

HAYMAN, W. K. and KENNEDY, P. B., *Subharmonic functions Vol. 1* (London Mathematical Society Monographs No 9, Academic Press, London, 1976), xvii+284 pp., £11.60.

This book, begun by the late Professor Kennedy and completed by Professor Hayman, is the first of a two-volume project giving a systematic introduction to the theory of subharmonic (s.h.) functions in  $\mathbf{R}^m$  ( $m \geq 2$ ), with some applications. It steers an intermediate course between potential theory and function theory. As the only book of its kind it will be indispensable to graduate workers in function theory, but it does not make excessive demands on the reader and could be read profitably by any analyst or by anyone with an interest in abstract potential theory. It is essentially self-contained, and consists of five chapters whose length increases steadily from 39 to 76 pages.

The first chapter, devoted to preliminary results, includes some properties of semi-continuous and convex functions, a minitreatment of Lebesgue integration, Green's theorem, harmonic functions in  $\mathbf{R}^m$ , Poisson's formula, the Dirichlet problem for a hyperball, and the  $m$ -dimensional forms of Harnack's inequality and theorem. Chapter 2 gives elementary properties of s.h. functions, Perron's solution of the Dirichlet problem, and Littlewood's concept of subordination for functions regular in a disc. In Chapter 3 F. Riesz's theorems on the representation of a positive linear functional by a measure, and the representation of a s.h. function locally as the sum of a potential and a harmonic function, are proved. The existence of the Green's function for a regular domain in  $\mathbf{R}^m$  is established, and is used to obtain a representation of a s.h. function which is a generalisation of the classical Poisson–Jensen formula. A version of Nevanlinna's first fundamental theorem for functions s.h. in an open ball leads to a characterisation of bounded s.h. functions in  $\mathbf{R}^m$  ( $m \geq 3$ ). Chapter 4 begins with theorems which generalise to  $\mathbf{R}^m$  the representation theorems of Weierstrass and Hadamard for a function as a product in terms of its zeros. The notions of asymptotic value and tract are then generalised to functions s.h. in  $\mathbf{R}^m$ . Chapter 5 is concerned with capacity and polar sets (which are the analogue in potential theory of sets of measure zero), and ends with an account of Choquet's theory of capacitability.

The whole book is beautifully written. Each chapter begins with a summary of its content, and the individual sections are also well motivated. To the end the pace seems admirably unhurried, and the book succeeds better than any other I have seen in making  $\mathbf{R}^m$  appear as simple as  $\mathbf{R}^2$ ; in part this may be due to the avoidance of a special notation for vectors. Printing and layout are excellent. A minor grumble is that it would be easier to refer back if theorems bore the numbers of the sections containing them; failing that, their headings might have been printed in bold type. There are some misprints, but the ones which I found were mainly non-mathematical and hence unimportant.

This is an addition to the mathematical literature which I welcome unreservedly.

PHILIP HEYWOOD

LANG, SERGE, *Introduction to Modular Forms* (Grundlehren der mathematischen Wissenschaften 222, Springer-Verlag, 1976), 261 pp., Cloth DM 54.00, U.S. \$22.20.

This is a valuable work providing a survey of different parts of the exceptionally active field of recent work on modular forms, their applications and their connexion with the arithmetic of number fields. It is, however, in no sense acceptable to ordinary mathematicians outside the Deligne–Langlands circle as an *introduction* to the study of modular forms. The fundamentals of the subject are very rapidly covered in the first 54 pages in a condensed form, and frequent reference is made to other authors in order to fill in details. Thus Hecke operators, which are normally met at a comparatively late stage in the theory, occur as early as p. 16, where they are defined as operators on the free abelian group generated by the lattices in the complex plane.

It would be wrong, however, to give the impression that the book is intended solely for the sophisticated reader and for those already having some familiarity with the subject. There are later chapters, such as Chapters 10, 11 and 14, where the going is comparatively easy and provided that the reader can remember the definitions of all the symbols (no defining list is included) and is prepared to use a little intelligent guess-work; for example, on p. 178 the undefined symbol  $B$  is presumably (and after one realises it, obviously) the standard Borel subgroup introduced on the previous page.

The author has clearly tried to give the reader a taste of a number of new fields in which modular forms make their appearance and has achieved some success in this objective. He goes far enough to provide the reader with the flavour of recent work, although in some cases one may feel that

additional piquancy would have been added if he had progressed slightly further; in particular, some explicit examples of the work of Swinnerton-Dyer and Serre on congruences might have been included.

Postscript: The reviewer was flattered to find himself included in the preface as a member of a quartet who kept the flag of modular function theory flying during the lean years before the recent explosion of interest in the subject. The author is incorrect, however, in assuming that H. Petersson was the only one of the four who did not contribute to the 1956 Bombay Colloquium on zeta-functions; all four were present there and contributed papers.

R. A. RANKIN

KALLENBERG, OLAV (editor), *Random Measures* (Akademie-Verlag and Academic Press, 1977), 104 pp., £6.

The mathematical foundations of point process theory have been developed rapidly in recent years; a monograph was urgently needed to collate the profusion of results. One such volume is the inaccessible German work of Kerstan, Matthes and Mecke. Kallenberg has provided a concise formal treatise incorporating many of his own improvements. One regret is that both works date essentially from 1974 and so do not include the continuing improvements and new concepts such as Papangelou's conditional intensity measures.

Kallenberg considers random measures on locally compact second countable Hausdorff spaces. The generalisation to random measures is mathematically natural and has technical applications, but most readers will find the point process case more intuitive. Few concessions are made to the novice who may well wonder why a point process is defined to be an integer-valued random measure, or what a Palm probability "means". The survey by Daley and Vere-Jones (in "Stochastic Point Processes" edited by P. A. W. Lewis, Wiley, 1972) is useful collateral reading.

"Random Measures" will remain for some years an invaluable reference work.

B. D. RIPLEY

KUSSMAUL, A. U., *Stochastic Integration and Generalized Martingales* (Pitman, 1977), xi+163 pp., £7.00.

The author states his aim "to imbed the theory of stochastic integration into a functional analytic framework". For right-continuous stochastic processes  $X$  and  $Z$ ,  $Z \rightarrow \int_0^\infty Z_t dX_t$  is a "measure" with values in  $L^p$ , the space of  $p$ -th power integrable random variables. Kussmaul defines this measure by extension theorems for vector-valued measures. Under localisation he finds the necessary and sufficient condition on  $X$  for its existence to be that  $X$  is a *quasimartingale*, i.e. the difference of two non-negative supermartingales.

I could not decide on the intended audience. A strong background in Banach spaces is needed, and uniform integrability is taken for granted, yet half the volume is devoted to well-known properties of martingales. I imagine very few readers will not be familiar with this material, so I recommend starting at Section 8.

I was irritated by the use of "modification" and "semimartingale" in new senses; at one point even a "finite set" has a new meaning! This is a photographically produced "Research note in mathematics"; commendably it has a (short) index and a list of symbols but the proofreading has been inadequate.

This treatise can only be recommended to experts in the field.

B. D. RIPLEY

CURTAIN, RUTH F. and PRITCHARD, A. J., *Functional Analysis in Modern Applied Mathematics* (Academic Press, London, 1977), ix+339 pp., £10.80.

Functional analysis has become a major tool in applied mathematics. Nevertheless, the authors point out in the Introduction, for an applied mathematician a "working knowledge of functional analysis . . . is not readily obtained by reading a standard text on functional analysis". Also, in spite of excellent books on applications in specific areas the authors felt "that there was a need for a book