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NUMERICAL APPROXIMATION OF STURM-LIOUVILLE EIGENVALUES

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This thesis examines some numerical methods for approximating a long sequence of eigenvalues of Sturm-Liouville problems. The aim is to analyse the convergence of these methods and to introduce modifications which improve the numerical performance. The results show that the use of the transformation to Liouville normal form helps considerably in obtaining eigenvalue estimates suitable for this type of eigenvalue problem.

After a brief introduction where notation and basic results on Sturm-Liouville problems are given, Chapter 1 reviews some of the different classes of numerical methods available and examines their limitations.

In Chapter 2 we develop simple eigenvalue error bounds for some initial value methods based on the standard, modified and scaled Prufer substitutions. However, in practice, it is found that these bounds are not always sharp. Improved bounds are then derived for the scaled and modified phase substitutions. These improved bounds show that the eigenvalue error is most uniform for the estimates obtained using the modified or scaled phase associated with an eigenvalue problem which is in Liouville normal form.

When the problem is in Liouville normal form viable numerical schemes for estimating the eigenvalues can be constructed by approximating the coefficient of the differential equation. In Chapter 3 we show that uniformly valid estimates of a long sequence of eigenvalues can be obtained when piecewise constant approximations are used.

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Finally, in Chapter 4, we propose a minor modification of a standard finite difference approximation to eigenvalue problems which are in Liouville normal form. The errors in the eigenvalue estimates obtained using this modification are shown to be greatly superior to those of the original problem.

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