are simpler than in previous models and more importantly, *they are singularity-free*. They are given by Eqs. (9a-f) of Cheng *et al.* (2005) which we don't reproduce here. Suffices it to say that in the stationary limit, the new model for the TOMs reads as follows:

$$\overline{w^3} = -A_1 \frac{\partial}{\partial z}\overline{w^2} - A_2 \frac{\partial}{\partial z}\overline{w\theta} - A_3 \frac{\partial}{\partial z}\overline{\theta^2} \tag{2.7}$$

$$\overline{w^2\theta} = -A_4 \frac{\partial}{\partial z}\overline{w^2} - A_5 \frac{\partial}{\partial z}\overline{w\theta} - A_6 \frac{\partial}{\partial z}\overline{\theta^2} \tag{2.8}$$

$$\overline{w\theta^2} = -A_7 \frac{\partial}{\partial z}\overline{w\theta} - A_8 \frac{\partial}{\partial z}\overline{\theta^2}, \qquad \overline{\theta^3} = -A_9 \frac{\partial}{\partial z}\overline{\theta^2} \tag{2.9}$$

Eqs. (2.7)–(2.9) exhibit the same structure of a linear combination of the z-derivatives of the SOMs first discussed in Canuto *et al.* (1994, 2001). In (2.7)–(2.9), the "diffusivities" A_i are given by ($\lambda = g\alpha$):

$$A_1 = (a_1\overline{w^2} + a_2\lambda\tau\overline{w\theta})\tau, \;\; A_2 = (a_3\overline{w^2} + a_4\lambda\tau\overline{w\theta})\lambda\tau^2, \;\; A_3 = (a_5\overline{w^2} + a_6\lambda\tau\overline{w\theta})\lambda^2\tau^3,$$

$$A_4 = a_7\tau\overline{w\theta}, \;\; A_5 = (a_8\overline{w^2} + a_9\lambda\tau\overline{w\theta})\tau, \;\; A_6 = (a_{10}\overline{w^2} + a_{11}\lambda\tau\overline{w\theta})\lambda\tau^2,$$

$$A_7 = a_{12}\tau\overline{w\theta}, \;\; A_8 = (a_{13}\overline{w^2} + a_{14}\lambda\tau\overline{w\theta})\tau, \;\; A_9 = a_{15}\tau\overline{w\theta} \tag{2.10}$$

The coefficients a_i in (2.10) are given in Appendix B of Cheng *et al.* (2005). In Figs. 2-3 we exhibit the new TOMs and FOMs compared with LES data and aircraft data.

Even though Eqs. (2.7)–(2.9) are relatively simple and have been successfully tested against LES data (Cheng *et al.* 2005), more recently we have succeeded in reducing them even further without deteriorating the comparison with LES data. In fact, we have found the following simplified version:

$$\overline{w^3} = -0.06 g\alpha\tau^2\overline{w^2}\,\frac{\partial\overline{w\theta}}{\partial z}, \qquad \overline{w^2\theta} = -0.3\tau\overline{w^2}\,\frac{\partial\overline{w\theta}}{\partial z}, \qquad \overline{w\theta^2} = -\tau\overline{w\theta}\,\frac{\partial\overline{w\theta}}{\partial z} \tag{2.11}$$

which are compared in Figs. 4a-c to the LES data of Mironov *et al.* (2000) and to the aircraft data of Hartmann *et al.* (1999). The data are reproduced quite well. The first of (2.11) correctly yields a negative skewness below the cooling ocean surface (or equivalently below the cloud top in the PBL case, see Stevens *et al.* 2005) where $\partial B/\partial z > 0$ (B is the buoyancy), while it yields a positive skewness near a surface heated from below where $\partial B/\partial z < 0$. By contrast, a down-gradient approximation which corresponds to retaining only the first term in (2.7):

$$\overline{w^3} \approx -\tau\overline{w^2}\,\frac{\partial\overline{w^2}}{\partial z} \tag{2.12}$$

yields the wrong sign of the skewness in both the above cases. To further highlight the

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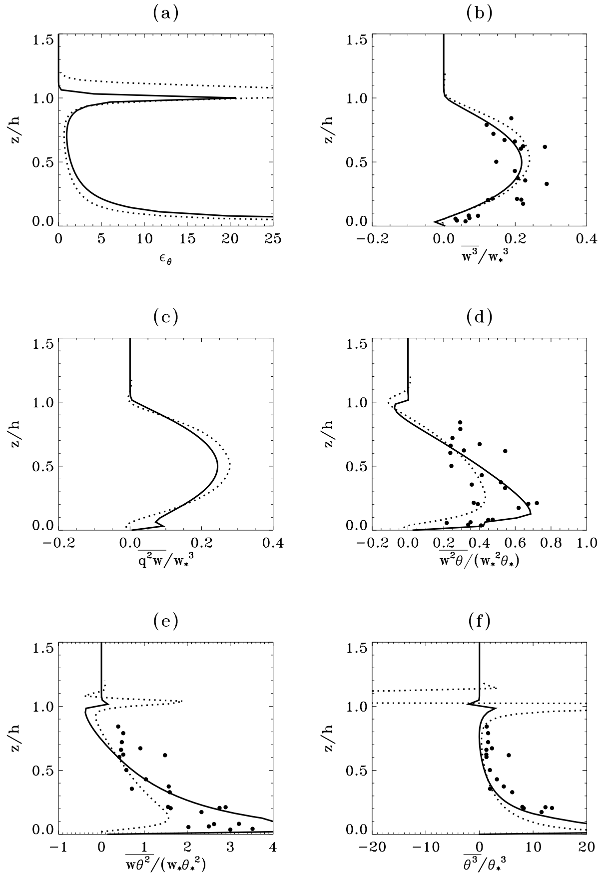

Figure 2. Temperature variance dissipation rate ε_θ and normalized third-order moments versus z/h resulting from the numerical model of a convective PBL. The solid lines represent the new model, the dot-dashed lines represent the LES data of Mironov *et al.* (2000) and the filled circles represent the aircraft data of Hartmann *et al.* (1999).

physical content of (2.11), we can re-write them as follows:

$$g\alpha\overline{w^2\theta} = 5\tau^{-1}(\overline{w^2})^{3/2}S_w, \qquad g\alpha\overline{w^2\theta} = \frac{50}{3}\tau^{-1}J(\overline{w^2})^{1/2}S_w, \tag{2.13}$$

so as to exhibit the skewness $S_w = \overline{w^3}/(\overline{w^3})^{3/2}$, as emphasized by previous authors (Wyngaard and Weil 1991; Hamba 1995). The development of this new model has also benefitted from the test of non-local models in stars (Kupka 1999; Kupka and Montgomery 2002; Montgomery and Kupka 2004).

3. Plumes and turbulence

Consider Eq. (2.2) of Canuto (2007) which, in the more general form, reads:

$$\frac{\partial T}{\partial z} + \cdots = \frac{\partial}{\partial x_j}(K_{ij}\frac{\partial T}{\partial x_i}) \tag{3.1}$$

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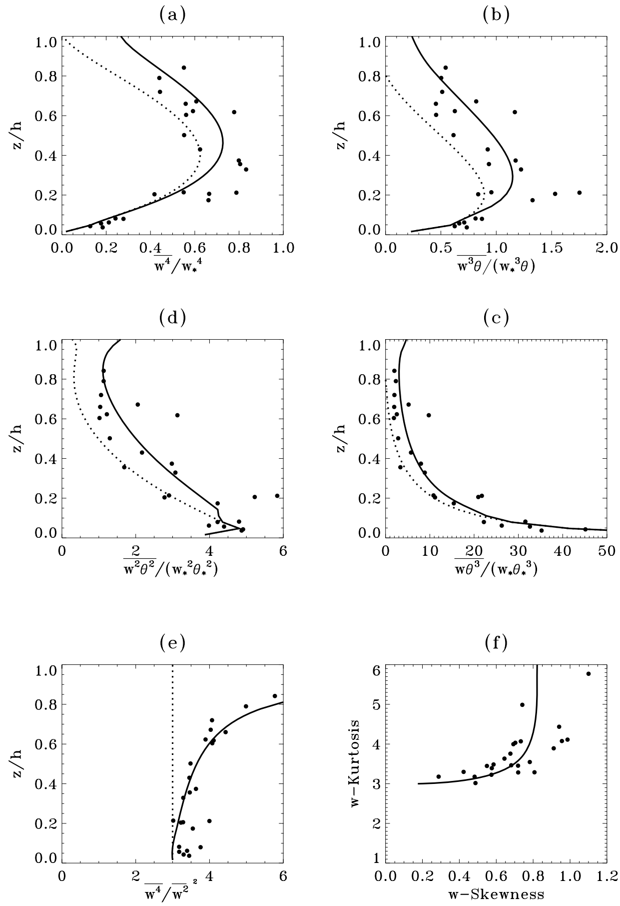

Figure 3. Normalized FOMs versus z/h according to the FOM model, as solid lines, using T, SOMs and TOMs resulting from the numerical simulation of a convective PBL as input, QN FOMs as dotted lines. The filled circles represent the aircraft data of Hartmann *et al.* (1999).

If we split the heat diffusivity tensor K_{ij} into symmetric and anti-symmetric parts:

$$K_{ij} = 1/2(K_{ij} + K_{ji}) + 1/2(K_{ij} - K_{ji}) \equiv K^{\mathrm{s}}_{ij} + K^{\mathrm{a}}_{ij} \tag{3.2}$$

and define the divergence-free velocity field:

$$u^{\star}_i = -\frac{\partial K^{\mathrm{a}}_{ij}}{\partial x_j}, \qquad \partial_i u^{\star}_i = 0 \tag{3.3}$$

Eq.(3.1) then becomes:

$$\frac{\partial T}{\partial z} + u^{\star}_i \frac{\partial T}{\partial x_i} = \frac{\partial}{\partial x_j}\left(K^{\mathrm{s}}_{ij}\frac{\partial T}{\partial x_i}\right) + \cdots \tag{3.4}$$

Thus, the symmetric part of K_{ij} yields *diffusion* while the anti-symmetric part yields *advection*. As Lappen and Randall (2001) pointed out, in diffusive transport, information flows both upward and downward; by contrast, in advective transport, the information is either up or down depending on the time evolution.

If one employs a local turbulence model, one only accounts for the rhs of (3.4) and the model is *purely diffusive*. On the other hand, the plume model that has been widely used

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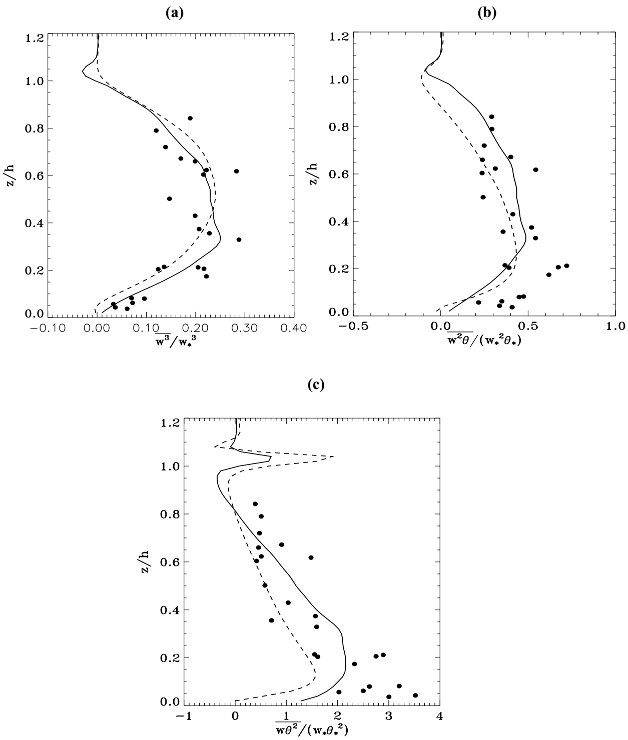

Figure 4. (a) The third moment $\overline{w^3}$ normalized by w_*^3 vs. height normalized by the PBL depth h. The filled circles represent the aircraft data of Hartmann *et al.* (1999). The dashed line shows the LES data of Mironov *et al.* (2000). The solid line represents the result of the new, simple model, using the lower order moments from LES data as input. (b) Same as Fig. 4a but for $\overline{w^2\theta}$ normalized by $w_*^2\theta_*$. (c) Same as Fig. 4a but for $\overline{w\theta^2}$ normalized by $w_*\theta_*^2$.

in the literature (Morton, Taylor and Turner 1956, cited as MTT) is *purely advective* since in fact the mean T equation reads:

$$\frac{\partial T}{\partial t} + w^\star \frac{\partial T}{\partial z} = 0 \tag{3.5}$$

Clearly, neither a purely diffusive nor a purely advective model is satisfactory since both advection and diffusion must be present for they represent different stages of the dynamical evolution of the system. Since the formal derivation just presented underlines the fact that advection and diffusion are not separate processes both being described by the same general diffusivity tensor, one must conclude that provided one includes *non-locality, turbulence models have all the ingredients necessary to account for both diffusion and advection.* The further task is to **"plumenize"** the model that is, to reformulate a non-local RSM so as to exhibit the up-down drafts (Canuto *et al.* 2007).

Plume models are attractive for they provide an intuitive visualization of narrow descending plumes and wide ascending plumes exhibited by LES studies of convection cooled from above. There are however difficulties, the first of which is that the MTT is purely advective and its extension to include diffusion is far from obvious.

The second problem is that MTT has equations for momentum and buoyancy but there are three unknowns, the third being the fraction of space occupied by the plumes that varies with z (or the plume's radius). Taylor suggested a phenomenological "*entrainment equation*" which contains an entrainment parameter α that MTT were unable to

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determine. The parameter α has thus far been treated as an adjustable coefficient but in reality is a function of the large scale features of the flow. Ellison and Turner (1959) used laboratory data to determine $\alpha = \alpha(\mathrm{Ri})$, where Ri is the Richardson number, but this function provides a poor fit to the Mediterranean outflow data (Price and Yang 1998; Wahlin and Cenedese 2006). A more complete formulation of α that includes non-local transport and leads to a better representation of the newest data has been recently proposed (Canuto *et al.* 2005).The third problem is that MTT assumes that σ, the fractional area occupied by a plume, is much smaller than unity:

$$\sigma \ll 1 \qquad \sigma = \Sigma^p(\Sigma^p + \Sigma^e)^{-1} \tag{3.6}$$

where $\Sigma^p(\Sigma^e)$ is the total cross-section of the plumes (environment) at a given depth. However, since during their evolution, plumes entrain fluid from the environment, σ increases with depth to the point where (3.6) becomes invalid. Specifically, *entrainment* causes the plume's mass flux $w_p\Sigma^p \propto \sigma w_p$ to grow while stable stratification decreases w_p, the net result being an increase of σ to the point where (3.6) breaks down. In addition, a small σ model cannot satisfy the *zero mass flux relation* ($\mathrm{w}_{u,d}$ are the velocities of the up-down drafts and z is considered upward):

$$\sigma w_d + (1-\sigma)w_u = 0 \tag{3.7}$$

which, in the small σ limit, implies:

$$|w_d| >> w_u \tag{3.8}$$

On the other hand, for the argument given above, when σ=1/2, Eq.(3.7) yields:

$$|w_u| = |w_d| \tag{3.9}$$

which is not allowed under (3.6). Finally, the mass conservation (3.7) is invariant under the transformation:

$$w_u \to w_d, \qquad \sigma \to 1-\sigma \tag{3.10}$$

and so should be any plume model. The MTT model is not invariant under (3.10) since it is valid only in the plumes' early development stages when the fraction of space occupied by the plumes is still small. In summary, the MTT model has the advantage of simplicity but: 1) it is restricted by Eq. (3.6), 2) it depends on the undetermined rate of entrainment α and 3) it is only advective since it leaves out diffusion.

4. New plume model

To correct the limitations of the MTT model, we proceed as follows.

(*a*) We employ the RSM in which non-locality is represented by the third-order moments for some of which we employ the new model discussed above.

(*b*) We write the non-local TOMs in the "plume approximation" which assumes a top hat profile that consists of two delta functions for the pdf of each state variable, corresponding to ascending and descending plumes. This implies (Lappen and Randall 2001) that such a profile has 100% probability of having one of just two possible values, the two allowed states being up-drafts and down-drafts. This introduces a considerable simplification to the problem since it reduces substantially the number of higher-order moments that are required, it assures the realizability condition of the higher order moments and requires fewer prognostic equations.

(*c*) The new turbulence-based plume model is such that all relations are invariant under (3.10) and thus the model is valid for the entire plume's lifetime,

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(d) In the small σ limit, the new model reproduces the MTT model.

To *"plumenize"* the TOMs using the up-down draft notation, we first employ the relations (Canuto and Dubovikov 1998):

$$\overline{w^2} = \sigma(1-\sigma)(w_u - w_d)^2 = \beta_\sigma w^2, \quad \beta_\sigma = \sigma(1-\sigma)^{-1} \tag{4.1}$$

$$J = \sigma(1-\sigma)(w_u - w_d)(\theta_u - \theta_d) \tag{4.2}$$

$$\overline{\theta^2} = \sigma(1-\sigma)(\theta_u - \theta_d)^2 = \beta_\sigma^{-1} w^{-2} J^2 \tag{4.3}$$

where $w \equiv w_d$. Analogous relations hold for the salinity field. These relations are invariant under (3.10). Correspondingly, the plumenized TOMs become:

$$\overline{w^3} = -\sigma(1-\sigma)(1-2\sigma)(w_u - w_d)^3 = \overline{w^2}^{3/2} S_w$$

$$\overline{w^2\theta} = -\sigma(1-\sigma)(1-2\sigma)(\theta_u - \theta_d)(w_u - w_d)^2 = \overline{w^2}^{1/2} S_w J$$

$$\overline{w\theta^2} = -\sigma(1-\sigma)(1-2\sigma)(\theta_u - \theta_d)^2(w_u - w_d) = \overline{\theta^2}^{1/2} S_\theta J \tag{4.4}$$

where the skewness is taken to be:

$$S_{\theta,w} \equiv (2\sigma - 1)[\sigma(1-\sigma)]^{-1/2} \tag{4.5}$$

With the additional relation:

$$\overline{w^3} = -0.06 g\alpha\tau^2 \overline{w^2} \frac{\partial J}{\partial z} \tag{4.6}$$

Eqs. (1.1)–(1.5), together with (4.4)–(4.5), constitute a new plume model.

5. Reynolds stress model with buoyancy and shear

In the case of stable stratification (e.g., the OV regimes), one must consider three fields, velocity, temperature and "salinity", where the latter is the name borrowed from oceanography for the third field which in stars is the mean molecular weight. The RSM prescribes the rules to derive the dynamic equations for the second-order moments. As discussed in detail in several papers (Canuto 1994; Canuto *et al.* 2001, 2002), the final results are:

Reynolds stresses, $R_{ij} = \overline{u_i u_j}$, $b_{ij} = R_{ij} - \frac{2}{3}\delta_{ij}K$, $\mathrm{D}/\mathrm{D}t = \partial/\partial t + \overline{u_i}\partial_i$:

$$\frac{\mathrm{D}b_{ij}}{\mathrm{D}t} = -\frac{8K}{15}\Sigma_{ij} - (1-p_1)\Omega_{ij} + (1-p_2)Z_{ij} + \frac{1}{2}g(\alpha_T L_{ij} - \alpha_s M_{ij}) - 5\,\tau^{-1} b_{ij} \tag{5.1}$$

All the terms in the rhs are traceless. They are defined as follows:

$$\Omega_{ij} = b_{ik}\Sigma_{jk} + b_{jk}\Sigma_{ik} - 2/3\,\delta_{ij} b_{km}\Sigma_{km}, \quad Z_{ij} = b_{ik}V_{jk} + b_{jk}V_{ik} \tag{5.2}$$

where the mean shear and vorticity were defined in Eqs. (8.5) of Canuto (2007). Furthermore,

$$L_{ij} = \lambda_i J_j^{\mathrm{h}} + \lambda_j J_i^{\mathrm{h}} - 2/3\,\delta_{ij}\lambda_k J_k^{\mathrm{h}}, \quad M_{ij} = \lambda_i J_j^{\mathrm{s}} + \lambda_j J_i^{\mathrm{s}} - 2/3\,\delta_{ij}\lambda_k J_k^{\mathrm{s}} \tag{5.3}$$

Here, $\lambda_i = -(g\overline{\rho})^{-1}\partial_i p$, $\alpha_{T,s}$ are the thermal expansion and haline contraction coefficients defined in Eqs. (3.2) of Part 1. Furthermore, $p_1 = 0.832$ and $p_2 = 0.545$.

Heat flux, $J_i^{\mathrm{h}} = \overline{u_i\theta}$:

$$\frac{\mathrm{D}J_{\mathrm{i}}^{\mathrm{h}}}{\mathrm{D}t} = -R_{ij}T_j - J_j^{\mathrm{h}}\overline{u}_{i,j} - (2\alpha_T\Psi - \alpha_s\overline{\sigma\theta})\partial_i\overline{p} - \pi_4^{-1}\tau^{-1}J_i^{\mathrm{h}} \tag{5.4}$$

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Salinity flux, $J_i^{\rm s} = \overline{u_i\sigma}$:

$$\frac{{\rm DJ}_{\rm i}^{\rm s}}{{\rm D}t} = -R_{ij}S_j - J_j^{\rm s}\overline{u}_{i,j} - (\alpha_T\overline{\sigma\theta} - 2\alpha_s\Phi)\partial_i\overline{p} - \pi_1^{-1}\tau^{-1}J_i^{\rm s} \tag{5.5}$$

Temperature variance, $\Psi = \frac{1}{2}\overline{\theta^2}$, *Salinity variance,* $\Phi = \frac{1}{2}\overline{\sigma^2}$:

$$\frac{{\rm D}\Psi}{{\rm D}t} = -J_i^{\rm h}T_i - 2\pi_5^{-1}\tau^{-1}\Psi, \quad \frac{{\rm D}\Phi}{{\rm D}t} = -J_i^{\rm s}S_i - 2\pi_3^{-1}\tau^{-1}\Phi \tag{5.6}$$

T-S correlation, $\overline{\theta\sigma}$:

$$\overline{\theta\sigma} = -\pi_2\tau(J_i^{\rm h}S_i + J_i^{\rm s}T_i) \tag{5.7}$$

where $T_i = \partial_i T$, $S_i = \partial_i S$, T and S being the mean temperature and salinity fields. To Eqs. (5.1)–(5.7) we must add the equation for K given by Eqs. (3.3) of Canuto (2007) while Eq. (1.5) must be generalized to include both shear and salinity which means that:

$$\frac{\partial\varepsilon}{\partial t} + \frac{\partial}{\partial z}\overline{w\varepsilon} = (c_1P_s + c_3P_b)\tau^{-1} - c_2\varepsilon\tau^{-1} \tag{5.8}$$

with (see Eqs. (3.1)–(3.2) of Canuto 2007):

$$P_s = -R_{ij}\overline{u}_{i,j}, \quad P_b = -g\rho_0^{-1}\overline{\rho w} \tag{5.9}$$

$$\rho_0^{-1}\overline{\rho w} = -\alpha_T J_h + \alpha_\mu J_\mu = -\rho_0^{-1}K_\rho\frac{\partial\overline{\rho}}{\partial z} = g^{-1}K_\rho N^2 \tag{5.10}$$

The dissipation time scales that appear in Eqs. (5.4)–(5.7) were written in terms of the dynamical time scale $\tau = 2K/\varepsilon$ and the proportionality coefficients were denoted by π_k.Without the help of an outside model, the RSM per se is incapable of determining such coefficients and that may be one of the reasons why in the past the RSM was not extended to include the salinity field. Without the knowledge of such constants, the above equations would be quite useless. In Canuto *et al.* (2001, 2002) it was shown that the outside model is provided by the RNG, the renormalization group, and we refer the reader to the lengthy discussion in the original papers. The numerical values are presented in Eq. (22d) of Canuto *et al.* (2002):

$$\pi_1 = \pi_4 = (27\,{\rm Ko}^3/5)^{-1/2}(1+\sigma_t^{-1})^{-1}, \quad \pi_3 = \pi_5 = \sigma_t, \quad \pi_2 = 1/3 \tag{5.11}$$

with a suggested valued of 0.72 for the turbulent Prandtl number σ_t.

Of course, it would be quite a task to hook up the above turbulence equations to a stellar code. Thus, we present the solutions of Eqs. (5.1)–(5.7) in the stationary case. In that limit, the equations become algebraic and can be solved analytically with the help of a symbolic algebra code. The results are quite simple:

$$\overline{w\theta} = -K_h\frac{\partial T}{\partial z}, \quad \overline{w\sigma} = -K_s\frac{\partial S}{\partial z}, \quad \overline{uw} = -K_m\frac{\partial\overline{u}}{\partial z}, \quad \overline{vw} = -K_m\frac{\partial\overline{v}}{\partial z} \tag{5.12}$$

where all the diffusivities have the general form:

$$K_\alpha = \frac{2K^2}{\varepsilon}S_\alpha, \quad S_\alpha = S_\alpha({\rm Ri}, R_\rho) \tag{5.13}$$

where Ri was defined in (3.4) of Canuto (2007) and the density ratio is given by:

$$R_\rho = \frac{\alpha_s S_z}{\alpha_T T_z} \tag{5.14}$$

the equivalent of which in the stellar case was introduced after (4.2) of Canuto (2007). The dimensionless "structure functions" S_α are algebraic expressions given in Canuto

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et al. (2002, see also Figs. 3-5). Relation (5.13) points to a clear division of labor, the K–ε equations must be solved to determine these two variables but they require (5.12)–(5.14). As an example of the role played by the dissipation time scale represented by the π_k, we consider the ratio heat/salinity diffusivities which turns out to have a rather simple form:

$$\frac{K_h}{K_s} = \frac{1 - \pi_1\pi_3 x R_\rho + \pi_1\pi_2 x(1 + R_\rho)}{1 + \pi_1\pi_3 x - \pi_1\pi_2 x(1 + R_\rho)} \tag{5.15}$$

where $x = (\tau N)^2(1 - R_\rho)^{-1}$. This expression exhibits the correct symmetry: when $R_\rho = -1$, heat and salt diffusivity coincide, as they indeed must. Furthermore, in the $\tau \gg N^{-1}$ limit, Eq. (5.15) becomes independent of x:

$$\frac{K_h}{K_s} = \frac{(\pi_2 - \pi_3)R_\rho + \pi_2}{\pi_3 - \pi_2(1 + R_\rho)} \tag{5.16}$$

In the strong turbulence limit of $\tau \ll N^{-1}$, we obtain instead:

$$K_h = K_s \tag{5.17}$$

as expected. In oceanography, Eq. (5.13) is written in the form:

$$K_\alpha = \Gamma_\alpha \frac{\varepsilon}{N^2}, \quad \Gamma_\alpha = \frac{1}{2}(\tau N)^2 S_\alpha \tag{5.18}$$

where the Γ_α are called *mixing efficiencies*. Lacking a predictive mixing model, in the past the practice has been to assume a unique Γ_α for momentum, heat and salinity taken to be $\Gamma_\alpha = 0.2$. The mixing model presented in Canuto *et al.* (2002) shows that the Γ_α are not universal constants and that they increase strongly near $R_\rho = 0.6$, a prediction consistent with the recent observations of much larger mixing at Barbados ($R_\rho = 0.6$) than at the NATRE location (Ledwell *et al.* 1998) where $R_\rho = 0.56$.

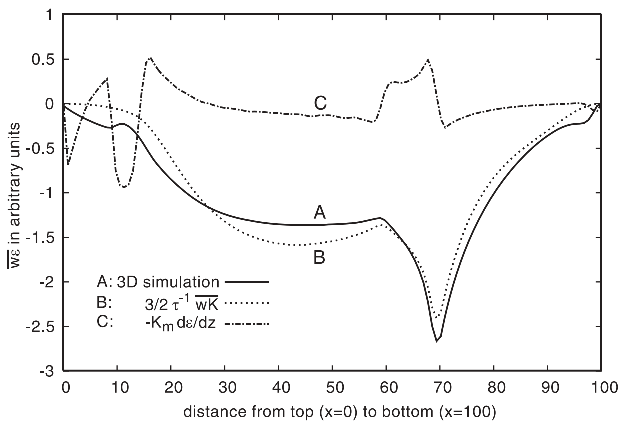

Figure 5. The function $\overline{w\epsilon}$ (in arbitrary units) from the 3D simulation of Kupka and Muthsam (2007). As one can observe, the down-gradient closure (6.5) fails to reproduce the data while (6.6) yields better results.

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6. Dissipation: an open problem

The dissipation ε in Eq. (3.1) of Canuto (2007) is defined as follows:

$$\varepsilon_{ij} = 2\nu\overline{\partial_i u_k \partial_j u_k} = \frac{2}{3}\delta_{ij}\varepsilon, \quad \varepsilon = 2\nu\int k^2 E(k)dk \tag{6.1}$$

The first consideration is that the largest contribution to the integral comes from the high wave number region corresponding to the smallest eddies which are difficult to model since they contain little energy but a large vorticity and have a short lifetime. As an example, it is easy to verify the inapplicability of the Kolmogorov spectrum $E(k) = \mathrm{Ko}\,\varepsilon^{2/3}k^{-5/3}$: if (6.1) were to be integrated over all wave numbers, it would diverge and the kinematic viscosity ν would not disappear while it is known that ε does not depend on ν. This is because the non-linear interactions enter the dynamic equations under a divergence and yield zero when one integrates over the whole volume leading to the overall relation: energy input=energy output which alternatively means that the non-linear interactions do not use energy, they simply transfer it from large to small eddies. Thus, what "arrives" to the small eddies is the same energy that was put into the system which is clearly independent of how viscous the system is, being an arbitrary external input. So, how should one read Eq. (6.1)? From left to right: given a fixed amount of energy input, which is identical to ε, the right hand side tells us at which k_d the dissipation occurs: the smaller the viscosity, the larger k_d has to be, that is, the smaller are the eddies that operate the dissipation process. Thus, one can say that Eq. (6.1) determines the upper limit of integration. For example, if one uses the Kolmogorov spectrum *only* up to k_d, integration of (6.1) gives $k_d = (\varepsilon\nu^{-3})^{1/4}$ which is just Eq. (1.2) from Canuto (2007).

Since the first of (6.1) entails only the velocity field, in principle one can derive a dynamic equation for ε. Davidov (1961) was the first to do so but the result was unmanageable. Over the years people tried to suggest an equation for ε and the most popular one has the following form (no shear):

$$\frac{\partial\varepsilon}{\partial t} + \frac{\partial}{\partial z}\overline{w\varepsilon} = c_1 g\alpha J\tau^{-1} - c_2\varepsilon\tau^{-1} \tag{6.2}$$

with $c_1 = 2.88$ and $c_2 = 3.8$. A discussion of (6.2), the variants that have been suggested and the attempts to derive it, can be found in Schiestel (1987), Rubinstein and Zhou (1996) and Kantha (2004). The two coefficients $c_{1,2}$ have also been the subject of discussion (Baumert and Peters 2000; Kantha 2004; Umlauf and Burchard 2005). Consider the K–ε equations in the homogeneous and stationary case. The first gives P=ε while the second gives $c_1 = c_2$, which contradicts the values just cited. Thus, the K–ε equations are inconsistent, at least in this limit. One can ask: are $c_{1,2}$ really constant or are they functions? Since we don't know the answer, we can only highlight how c_2 is arrived at. Consider freely decaying turbulence in which case the K–ε equations become $\dot{K} = -\varepsilon$ and $\dot{\varepsilon} = -c_2\varepsilon\tau^{-1}$ where $\tau = 2K\varepsilon^{-1}$. Using power law solutions $K \sim t^{-n}$, $\varepsilon \sim t^{-m}$ one obtains $m = n + 1$ and $c_2 = 2(1 + n^{-1})$. Since numerical simulations (Chasnov 1995, 1997a,b) yield $n = 6/5 - 10/7$, one obtains the value of c_2 cited above. Once that value is accepted, the coefficient c_1 is determined using data that include a non-zero production term. The procedure is hardly without fault since there is no reason why the c_2 determined in the freely decaying case should also hold in the case when production is present. For example, one could generalize the last result to:

$$c_2 = 2(1 + n^{-1})(1 + aP/\varepsilon)^{-1} \tag{6.3}$$

that reduces to the previous result when there is no production $P = 0$ while in the $P = \varepsilon$ case considered before, a proper choice of the parameter "a" could yield $c_1 = c_2$ and

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make the inconsistency disappear. Since this procedure is hardly satisfactory, Eq. (6.2) must be viewed as the weakest point in any turbulence model. Often it is not even used and instead one employs a heuristic approach on the grounds that to fix the problems just mentioned one must resort to empirical models anyway and thus (6.2) does not seem to offer great advantages. A common approach is to use a Kolmogorov's type expression:

$$\varepsilon = K^{3/2}L^{-1}, \quad L = c\,l(z), \quad \frac{1}{l(z)} = \frac{1}{\kappa\,z} + \frac{1}{l_0} + \frac{1}{l_s} \tag{6.4}$$

with $c = 5.87$ and $l_s = 0.2(KN^{-2})^{1/2}$. The expression for l was suggested by Blackadar (1962) and Deardorff (1980) and is still widely used (André *et al.* 1978; Galperin *et al.* 1988). Here z is the distance from the nearest "wall" and κ is the von Karman constant whose value is around 0.4. One can observe that for small z's, $l(z) = \kappa z$ which is the so-called "law of the wall", while for larger z's, one obtains $l(z) = l_0$ which, on the basis of LES data, is taken to be $0.17H$, where H is the extent of the mixing zone. Furthermore, since in the case of stable stratification, turbulent kinetic energy transforms into eddy potential energy, Deardorff introduced the length scale l_s. Needless to say, the universality of these relations is doubtful, at best. To this, we may add that the non-local term in (6.2) is a further source of uncertainty. A down-gradient approximation:

$$\overline{w\varepsilon} = -K_m\,\partial_z\varepsilon \tag{6.5}$$

has been adopted by several authors (Burchard and Bolding 2001; Burchard and Deleersnijder 2001; Umlauf and Burchard 2005) while Canuto (1992, eq. 49) suggested:

$$\overline{w\varepsilon} = 3/2\tau^{-1}F_{\mathrm{KE}}, \quad F_{\mathrm{KE}} = \overline{wK} \tag{6.6}$$

Eqs. (6.5, 6.6) have been recently assessed by Kupka and Muthsam (2007) using DNS data for buoyant convection. In Fig. 5 (kindly provided by F. Kupka) one observes quite clearly that (6.5) is far from being acceptable whereas (6.6) is much closer to the data. We can only conclude that for the time being we must live with these uncertainties about the dissipation ε.

7. Conclusions

While atmospheric and oceanic mixing problems have been treated for years with RSM, astrophysical problems by and large have not. It is the goal and the hope of this review to suggest that it may be time to forgo heuristic models since the RSM is capable of including great many physical processes in a unified and manageable way.

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Discussion

Stein: You talk of eddies, but the numerical simulations show that this is a bad picture. One should instead use a picture in terms of downdrafts and upflows.

Canuto: You are referring to your numerical solutions of a physical situation that is quite limited in two ways: it deals with 3% of the Sun's dimension and is driven by cooling from above. I very much doubt whether such circumstances can be generalized to for example shear driven turbulence in the overshooting zone, mixing driven by double-diffusion processes like semi-concevtion, mixing driven by gravity waves etc.

Cooling from above exists not only in the case of your study, but both in the ocean and in the atmosphere. To show you how rare those events are and so the process that drives them, let me say that in the ocean such processes occur essentially in three places: Labrador, Gulf of Lyon and the Weddeell Sea where indeed loss of surface buoyancy drives mixing. However, the rest of the ocean and the majority of it is not driven by cooling from above, but from either shear at the surface or mixing from the bottom processes that are extremely more common. In the atmosphere, cooling from above also occurs in the case of cloud-capped PBL (planetary boundary layer), but in the vast majority of cases that is not the main physical process responsible for mixing.

Having said that, let me also recall that the Reynolds Stress Models (RSMs) has a long and successful pedigree that began some 50 years ago, but only recently has found its way into astrophysics. The other communities have used it for more than a quarter of a century. It solves the basic Navier-Stokes equations and scalar equations exactly as you do numerically. It is the only approach that I know of that gives results that can be used in a stellar (or oceanic, atmospheric) code, that is resilient enough to accomodate a variety of driving mechanisms exactly as it is needed in dealing with stellar interiors. Since LES cannot be hooked up to a stellar code, RSMs are the only alternative there is and finally let us not forget that we routinely used LES/DNS/lab data to double check the RSM results before we offer them to stellar structure-evolution people for their use in their codes.

One last remark. I have recently recast the non-local RSM equations into a form that in the special cooling-from-above situation, exhibits the plume behavior that you and others have observed, that being an example of the resiliency of the RSM method.

[editorial remark]: Only the above question was also recorded on paper for the introductory review of the conference, the contents of which is presented in this volume in two separate contributions by V.M. Canuto. This discussion was continued as the first main topic in the round table discussion for session A, also published in this volume.

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