paper by Brown [1] but for completeness sake we give some details here, taking a more elementary approach than Brown's.

First let $\alpha = k_0 + k_1 a + k_2 a^2 + k_3 a^3 + k_4 b + k_5 ab + k_6 a^2 b + k_7 a^3 b$ be an arbitrary element of R, with $k_i \in \mathbb{Z}_2$ for $i = 0, \ldots, 7$. Then $\alpha \in C$ precisely when α commutes with both generators a and b of G. Comparing αa, αb with $a\alpha$, $b\alpha$ respectively, gives that $\alpha \in C$ precisely when $k_1 = k_3$, $k_4 = k_6$ and $k_5 = k_7$. Thus $\alpha \in C$ if and only if

$$(*) \qquad \alpha = k_0 + k_1(a + a^3) + k_2 a^2 + k_4(b + a^2 b) + k_5(ab + a^3 b).$$

It follows that C has 32 elements and the 16 non-units, that is those with even support, form the unique maximal ideal M of C. Moreover, using the expression (*) above, an easy calculation shows that $M^2 = 0$ and for any nonzero element x of M the principal ideal Cx is simply $\{0, x\}$. It follows that C is its own classical quotient ring (since every element of M is a zero-divisor) and given any two distinct nonzero elements x, y of M we may define a C-homomorphism $f: Cx \to C$ by $f(x) = y$. Since $Cx \cap Cy = 0$, f can not be extended to an endomorphism on C. Hence, by Baer's criterion for injectivity, C is not self-injective.

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