Introduction

At last, some remarks are made about the transfer of momentum from the sun to the planets, which is fundamental to the theory. The importance of magnetohydrodynamic waves in this respect are [sic] *pointed out.*

> First published mention of the term *magnetohydrodynamic*, from "On the cosmogony of the solar system III" by Hannes Alfvén, 1942, *Stockholm's Observatoriums Annaler*, v. 14, 9.1–9.29.

THE ANCIENT GREEKS knew the universe to be made up of the four elements: earth; water; wind; and fire. Today, we know these as the four states of matter: solid; liquid; gas; and plasma, three of which fall into the realm of *fluid dynamics*. Indeed, more than 99.9% of "ordinary matter" in the universe is in the fluid state and, in particular, the plasma (magnetohydrodynamical) state.[1]

Yet, as a pure science, fluid dynamics has often been omitted from many university undergraduate physics curricula. In fact, if you want to find regularly offered courses in fluid dynamics in a university calendar, you're more likely to find them among the engineering or applied mathematics offerings than physics.

One could come up with a number of reasons for this:

- areas of physics such as classical mechanics, electrodynamics, and quantum mechanics are deemed more "fundamental" and courses such as fluid dynamics get relegated as "optional", if offered at all;
- analytical progress generally requires mathematics not typically understood by most undergraduate students of physics until their fourth year; and
- historically, the really interesting problems required the use of major laboratory facilities (such as those available in a large engineering department) or theorems of advanced applied mathematics.

An alternative to expensive laboratories or a degree in Applied Mathematics is computing. While supercomputers capable of solving interesting problems in fluid dynamics have been available since the mid 1980s, it is only since the turn of the 21st century that *cheap* supercomputing has become widely available so that "ordinary" physicists and astrophysicists can once again do interesting problems in the subject.

Indeed, many of the more "interesting" problems in astrophysics such as those

[1] www.plasma-universe.com/99-999-plasma/.

in star formation, planetary discs, stellar evolution, the interstellar medium, formation of galaxies, galactic and extragalactic outflows and accretion, the early universe, cosmology, even the Big Bang itself have awaited this "promised land" of cheap supercomputing. Now that it has "arrived", more and more of the literature in astrophysics is being devoted to applications of fluid dynamics and, in particular, magnetohydrodynamics. More than for any other practitioner of physics, astrophysicists are finding the role of fluid dynamics is becoming *increasingly* important with time, not less. For this reason alone, I would argue, university physics curricula should be offering more courses in fluid dynamics, lest the discipline be taken over completely by the engineers and applied mathematicians!

Before we start, let us agree on some basic terms and their uses.

1. A *fluid* is a state of matter that can flow. A liquid is an *incompressible* fluid, while gas and plasma (ionised gas) are *compressible* fluids. A more technical definition of a fluid involves the notion of *granularity*, where the *mean free path* (or *collision length* defined as the distance a particle in the fluid can travel, on average, before colliding with another particle), δl, is much less than any measurable scale length of interest ($\mathcal{L}$). When $\delta l \ll \mathcal{L}$, a fluid can be treated as a *continuum* rather than as an *ensemble of particles* which simplifies the governing equations enormously.

2. *Fluid Dynamics*, a term which is interchangeable with *hydrodynamics* (HD), is the physics of fluid flow (compressible or incompressible), and involves the concepts of mass and energy conservation, Newton's second law, and an *equation of state*.

3. *Fluid Mechanics* has come to refer to fluid dynamics from an engineering vantage point, with more emphasis on experimentation than on theory. Typically (but not always), a text entitled *Fluid Mechanics* will be an engineering text, while a text entitled *Fluid Dynamics* will be a physics text. A notable exception is Landau and Lifshitz' classic text *Fluid Mechanics*, which, in many ways, is the definitive treatment of the subject from a theoretical physicist's perspective.

4. *Gas Dynamics* is compressible fluid dynamics in which all the fluid particles are neutral.

5. *Magnetohydrodynamics (MHD)* is compressible or incompressible fluid in which an appreciable fraction of the particles are charged (ionised) and where charge neutrality is observed at all length scales of interest. Thus, within any volume element however small, there must be as many negative charges as positive. In an MHD fluid, circulation of charged particles at the sub-fluid length scale implies a current and thus a magnetic field which, in turn, interacts with ionised particles on the post-fluid length scale. Note that an MHD fluid need not be 100% ionised for the equations of MHD to apply (*e.g.*, Chap. 10 on

non-ideal MHD). Neutrals in a partially (even a few percent) ionised fluid can couple to the magnetic field via collisions with charged particles. By contrast, a completely neutral gas can neither generate nor interact with a magnetic field.

An MHD fluid can be created from an HD fluid by increasing the ionisation fraction. For a gas, this can be done by increasing its temperature and thus compressible MHD fluids are plasmas. For a liquid such as water, the ionisation fraction can be increased by dissolving salts. While the earth's oceans permeated by the earth's magnetic field technically constitutes an MHD fluid, the weakness of the earth's magnetic field ($\sim 4 \times 10^{-5}$ T, $\beta \sim 10^9$ defined in §5.2)[2] and the extremely low fraction of particles that are ionised renders the MHD effects just about immeasurable.

6. *Plasma Physics* is the study of the collective behaviour of an ensemble of charged particles at length scales smaller than the fluid length scale thereby rendering the MHD equations inapplicable. Plasma physics is generally described by the *Vlasov–Boltzmann equation* which can account for non-fluid-like behaviour such as charge separation and plasma oscillations. An MHD fluid can be described as a plasma in which charge neutrality is observed at all length scales of interest, and thus MHD is an important special case of plasma physics. An excellent first text on plasma physics, which is beyond the scope of this text, is Volume 1 of Francis Chen's now-classic text *Plasma Physics and Controlled Fusion* (1984).

The equations of MHD reduce to the equations of HD when the magnetic induction ($\vec{B}$) is set to zero. As we shall see, HD becomes MHD by adding the Lorentz force to the hydrodynamic version of Newton's second law, and by introducing Faraday's law of induction that governs how the magnetic induction evolves. These modifications, which will seem rather elementary when first introduced, belie the incredible complexity magnetism provides an ionised fluid. For example, while a hydrodynamical fluid can support compressive waves only (and thus, much of HD can be understood in one dimension), the tension along lines of magnetic induction allow a magnetohydrodynamical fluid to support transverse waves as well, thus requiring all three dimensions to describe.

To understand MHD is to understand wave mechanics, and much of this text is devoted to building the students' mathematical skills and physical intuition in this area. By the end of Part I, the student will be able to solve the most complex MHD problem one can do exactly (albeit, semi-analytically), namely the MHD Riemann problem. And while the development of a general, multidimensional computer code

[2]Strictly speaking, it is the magnetic *induction*, $\vec{B}$, that has units tesla while the magnetic *field*, $\vec{H} = \vec{B}/\mu$ (App. B), has units ampere/metre. Thus, the earth's magnetic *field* is about 30 A/m. In this book, I attempt to be consistent with this distinction by using the term *magnetic field* when referring to magnetism generically, and *magnetic induction* when reference is to $\vec{B}$ specifically although, for the most part and especially in astrophysics, this difference is largely academic since all that separates them is the constant μ_0.

to solve more complex problems in MHD is beyond the scope of this book, the 1-D Riemann problem and the ideas upon which it is based are at the core of virtually every general computer program written and with which a whole host of interesting (astro)physical problems become accessible.