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FINITE SYMMETRIC GRAPHS WITH 2-ARC-TRANSITIVE QUOTIENTS: AFFINE CASE

M. REZA SALARIAN

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Dedicated to Professor Praeger

Abstract

Let G be a finite group and Γ a G-symmetric graph. Suppose that G is imprimitive on $V(\Gamma)$ with B a block of imprimitivity and $\mathcal{B} := \{B^g; g \in G\}$ a system of imprimitivity of G on $V(\Gamma)$. Define $\Gamma_{\mathcal{B}}$ to be the graph with vertex set $\mathcal{B}$ such that two blocks $B, C \in \mathcal{B}$ are adjacent if and only if there exists at least one edge of Γ joining a vertex in B and a vertex in C. Xu and Zhou ['Symmetric graphs with 2-arc-transitive quotients', *J. Aust. Math. Soc.* **96** (2014), 275–288] obtained necessary conditions under which the graph $\Gamma_{\mathcal{B}}$ is 2-arc-transitive. In this paper, we completely settle one of the cases defined by certain parameters connected to Γ and $\mathcal{B}$ and show that there is a unique graph corresponding to this case.

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1. Introduction

Let G be a finite group. A graph Γ is called G-*symmetric* if Γ admits G as a group of automorphisms acting transitively on the set of vertices and the set of arcs of Γ, where an *arc* is an ordered pair of adjacent vertices. Suppose that G is imprimitive on $V(\Gamma)$ with B a block of imprimitivity. Then

$$\mathcal{B} := \{B^g; g \in G\}$$

is a system of imprimitivity of G on $V(\Gamma)$. Define $\Gamma_{\mathcal{B}}$ to be the graph with vertex set $\mathcal{B}$ such that two blocks $B, C \in \mathcal{B}$ are adjacent if and only if there exists at least one edge of Γ joining a vertex in B and a vertex in C. We call $\Gamma_{\mathcal{B}}$ the *quotient graph* of Γ with respect to $\mathcal{B}$. A graph Γ is called $(G, 2)$-arc-transitive if it admits G as a group of automorphisms acting transitively on the set of vertices and the set of 2-arcs of Γ, where a 2-arc is an oriented path of length two. In [1] the following question was asked:

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QUESTION 1.1. Under the assumptions above, when is the quotient $\Gamma_{\mathcal{B}}$ a $(G,2)$-arc-transitive graph?

Notation. Fix $B \in \mathcal{B}$. Let $\mathcal{U} := \Gamma_{\mathcal{B}}(B)$ be the set of blocks of $\mathcal{B}$ adjacent to B in $\Gamma_{\mathcal{B}}$. For $\alpha \in B$, let $\Gamma_{\mathcal{B}}(\alpha)$ be the set of blocks in $\mathcal{U}$ containing at least one neighbour of α in Γ and let $r := |\Gamma_{\mathcal{B}}(\alpha)|$. For $C \in \mathcal{U}$, let $\Gamma(C)$ denote the set of vertices of Γ adjacent to at least one vertex in C. Define $v := |B|$ and $k := |\Gamma(C) \cap B|$ for $C \in \mathcal{U}$. Since Γ is G-symmetric and $\mathcal{B}$ is G-invariant, r, v and k are independent of the choice of α, B and C, respectively. Denote by G_B the setwise stabiliser of B in G, and define $H := G_B^{\Gamma_{\mathcal{B}}(B)}$ to be the quotient group of G_B relative to the kernel of the induced action of G_B on $\mathcal{U}$.

In [2], necessary conditions for $\Gamma_{\mathcal{B}}$ to be $(G,2)$-arc-transitive were obtained in the case when $k = v - p \geq 1$, where p is an odd prime. The following result is extracted from [2, Theorem 1.1]. (It corresponds to the third case in the theorem.)

THEOREM 1.2. *Assume, in the context of the notation above, that $G \leq \mathrm{Aut}(\Gamma)$, $\Gamma_{\mathcal{B}}$ is $(G,2)$-arc-transitive and $\Gamma_{\mathcal{B}}$ is connected with valency $b \geq 2$. Assume further that $k = v - p \geq 1$, $p = 2^n - 1$ is a Mersenne prime, $v = 2^m p$ is a multiple of p and $r = (2^m - 1)t$, where $n - 1 \geq m \geq 1$ and $t \geq 2$ are integers. Then H is isomorphic to a 2-transitive subgroup of* $\mathrm{AGL}(n,2)$.

We show that $p = 3$ in this situation and that there is a unique graph satisfying the conditions of Theorem 1.2. More explicitly, we prove the following theorem.

THEOREM 1.3. *With the assumptions of Theorem 1.2, we have $p = k = 3$ and $v = 6$.*

Theorem 1.3 shows that the graph which appears in [2, Theorem 3] is the only graph satisfying the conditions of Theorem 1.2.

2. Proof of the main theorem

In what follows we use the notation and assumptions in Theorem 1.2. By [2], $|\mathcal{U}| = p + 1$ and so we may set

$$\mathcal{U} := \{C, C_1, \ldots, C_p\}, \quad W := \Gamma(C) \cap B.$$

Then $|B \backslash W| = p$ by our assumption and $r = 2^n - 2^{n-m}$ by [2]. Let H_C be the stabiliser of C in H. Then H_C leaves W and $B \backslash W$ invariant. Since $\Gamma_{\mathcal{B}}$ is assumed to be $(G,2)$-arc-transitive, H is 2-transitive on $\mathcal{U}$ and so H_C is transitive on $\mathcal{U} \backslash \{C\}$. In fact, Γ, $\Gamma_{\mathcal{B}}$ and H satisfy the conditions in the third row of [2, Table 2]. So we assume that $H = N \rtimes H_C$ is an affine group (isomorphic to a subgroup of $\mathrm{AGL}(n,2)$). Here $N \cong \mathbb{Z}_2^n$ is an elementary abelian group of order $p + 1 = 2^n$ and is the minimal normal subgroup of H acting regularly on $\mathcal{U}$ with centraliser $C_H(N) = N$. Further, H_C is isomorphic to a subgroup of $\mathrm{GL}_n(2)$ and acts transitively on the set of involutions of N.

Since N has exactly p involutions and H_C is transitive on the set of them, p divides the order of H_C. Since p is a prime, H_C contains an element of order p, say, x. Define

$$X := \langle x \rangle \leq H_C, \quad P := \langle N, x \rangle = N \rtimes X \leq H.$$

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Lemma 2.1. *The following hold:*

(i) *X is of order p and is regular on $\mathcal{U}\backslash\{C\}$;*
(ii) *X fixes W and $B\backslash W$ setwise and is fixed-point-free on each of them;*
(iii) *X is regular on $B\backslash W$.*

Proof. (i) Obviously, X has order p. Since $|X| = |\mathcal{U}\backslash\{C\}| = p$ is a prime, by the orbit-stabiliser lemma X must be regular on $\mathcal{U}\backslash\{C\}$.

(ii) Since $X \leq H_C$, it fixes W and $B\backslash W$ setwise. If a vertex $\alpha \in B\backslash W$ is fixed by a nonidentity element of X, then it is fixed by every nonidentity element of X. Since by (i), X is transitive on $\mathcal{U}\backslash\{C\}$, we then have $\alpha \in \Gamma(C_i) \cap B$ for $i = 1, 2, \ldots, p$, which yields $r = p$ and so $m = 0$, a contradiction. Therefore, X is fixed-point-free on $B\backslash W$. A similar argument shows that X is fixed-point-free on W.

(iii) Since $|X| = |B\backslash W| = p$ is a prime and X acts fixed-point-freely on $B\backslash W$, X must be regular on $B\backslash W$. □

Lemma 2.2. *No nonempty subset of W is N-invariant.*

Proof. Suppose to the contrary that $\emptyset \neq Y \subseteq W$ is N-invariant. Since N is regular on $\mathcal{U}$, for each i there exists a unique element $g_i \in N$ such that $C^{g_i} = C_i$. Hence $W^{g_i} = \Gamma(C_i) \cap B$. Since Y is N-invariant, we have $Y = Y^{g_i} \subseteq W^{g_i}$ for $i = 1, 2, \ldots, p$, which implies $r = p + 1 = 2^n$, a contradiction. □

Lemma 2.3. *The subgroup P is transitive on B.*

Proof. Let α^N be an N-orbit on B, where $\alpha \in B$, and set $A = \bigcup_{g\in P}(\alpha^N)^g$. Since $N \trianglelefteq P \leq H$, $A \subseteq B$ and P is transitive on A, both A and $B\backslash A$ are P-invariant. In particular, both A and $B\backslash A$ are N-invariant and X-invariant. Since $A \neq \emptyset$, by Lemma 2.2 we have $A \cap (B\backslash W) \neq \emptyset$. On the other hand, by Lemma 2.1, X is transitive on $B\backslash W$. Since A is X-invariant and $A \cap (B\backslash W) \neq \emptyset$, it follows that $B\backslash W \subseteq A$. Now $B\backslash A \subseteq W$ and $B\backslash A$ is N-invariant, by Lemma 2.2, so $B\backslash A = \emptyset$ and hence P is transitive on $B = A$. □

Since N is regular on $\mathcal{U}$, it contains a unique involution z which interchanges C and C_1. Write

$$W_1 := \Gamma(C_1) \cap B, \quad B_z := \{\alpha \in B : \alpha^z = \alpha\}.$$

Then z interchanges W and W_1 and both $W \cap W_1$ and $W \cup W_1$ are z-invariant. Note that $|W \cap W_1| = \lambda = (a-1)(p - 2^{n-m}) \neq 0$ by [2] with $a = 2^m$. Therefore $B_z \cap (W \cap W_1) \neq \emptyset$. Since N is abelian, B_z is N-invariant. Fix an N-orbit α^N contained in B_z, where $\alpha \in B_z$, and set

$$\mathcal{F} := \{(\alpha^N)^g : g \in X\}.$$

Since N is normal in P, $\mathcal{F}$ is a system of imprimitivity for P. Then $|\alpha^N| = 2^m = a$, $|\mathcal{F}| = p$ and $\mathcal{F}$ is the set of all N-orbits on B. We note that if $(\alpha^N)^{g_1} = (\alpha^N)^{g_2}$ for distinct $g_1, g_2 \in X$, then since $X = \langle g_1 g_2^{-1}\rangle$, we see that α^N is P-invariant. By Lemma 2.3, N is regular on B which implies that $v = |N| = p + 1 = 2^n$, a contradiction.

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Lemma 2.4. *We have* $|\alpha^N \cap (B\backslash W)| = 1$. *In fact, each element of* $B\backslash W$ *is in a unique element of* $\mathcal{F}$ *and each element of* $\mathcal{F}$ *contains a unique element of* $B\backslash W$.

Proof. Let $x \in B\backslash W$. Since $\mathcal{F}$ is a system of imprimitivity for P, there is $(\alpha^N)^g \in \mathcal{F}$, $g \in X$, such that $(\alpha^N)^g \cap (B\backslash W) \neq \emptyset$ and we may asssume that $x \in \alpha^N$. Since X fixes $B\backslash W$ setwise and is transitive on $B\backslash W$, we have $x^X = B\backslash W$. Using the fact $|\mathcal{F}| = p = |B\backslash W|$, we have $\alpha^N \cap (B\backslash W) = \{x\}$. This shows that each element of $B\backslash W$ is in a unique element of $\mathcal{F}$ and each element of $\mathcal{F}$ contains a unique element of $B\backslash W$. □

Since z is fixed-point-free on $\mathcal{U}$, z has $(p+1)/2 = 2^{n-1} = q$ orbits on $\mathcal{U}$ each of length 2. Let R_i, $i = 1, 2, \ldots, q$, be the orbits of z on $\mathcal{U}$, $R_1 = \{C = C_0, C_1\}$ and $R_i = \{C_{2i-2}, C_{2i-1}\}$, $i = 2, \ldots, q$. For $i = 1, \ldots, p$, set $W_i = \Gamma(C_i) \cap B$ and $W = W_0$. Then for $i \neq j$, we have $|W_i \cap W_j| = \lambda = (a-1)(p - 2^{n-m})$. We note that since $\langle z \rangle$ is normal in N, $\{R_i,\ i = 1, 2, \ldots, q\}$ is a system of imprimitivity for N. We recall that $r = 2^n - 2^{n-m}$ and $a = 2^m$.

Lemma 2.5. *For* $i = 1, 2, \ldots, q$, *we have:*

(i) *N has* $|B\backslash(W_{2i-2} \cup W_{2i-1})| = 2p - v + \lambda = 2^{n-m} - 1$ *orbits on* B_z *and* $|B_z| = 2^n - 2^m$;

(ii) $B_z = (B_z \cap W_{2i-2} \cap W_{2i-1}) \cup (B\backslash(W_{2i-2} \cup W_{2i-1}))$;

(iii) $|B_z \cap W_{2i-2} \cap W_{2i-1}| = |B\backslash(W_{2i-2} \cup W_{2i-1})|(a-1)$.

Proof. Without loss of generality, we can take $i = 1$. Since both $W \cap W_1$ and $W \cup W_1$ are z-invariant, $B\backslash(W \cup W_1) \subseteq B\backslash W$ is also z-invariant. This and Lemma 2.4 together imply that $B\backslash(W \cup W_1) \subseteq B_z$ and N has $|B\backslash(W \cup W_1)| = 2p - v + \lambda = 2^{n-m} - 1$ orbits on B_z. Therefore $|B_z| = 2^m(2^{n-m} - 1) = 2^n - 2^m$ and the lemma holds. □

We know that N has p orbits on B each of length a and we denote these orbits by B_i, $i = 1, \ldots, p$. By Lemma 2.4, we can write $B\backslash W = \{\alpha_1, \ldots, \alpha_p\}$ where $\alpha_i \in B_i$, $i = 1, 2, \ldots, p$.

Lemma 2.6. *For* $i = 1, \ldots, p$ *and* $j = 1, 2, \ldots, p+1$:

(i) $B_i \cap (B\backslash W_j)| = 2$ *and* $B_i \cap (B\backslash(W_{2j-2} \cup W_{2j-1})| = 1$;

(ii) *either* $B_i \subset B_z$ *or* $B_i \cap B_z = \emptyset$;

(iii) *if* $B_i \subset B_z$, *then* $|B_i \cap W_{2s-2} \cap W_{2s-1}| = a - 1$ *for each* $s = 1, \ldots, q$;

(iv) *if* $B_i \cap B_z = \emptyset$, *then* $|B_i \cap W_{2s-2} \cap W_{2s-1}| = a - 2$.

Proof. Since B_z is N-invariant, (ii) holds. Note that $|B_i| = a$, $i = 1, \ldots, p$. Let $B_1 \subseteq B_z$ and $B_2 \subseteq (W \cup W_1)\backslash B_z$. By Lemmas 2.4 and 2.5(i), we see that $B_1\backslash\{\alpha_1\} \subseteq (W \cap W_1)$, z acts fixed-point-freely on B_2 and $B_2\backslash\{\alpha_2, \alpha_2^z\} \subseteq (W \cap W_1)$. Observe that for each orbit $R_i = \{C_{2i-2}, C_{2i-1}\}$, $i = 1, \ldots, q$, and each orbit B_j, $j = 1, \ldots, p$, either $B_j \subseteq B_z$ and there is an element $x_i \in B_j$ such that $x_i \notin W_{2i-1} \cup W_{2i-2}$ and $B\backslash\{x_i\} \subseteq (W_{2i-1} \cap W_{2i-2})$, or z is fixed-point-free on B_j and $(B_j\backslash\{\alpha_j, \alpha_j^z\}) \subseteq (W_{2i-1} \cap W_{2i-2})$. This proves the lemma. □

Set $O = W\backslash W_1$.

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Lemma 2.7. *For each* $R_i, i = 2, \dots, q$, *we have:*

(i) $O = (O \cap W_{2i-2} \cap W_{2i-1}) \cup ((O \cap W_{2i-1}) \backslash W_{2i-2}) \cup ((O \cap W_{2i-2}) \backslash W_{2i-2})$;

(ii) $|((O \cap W_{2i-1}) \backslash W_{2i-2})| = |((O \cap W_{2i-2}) \backslash W_{2i-2})|$.

Proof. Without loss of generality, we can take $i = 2$. To prove (i), we show that $O = (O \cap W_2 \cap W_3) \cup ((O \cap W_2) \backslash W_3) \cup ((O \cap W_3) \backslash W_2)$. For this it is enough to show that $O \subseteq W_3 \cup W_2$. Assume not and let $x \in O \backslash (W_2 \cup W_3)$. Then there is $j \in \{1, \dots, p\}$ such that $x \in B_j$. Since $O \cap B_z = \emptyset$, we conclude that z acts fixed-point-freely on B_j and then $\{x, x^z\} \subseteq (B_j \backslash (W_2 \cup W_3))$. But by Lemma 2.6(i), N has no such orbit, a contradiction. Hence (i) holds. Since $|W \cap W_2| = |W \cap W_3| = \lambda$, $|W_2 \cap W \cap W_1| = |W_3 \cap W \cap W_1|$ and, by (i), $|(O \cap W_2) \backslash W_3| = |(O \cap W_3) \backslash W_2|$, the lemma is proved. □

Lemma 2.8. *We have:*

(i) $W_{2i-2} \cap W_{2i-1} \cap B_z = B \backslash (W_{2j-2} \cup W_{2j-1})$ *for* $i, j = 1, \dots, q$, $i \neq j$;

(ii) $m = 1$;

(iii) $n = 2, v = 6$ *and* $k = 3 = p$;

(iv) $P \cong A_4$.

Proof. To prove (i), we may assume that $i = 1$ and $j = 2$. By Lemma 2.7(i) we have $O = (O \cap W_2 \cap W_3) \cup ((O \cap W_2) \backslash W_3) \cup ((O \cap W_3) \backslash W_2)$. Let $O_1 = O^z = W_1 \backslash W$ and $|O \cap W_2 \cap W_3| = a_1 = |O_1 \cap W_2 \cap W_3|$. By Lemma 2.7(ii), $|(O \cap W_2) \backslash W_3| = a_2 = |(O \cap W_3) \backslash W_2|$. Next, set $|(W_1 \cap W \cap W_2) \backslash W_3| = b_2 = |(W_1 \cap W \cap W_3) \backslash W_2|$, $|(W_1 \cap W) \backslash (W_2 \cup W_3)| = c = |(W_2 \cap W_3) \backslash (W \cup W_1)|$ and $b_1 = |W_2 \cap W_3 \cap W_1 \cap W|$. We note that $B \backslash (W \cup W_1) \subseteq B_z$, so $W_2 \cap (B \backslash (W \cup W_1)) = W_3 \cap (B \backslash (W \cup W_1)) = c$. Now,

$$2a_1 + b_1 + c = |W_2 \cap W_3| = \lambda, \tag{2.1}$$

$$2b_2 + b_1 + c = |W \cap W_1| = \lambda, \tag{2.2}$$

$$a_1 + a_2 + b_2 + b_1 = |W \cap W_3| = \lambda, \tag{2.3}$$

$$a_1 + 2a_2 = |O| = k - \lambda, \tag{2.4}$$

$$\lambda + a_2 + a_1 + c = |W_3| = k. \tag{2.5}$$

From (2.1) and (2.2), $a_1 = b_2$, and from (2.4) and (2.5), $c = a_2$. From this and (2.3) and (2.4), $2(k - \lambda - 2c) + c + b_1 = \lambda$. This implies that $b_1 = 3\lambda + 3c - 2k$. Again, from (2.3) and (2.4), $(a_1 + 2c) - (2a_1 + c + b_1) = k - 2\lambda$. Hence $c - a_1 - b_1 = k - 2\lambda$. By this and (2.4), we conclude that $a_1 + 2c - 2(c - a_1 - b_1) = k - \lambda - 2(k - 2\lambda)$. Therefore $3a_1 + 2b_1 = 3\lambda - k$. Thus,

$$b_1 = 3\lambda + 3c - 2k \tag{2.6}$$

$$3a_1 + 2b_1 = 3\lambda - k. \tag{2.7}$$

Set $d = |W \cap W_1 \cap W_2 \cap W_3 \cap B_z|$ and $t_1 = |B \backslash (W \cup W_1)| = 2p - v + \lambda = 2^{n-m} - 1$. We need the following claim.

Claim. $b_1 - d = 2a_2(a - 2) + a_1(a - 4)$ and $d = (t_1 - c)(a - 1) + c(a - 2)$.

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Proof. By Lemmas 2.4 and 2.5(i), N has $p - t_1 = k - \lambda = |O| = 2a_2 + a_1$ orbits on $B \backslash B_z$ and t_1 orbits on B_z. So assume that $B \backslash B_z = \bigcup_{i=1}^{2a_2+a_1} B_i$. By Lemma 2.6(i), $|B_i \cap O| = |B_i \cap O_1| = |B_i \cap (W_2 \backslash W_3)| = |B_i \cap (W_3 \backslash W_2)| = 1$, $i = 1, \ldots, 2a_2 + a_1$. So by Lemma 2.7(i), we may assume that $|B_i \cap (O \backslash (W_2 \cap W_3))| = 1$ for $i = 1, 2, \ldots, 2a_2$, and $|B_i \cap O \cap W_2 \cap W_3| = 1$ for $i = 2a_2 + 1, \ldots, 2a_2 + a_1$. Assume that $B_i \cap O = \{x_i\}$, $i = 1, \ldots, 2a_2 + a_1$. Since z is fixed-point-free on B_i, by Lemma 2.6(iv), for $i = 1, \ldots, 2a_2$, we have $B_i \backslash \{x_i, x_i^z\} \subseteq W \cap W_1 \cap W_2 \cap W_3$, and for $i = 2a_2 + 1, \ldots, 2a_2 + a_1$, we have $|(B_i \backslash \{x_i, x_i^z\}) \cap W \cap W_1 \cap W_2 \cap W_3| = a - 4$. Thus $b_1 - d = 2a_2(a - 2) + a_1(a - 4)$. Since $B_z = \bigcup_{i=|O|+1}^{p} B_i$ by Lemma 2.6(iii), for $i = |O| + 1, \ldots, p$, we have

$$|B_i \cap (B \backslash (W_2 \cup W_3))| = 1 = |B_i \cap (B \backslash (W \cup W_1))|.$$

Hence $d = (t_1 - c)(a - 1) + c(a - 2)$ and the claim holds. □

Recall that $a_2 = c$, $k = (a - 1)p$ and $\lambda = (a - 1)(p - 2^{n-m})$. By (2.6) and our claim above, we conclude that

$$3\lambda + 3c - 2k = 2c(a - 2) + a_1(a - 4) + (t_1 - c)(a - 1) + c(a - 2).$$

From this, since $t_1 = 2^{n-m} - 1$,

$$c(8 - 2a) = (a - 1)(-2^n + 2^{n-m+2}) + a_1(a - 4).$$

Therefore,

$$\begin{aligned} c &= (2^m - 1)2^{n-m+2}(1 - 2^{m-2})/(8 - 2^{m+1}) - a_1/2 \\ &= (2^m - 1)2^{n-m}(1 - 2^{m-2})/(2 - 2^{m-1}) - a_1/2 = (a - 1)2^{n-m-1} - a_1/2. \end{aligned}$$

From this and equation (2.1), $3a_1 + b_1 = 2\lambda - (a - 1)2^{n-m}$. Now by equation (2.7), $b_1 = 3\lambda - k - (2\lambda - (a - 1)2^{n-m}) = \lambda - k + (a - 1)2^{n-m} = 0$. This and Lemma 2.5(ii) imply that $(W_2 \cap W_3 \cap B_z) \subseteq (B \backslash (W \cup W_1))$. Next, we note that by Lemma 2.5(iii), $|W_2 \cap W_3 \cap B_z| = |W \cap W_1 \cap B_z| = t_1(a - 1)$. So $t_1 \geq t_1(a - 1)$ and then $a = 2$. This gives $m = 1$, and (i) and (ii) hold. By (i) and Lemma 2.5(ii), we have $q = 2$. Thus, we see that $2^{n-1} = 2$ and $n = 2$. This gives (iii) and (iv) and the lemma is proved. □

Finally, Theorem 1.3 follows from Lemma 2.8.

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M. REZA SALARIAN, Department of Mathematics,
Kharazmi University, Karaj/Tehran, Iran
e-mail: salarian@khu.ac.ir

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