

RESEARCH ARTICLE

Greenhush and greenwash: a signalling game analysis of strategic environmental disclosure

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Abstract

This paper presents a theoretical framework to explain how firms strategically choose between truthful disclosure, greenwash (overstating environmental performance) or greenhush (deliberately under-communicating positive environmental actions). The analysis reveals that greenhush arises as an equilibrium when signalling costs exceed benefits from investor support, particularly when firms can secure sales without environmental claims. Greenwash emerges when penalties for false claims are insufficient relative to market premiums. Notably, increasing investor support for environmental initiatives reduces greenhush but may unintentionally promote greenwash rather than truthful disclosure without complementary regulatory mechanisms. The results suggest several policy strategies to promote truthful labeling: strengthening certification credibility by increasing the cost differential between legitimate and fraudulent certification, calibrating penalties to ensure separating equilibria and developing coordinated approaches that simultaneously target investor preferences andverification mechanisms.

Keywords: CSR; eco-labels; ESG; greenhush; greenwash

JEL classification: D8; L1; Q5

1. Introduction

Environmental sustainability has become an essential facet of corporate strategy. Against a backdrop of heightened scrutiny, firms carefully orchestrate their environmental communications through sustainability reports, press releases, corporate websites and eco-labels (Lyon and Montgomery, 2013). The strategic decisions firms make about how to communicate their environmental initiatives have led to two contrasting phenomena: greenwash and greenhush.

Greenwash involves firms overstating their environmental performance to appear more sustainable than they actually are Lenox (2006), Delmas and Montes-Sancho (2009), Lyon and Montgomery (2015). This practice is particularly common in jurisdictions with weak eco-label regulation, such as the United States, where firms face

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a low likelihood of penalties for misleading claims (Negash and Lemma, 2020). The literature on greenwash is vast, examining its determinants, prevalence and consequences across various industries and regulatory contexts.

In contrast, greenhush (also known as 'brownwashing', 'strategic silence', 'green blushing', 'green muting' or staying 'quiet' about corporate social responsibility activities) refers to the paradoxical behaviour where firms deliberately under-communicate or even hide their positive environmental initiatives (Makower and Pike, 2008, Kim and Lyon, 2015, Carlos and Lewis, 2018, Ginder *et al.*, 2021, Wang *et al.*, 2021). This challenges the conventional wisdom that firms exceeding regulatory requirements would eagerly publicize their actions to regulators, customers and the public (Denicolò, 2008).

Recent evidence suggests that greenhush is becoming a widespread phenomenon. A 2022 report from sustainability consultancy South Pole surveyed over 1,200 executives and found that nearly 25 per cent say they do not plan to publicize their net-zero emissions reduction targets 'beyond the bare minimum or as required' (Kähkönen *et al.*, 2022). This finding has catalysed media and industry attention to the phenomenon (Letzing, 2022, Makower, 2022, Chen, 2023, Dhanani, 2023).

Specific examples of greenhush, though difficult to identify by their nature, have been documented. The Guardian reported in 2019 that a Levi Strauss factory reduced water usage from 316 litres to 1 litre per pair of jeans, yet kept the technique a trade secret (Coburn, 2019). Similar cases include Portuguese wineries quietly switching from conventional to organic practices and a well-known car manufacturer reducing energy consumption by 75 per cent without public disclosure. More recently, The Washington Post reported that BlackRock removed its previous declaration of commitment to net-zero emissions from its sustainable investing webpage, while Anheuser-Busch InBev discontinued advertisements about its environmental goals (Joselow, 2023).

Empirical investigation of greenhush can be traced to Font *et al.* (2017). They found that tourism businesses in England communicated only 30 per cent of their sustainability actions, and attributed this finding to fear that advertising sustainability practices would lead customers to anticipate a worse experience. Similarly, Delmas and Grant (2014) found that wineries often choose not to disclose that their products are organic, as these eco-labels have been shown to have a negative effect on wine prices.

The existence of greenhush raises important questions about corporate communication strategies. Why would firms deliberately under-communicate their positive environmental initiatives? When is greenhush preferred to greenwash, or vice-versa? How do stakeholder preferences influence this strategic choice? Understanding these decisions is critical not only for environmental scholars but also for managers navigating sustainability communications and policymakers designing effective eco-labelling programs.

1.1. Stakeholder theory and environmental communication

This paper addresses these questions by applying stakeholder management theory (Freeman, 1984) within a signalling game framework. Stakeholder theory postulates that firms must cater to multiple stakeholder interests beyond merely maximizing

shareholder wealth (Donaldson and Preston, 1995). According to Freeman (1984), stakeholders include any group or individual who can affect, or is affected by, the achievement of the corporation's purpose, such as shareholders, employees, customers, suppliers, local communities and governmental groups.

Mitchell *et al.* (1997) posit that stakeholder salience (which groups have the most power) is dynamic and determined by three attributes: (i) Power: The ability to influence the firm, (ii) Legitimacy: Whether the stakeholder's claims are seen as proper or appropriate and (iii) Urgency: The degree to which stakeholder claims call for immediate attention. In our model, investors exert power through their impact on the firm's cost structure, while consumers exert power through purchasing decisions affecting revenue.

While there is substantial literature about the divergence of stakeholder interests (Wolfe and Putler, 2002), formal modelling of how these divergent preferences shape environmental communication strategies remains limited. To our knowledge, only Kim and Lyon (2015) have applied stakeholder theory to explaining greenhush, describing it as context-sensitive and dynamic; for instance, regulators become more salient when a firm is growing, while shareholders gain salience when profits are low or during economic deregulation.

It is well-established that consumers generally respond positively to sustainability cues from corporations, as evidenced by higher willingness to pay for sustainably marketed products (Accenture, 2019). Consumers are increasingly demanding more sustainable and environmentally responsible practices, though they are highly sensitive to a firm's perceived motives, often distinguishing between self-serving and genuinely socially driven environmental, social and governance (ESG) activities (Chernev and Blair, 2015).

In contrast, investor preferences regarding sustainability are more heterogeneous and context-sensitive. While some investors prioritize ESG factors and support firms with strong environmental performance, others focus primarily on financial returns and may view environmental initiatives as detracting from profitability (Kraik, 2020). This variability derives from political differences (Saad, 2023) and differing perceptions of how ESG efforts impact financial performance or fiduciary duties (Schanzenbach and Sitkoff, 2020).

Malshe *et al.* (2023) posit that stakeholders not only diverge on environmental issues but also assign different weights to social and governance aspects of ESG. Their research shows that consumer brand equity is positively correlated with environmental performance, while institutional investor ownership is negatively correlated. Institutional investors may prioritize governance aspects because these directly correlate with long-term value creation, sometimes at the expense of environmental considerations. In some regions, particularly the United States, there has been growing anti-ESG sentiment favouring institutional divestment from ESG initiatives (Malone *et al.*, 2023, Welsh, 2023, Hilton, 2025).

1.2. Eco-labels as signals

Green firms face a significant challenge when attempting to capitalize on consumer demand for sustainable products due to information asymmetry: consumers typically

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cannot discern the production process, making it difficult to identify genuinely green products. Since environmentally friendly techniques are generally more costly, firms may be disinclined to adopt them without a means to credibly signal their greenness.

Eco-labels represent one means to ameliorate this informational asymmetry. These labels advertise positive environmental attributes (or the absence of negative ones) associated with a product or its production process. They may be provided by a third party as the result of a certification process or developed by firms themselves. The signalling aspect of eco-labels is well-suited to game-theoretic analysis, and such models have previously been used to analyse the interaction of eco-labels with pollution levels (Mattoo and Singh, 1994, Ibanez and Grolleau, 2008, Sengupta, 2024), mandatory environmental disclosure (Garrido *et al.*, 2020) and tax incentives for disclosure (Jamalpuria, 2012).

1.3. Contribution and organization

This paper contributes to the environmental communication literature in several important ways. Firstly, it provides the first formal model of greenhush as a strategic communication choice within a signalling game framework, addressing a significant gap in theoretical understanding of this increasingly documented phenomenon. Secondly, it integrates the influence of different stakeholders – specifically consumers and investors – to explain how firms choose truthful disclosure, greenwash or greenhush. Thirdly, it derives conditions for the coexistence of multiple equilibria and analyses their welfare implications, showing how market outcomes depend on the interplay between regulatory parameters, stakeholder preferences and firm characteristics.

The results reveal that greenhush emerges as a rational equilibrium strategy when the costs of environmental signalling exceed the benefits, particularly when investor support for environmental initiatives is weak. We identify specific parameter conditions under which greenhush dominates other strategic options, showing that it is not an anomaly, but rather a rational outcome in certain market environments. Additionally, our model identifies three distinct equilibrium regimes – separating (truthful disclosure), greenwash pooling and greenhush pooling – and precisely characterizes the boundaries between these regimes based on certification costs, penalties for false claims and investor preferences.

The remainder of the paper is organized as follows. Section 2 presents the model setup, including the firm, consumer and investor characteristics, with firm types explicitly defined as green (*G*) or brown (*B*). Section 3 analyses the various equilibria that can emerge, including truthful separation, greenwash pooling and greenhush pooling, with economic intuition provided for each result. Comparative statics are provided to illustrate how changes in key parameters affect equilibrium outcomes. Finally, section 4 discusses the implications of the findings and suggests directions for future research.

2. Model

2.1. Overview and players

Consider a signalling game involving two players: a firm producing a single good and a consumer with unit demand. The firm has a standing relationship with an

institutional investor – an external capital provider whose preferences can influence the firm's costs. The firm possesses private information about its production type, which is unobservable to either the consumer or investor.

The investor harbors preferences concerning the firm's environmental strategy, quantified by a parameter $\gamma_I \in [0,1]$. A value of zero signifies complete opposition or indifference to sustainable production, whereas a value of one indicates maximum support. We assume the firm knows γ_I deterministically.

The firm offers its product to the consumer at a fixed price p > 0, and the consumer's demand is inelastic for one unit, conditional on purchase. The firm's production process falls into one of two categories: it is either environmentally sustainable, designated as the green type (G), or it employs conventional methods, designated as the brown type (B). Nature determines the firm's type $g \in \{G, B\}$, with the prior probability of the firm being green denoted by q, where $q \in (0, 1)$.

2.2. Consumer preferences and decision

The representative consumer derives utility V_G from consuming the green product and utility V_B from consuming the brown product. The empirical evidence of a green price premium in many markets supports the idea that consumers generally value the green product more. Therefore, we assume positive perceptions of the green good – the price must be such that a perfectly identified green product is desirable, while a perfectly identified brown product is less so (or not desirable above its utility value). Formally, we assume $V_G > p > V_B \ge 0$.

The consumer does not observe the firm's true type g. After observing the firm's signal s, the consumer forms a posterior belief $\mu_s = Pr(G|s)$ representing the probability that the firm is green. The consumer's expected utility from purchasing the product is then:

$$E[U_C|s] = \mu_s V_G + (1 - \mu_s) V_B - p.$$

The consumer maximizes expected utility and will purchase the good if $E[U_C|s] \ge 0$. This condition is met if:

$$\mu_s V_G + (1 - \mu_s) V_B \ge p. \tag{1}$$

Rearranging inequality (1) gives the threshold belief $\bar{\mu}$:

$$\mu_s(V_G - V_B) \ge p - V_B \implies \mu_s \ge \frac{p - V_B}{V_G - V_B} \equiv \bar{\mu}.$$

Given the assumption $V_G > p > V_B$, it follows that $0 < \bar{\mu} < 1$. The consumer purchases if his posterior belief μ_s meets or exceeds this threshold $\bar{\mu}$. The purchase decision, given signal s and posterior belief μ_s , is therefore deterministic for the representative consumer. We denote the purchase outcome (1 for purchase, 0 for no purchase) by θ_s :

$$\theta_s = \mathbf{1}_{\mu_s \ge \bar{\mu}} = \begin{cases} 1 & \text{if } \mu_s \ge \bar{\mu} \\ 0 & \text{if } \mu_s < \bar{\mu} \end{cases}$$

where 1 is the indicator function.

2.3. Signals and costs

To convey information about its environmental practices, the firm chooses one of three distinct signals, $s \in \{\text{Cert}, \text{Uncert}, \text{None}\}.$

- s = Cert: A certified claim using a third-party verified eco-label.
- *s* = Uncert: An uncertified, self-declared claim about environmental performance.
- s = None: No environmental claim is made.

Associated with these signals are fixed costs incurred by the firm upon choosing the signal. The cost of making no claim is $L_{\rm None}=0$. Making an uncertified claim incurs a fixed cost $L_{\rm Uncert}\geq 0$. Obtaining and using a certified label incurs a fixed cost which depends on the firm's type: L_G for the green firm and L_B for the brown firm. This creates a cost hierarchy: $p>L_B>L_G>L_{\rm Uncert}>L_{\rm None}=0$; the price is assumed to be greater than the label cost, so that both types have an incentive to certify. The cost of certification is higher for the brown firm because the firm must expend time and resources on deceiving the certifying body.

Penalties arise if the brown firm makes a false-positive environmental claim ($s \in \{\text{Cert}, \text{Uncert}\}$). Let K represents the expected costs from sanctions or reputation damage. The penalty is only incurred if the consumer purchases the product ($\theta_s = 1$), and thus incurs some harm from deception. The green firm, whose claims are truthful, faces no such penalties. Penalties are assumed to be deterministic, conditional on purchase.

Both firm types incur a base production cost C. Additionally, the green firm incurs costs related to its sustainable practices. This green investment cost is influenced by the investor's preference, γ_I , and is contingent on the disclosure of the green initiative ($s \in \{\text{Cert}, \text{Uncert}\}$). The green investment cost, $C_{\text{green}}(g, s)$, is assumed linear in γ_I and is conditional on the firm type and the signal decision, therefore:

$$C_{\text{green}}(g, s) = \begin{cases} a - \gamma_I \text{ if } G \text{ and } s \in \{\text{Cert, Uncert}\} \\ a \text{ if } G \text{ and } s = \text{None} \\ 0 \text{ if } B \end{cases}$$

Here, a>0 is the base investment cost. We assume $a\geq \gamma_I$ to ensure non-negative net investment cost. As investor support increases, the net cost $a-\gamma_I$ decreases.

2.4. Game structure

The sequence of events unfolds as follows:

- (1) Nature assigns the firm's type $g \in \{G, B\}$ according to the prior probability $q = \Pr(g = G)$.
- (2) The firm observes its own type g, the investor preference γ_I , the consumer valuations V_G , V_B and the market price p. Based on this information, the firm decides whether to produce and selects its signal $s \in \{\text{Cert}, \text{Uncert}, \text{None}\}$.

- (3) The consumer observes the firm's chosen signal s, but not its actual type g or the investor preference γ_I . The consumer updates his prior belief q to a posterior belief $\mu_s = \Pr(g = G|s)$ using Bayes' rule where possible.
- (4) The consumer decides whether to purchase the product based on comparing μ_s to the threshold $\bar{\mu}$.

2.5. Payoffs

The firm's expected profit, $E[\pi_s^g]$, depends on its type g and chosen signal s. The structure is revenue (if purchase occurs) net of base production cost, environmental investment cost (if any), signalling cost and penalty (if any and purchase occurs).

For the green firm, the profits for each signal are:

$$\begin{split} \pi_{\text{Cert}}^G &= \theta_{\text{Cert}} p - C - (a - \gamma_I) - L_G \\ \pi_{\text{Uncert}}^G &= \theta_{\text{Uncert}} p - C - (a - \gamma_I) - L_{\text{Uncert}} \\ \pi_{\text{None}}^G &= \theta_{\text{None}} p - C - a. \end{split}$$

For the brown firm, the profits are:

$$\begin{split} \pi_{\text{Cert}}^B &= \theta_{\text{Cert}} p - C - L_B - \theta_{\text{Cert}} K \\ &= \theta_{\text{Cert}} (p - K) - C - L_B \\ \pi_{\text{Uncert}}^B &= \theta_{\text{Uncert}} p - C - L_{\text{Uncert}} - \theta_{\text{Uncert}} K \\ &= \theta_{\text{Uncert}} (p - K) - C - L_{\text{Uncert}} \\ \pi_{\text{None}}^B &= \theta_{\text{None}} p - C. \end{split}$$

Note that the brown firm incurs no investment cost and faces potential penalties only if its signal induces purchase ($\theta_s = 1$). The game structure can now be visualized as in figure 1.

The equilibrium concept employed is Perfect Bayesian Equilibrium (PBE). A PBE consists of a strategy profile for the firms (choice of s for each type g) and a belief system for the consumer (μ_s for each s) such that strategies are sequentially rational given beliefs, and beliefs are consistent with strategies using Bayes' rule on the equilibrium path. Off-path beliefs must also be specified. Because the equilibria are derived by ruling out profitable deviations, all equilibria in the next section survive the Cho and Kreps (1987) Intuitive Criterion by construction.

3. Equilibrium analysis

In a separating PBE, the green firm and brown firm must choose different signals, $s_G \neq s_B$. By Bayes' rule, this leads to posterior beliefs $\mu_{s_G} = 1$ and $\mu_{s_B} = 0$. Since $\bar{\mu} = \frac{p-V_B}{V_G-V_B} \in (0,1)$, it follows that $\mu_{s_G} = 1 > \bar{\mu} > 0 = \mu_{s_B}$. Therefore, the consumer will purchase after observing a label ($\theta_{s_G} = 1$) and will not purchase in the absence of a label ($\theta_{s_B} = 0$). We assume that the certified label is always the more informative, so that $\mu_{\text{Cert}} \geq \mu_{\text{Uncert}}$.

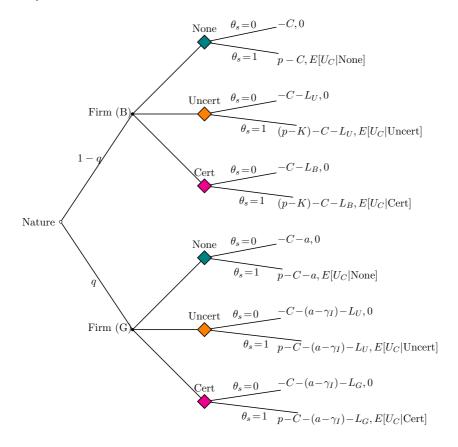


Figure 1. Game tree representation. Note: L_U denotes L_{Uncert} .

3.1. Separating equilibria

Proposition 1. Assume $V_G > p > V_B \ge 0$, implying $\bar{\mu} = \frac{p - V_B}{V_G - V_B} \in (0, 1)$, and $\mu_{Cert} \ge \mu_{Uncert}$. The following equilibria can be sustained:

Truthful Certification (TC): The green firm chooses a certified claim ($s_G = Cert$) and the brown firm chooses no claim ($s_B = None$) if $p \le L_B + K$. This equilibrium is supported by consumer beliefs $\mu_{Cert} = 1$, $\mu_{None} = 0$, and any off-equilibrium path belief $\mu_{Uncert} < \bar{\mu}$.

Truthful Self-Declaration (TSD) equilibrium: The green firm chooses an uncertified claim ($s_G = Uncert$) and the brown firm chooses no claim ($s_B = None$), if $p \le L_{Uncert} + K$. This equilibrium is supported by consumer beliefs $\mu_{Uncert} = 1$, $\mu_{None} = 0$, and off-equilibrium path belief $\mu_{Cert} = 1$.

Proposition 1 identifies the conditions under which the green firm can successfully differentiate itself from the brown type, leading to full information revelation in the marketplace. In both separating equilibria the green firm makes a claim that induces

purchase, while the brown firm remains silent and does not sell. The core mechanism enabling separation follows standard signalling logic: the signal must be sufficiently costly or risky for the brown type to mimic, creating a separating equilibrium where communication credibly reveals type.

In the TC equilibrium, separation hinges on the combined cost of obtaining certification (L_B) and the penalty for greenwash (K). The certification process imposes higher costs on brown firms attempting to falsely verify environmental credentials $(L_B > L_G)$, making mimicry unattractive when penalties or certification costs are sufficiently high. Additionally, the consumer's pessimistic belief about uncertified claims $(\mu_{\text{Uncert}} < \bar{\mu})$ serves as a crucial off-equilibrium deterrent, preventing both types from deviating to the cheaper uncertified signal.

The investor preference parameter γ_I influences the green firm's profitability but does not directly determine the separation condition itself. Higher values of γ_I reduce the effective environmental investment cost for green firms that signal, thereby making signalling more attractive. This can expand the parameter space where green firms satisfy their participation constraint and willingly enter the market with environmental claims. However, the actual separation condition depends primarily on the brown firm's incentives.

3.2. Pooling equilibria

Proposition 2. (Pooling Equilibria) Assume $V_G > p > V_B \ge 0$, implying $\bar{\mu} = \frac{p - V_B}{V_G - V_B} \in (0,1)$, and $\mu_{Cert} \ge \mu_{Uncert} \ge \mu_{None}$. The following mutually exclusive pooling PBEs can be sustained:

- (1) **Certification Greenwash (CGW):** Both firm types choose $s^* = Cert$ if $\mu_{None} \le \mu_{Uncert} < \bar{\mu} \le q$ (purchase occurs on path, not off-path) and $p \ge L_B + K$. This equilibrium is supported by consumer belief $\mu_{Cert} = q$.
- (2) **Self-Declaration Greenwash (SGW):** Both firm types choose $s^* = Uncert$ if $\mu_{None} < \bar{\mu} \le q \le \mu_{Cert}$ (purchase occurs on path and off-path) and $p \ge L_{Uncert} + K$. This equilibrium is supported by consumer belief $\mu_{Uncert} = q$.
- (3) **Greenhush (GH):** Both firm types choose $s^* = N$ one if:
 - (a) $\bar{\mu} \leq q < \mu_{Uncert}$ (purchase occurs on path and off-path) and $L_{Uncert} \geq \gamma_I$.
 - (b) $q < \mu_{Uncert} \leq \mu_{Cert} < \bar{\mu}$ (purchase does not occur on or off-path) and $L_{Uncert} \geq \gamma_I$.

This equilibrium is supported by consumer beliefs $\mu_{None} = q$, and off-path beliefs $\mu_{Uncert}, \mu_{Cert} \geq q$.

In greenwash equilibria, both green and brown firms make identical environmental claims, creating information asymmetry that prevents consumers from distinguishing between genuine and false environmental attributes. Proposition 2 reveals that this manifests in two distinct forms.

The first, CGW, emerges when the potential profit from false certification exceeds the costs ($L_B + K$). Even with third-party verification, if certification costs for brown

firms are not prohibitively high or penalties for false claims are insufficient, brown firms can profitably imitate green firms.

The second, SGW, occurs when both types use uncertified environmental claims. This is possible when the profit from false claims exceeds the cost of making uncertified claims ($L_{\rm Uncert}+K$). Since uncertified claims are typically cheaper than certification, this form of greenwash represents a more accessible deception strategy: the proliferation of vague, unverified environmental claims in markets where making such claims is relatively inexpensive and penalties for misleading communications are weak. The model predicts that as the costs of making environmental claims decrease, the likelihood of SGW increases unless correspondingly stronger enforcement mechanisms are implemented.

Both greenwash equilibria require consumers to have sufficiently optimistic prior beliefs $(q \geq \bar{\mu})$ to ensure purchase occurs on the equilibrium path. This reflects an important economic insight: greenwash thrives in markets where consumers have favourable predispositions towards firms' environmental credentials, making them willing to purchase based on average environmental quality rather than verified performance.

Greenhush manifests as a pooling equilibrium where both green and brown firms remain silent about environmental attributes. Proposition 2 reveals two distinct economic rationales for greenhush, depending on consumer prior beliefs.

Market-Driven Greenhush: With optimistic priors $(q \geq \bar{\mu})$, green firms may still choose silence if signalling costs outweigh benefits from investor sentiment. This type of greenhush arises when the market does not require an environmental signal to generate sales, so a purchase will occur both on and off the equilibrium path. The green firm can secure revenue simply by existing in a market with a sufficiently high average quality perception. Making an environmental claim offers the potential benefit of reduced costs due to investor support (γ_I) but incurs a signalling cost (L_{Uncert}) . If the guaranteed revenue plus the saved signalling cost is greater than or equal to the potential investor benefit, the firm rationally chooses silence. This highlights that greenhush can occur even when firms have positive environmental stories to tell, simply because the cost of telling the story outweighs the incremental benefit, especially when the primary benefit is already secured without signalling.

High prices (p), high signalling costs $(L_{\rm Uncert})$ and low investor support (y_I) favour this type of greenhush. The brown firm naturally prefers silence as well, as it gets revenue p without incurring signalling costs or penalties. In this model, the firm types are fixed, so a green firm will simply remain silent rather than retooling its production processes. Thus, this explanation is compelling for explaining greenhush in the short run, but not in the long run. Since the empirical literature on this topic is relatively recent, and mostly cross-sectional, it is difficult to say whether greenhush is a transitory phenomenon.\frac{1}{2}

Reputation-Constrained Greenhush: With pessimistic priors $(q < \bar{\mu})$, green firms remain silent because signalling alone will not convince skeptical consumers to

¹Even the few longitudinal studies (Huang *et al.*, 2022) have little to say about the long-run dynamics of greenhush.

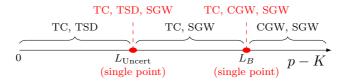


Figure 2. Equilibrium regions based on brown firm's net gain (p - K).

purchase. The low q means the baseline level of market skepticism is too high for the consumer to purchase based on the probable firm type (i.e., without a perfectly revealing signal). If the industry has a history of greenwashing, weak regulation, or operates in a sector perceived as inherently polluting (e.g., fossil fuels, certain types of manufacturing), consumers might rationally start with a low prior belief ($q < \bar{\mu}$). They are skeptical by default that any firm they encounter in this space is genuinely green.

Since no signal can credibly convince the consumer to buy (within this equilibrium structure), the communication decision again hinges on the trade-off between the signalling cost ($L_{\rm Uncert}$) and the non-market benefit from investors (γ_I). If the cost of making even the cheapest claim exceeds the benefit derived from investor support, the green firm chooses silence. This captures situations where firms possess genuine environmental credentials but face such profound consumer distrust that communication efforts are perceived as futile for market access. Silence becomes the cost-minimizing strategy when market channels are effectively closed, unless non-market stakeholder benefits are sufficiently high.

3.3. Multiple equilibria

Theorem 1 (Multiple Equilibria). Assume $V_G > p > V_B \ge 0$, and $q \ge \bar{\mu}$, where $\bar{\mu} = \frac{p - V_B}{V_G - V_B}$. The following statements characterize the price ranges that admit multiple equilibria:

- (1) Neither separating equilibrium can coexist with its corresponding pooling equilibrium except at knife-edge cases:
 - TC and CGW coexist only when $p = L_B + K$.
 - TSD and SGW coexist only when $p = L_{Uncert} + K$.
- (2) TC and SGW coexist if and only if:

$$L_{Uncert} \leq p - K \leq L_B$$

(3) TSD and Certified Greenwash cannot coexist under any parameters.

Figure 2 shows graphically the regions described in theorem 1. The Greenhush equilibrium can exist in any of these regions if $L_{\text{Uncert}} \ge \gamma_I$, but under different beliefs.

3.4. Comparative statics

Understanding how changes in key parameters shift the equilibrium behaviour provides crucial insights into the drivers of greenwash, greenhush and truthful

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disclosure. We focus on two parameters: investor preferences (γ_I) and the labelling costs.

3.4.1. Impact of investor preferences

The parameter y_I represents the extent to which institutional investors provide cost relief or other benefits to the green firm conditional on the firm signalling as green (via s = Cert or s = Uncert). It directly captures the influence of this specific stakeholder group on the firm's communication incentives.

Recall that the Greenhush equilibrium, where both firm types choose s = None, requires $L_{\text{Uncert}} \geq \gamma_I$; in essence, Greenhush occurs when the benefit from investor support is not large enough to overcome the relevant signalling cost. An increase in investor preference for green initiatives (higher γ_I) has the following effects:

- (1) **Reduces likelihood of Greenhush:** As y_I increases, the conditions required to sustain the Greenhush equilibrium become harder to satisfy. The opportunity cost for the green firm choosing silence increases, as it foregoes a larger potential cost reduction. This shrinks the parameter space where Greenhush is the stable outcome.
- (2) **Encourages labelling:** A higher γ_I makes the green firm's participation constraints for signalling equilibria (TC: $p \ge L_G \gamma_I$; TSD: $p \ge L_{\text{Uncert}} \gamma_I$) easier to satisfy. It increases the net benefit of truthfully revealing its type.

This highlights a key channel through which stakeholder pressure can influence corporate environmental communication. Stronger pro-ESG sentiment among investors directly increases the incentive for genuinely green firms to overcome signalling costs and disclose their practices, thereby reducing the prevalence of greenhush. However, it does not, in itself, solve the problem of greenwashing, which depends more on the credibility of signals and penalties.

Next consider the impact of increasing the total cost $L_B + K$, which makes it more difficult for brown firms to falsely certify:

- (1) **Expands region for TC:** The TC equilibrium requires $p \le L_B + K$. Increasing $L_B + K$ relaxes this condition, making it possible to exist over a wider range of prices. Certification becomes a more robust separating signal as the cost hurdle for the brown firm increases.
- (2) **Shrinks region for CGW:** The CGW equilibrium requires $p \ge L_B + K$. Increasing $L_B + K$ makes this condition harder to satisfy. Mimicking the certified signal becomes less profitable for the brown firm, thus reducing the likelihood of certified greenwash.

What matters in determining the equilibria outcomes is the differential between $L_B + K$ and $L_{\rm Uncert}$, not L_G . The green firm's certification cost only affects: i) The green firm's participation constraint and ii) the green firm's profits in equilibrium. It does not affect whether an equilibrium can be sustained because the green firm always prefers to signal when it leads to a sale. What prevents pooling is the brown firm's cost-benefit calculation, not the green firm's. By increasing either the cost to the brown firm to receive

a certification, or increasing the penalties for greenwashing, the incentive shifts from greenwash to TC. However, increasing L_B without a corresponding increase in $L_{\rm Uncert}$ will increase the range of prices that support both TC and SGW.

4. Discussion

This paper has developed a theoretical model that explains when and why firms choose different environmental communication strategies – truthful disclosure, greenwash or greenhush – in the presence of stakeholders with potentially divergent preferences. By integrating stakeholder theory with a signalling game framework, we identify specific conditions under which different equilibria emerge and coexist, highlighting the critical role of certification mechanisms, regulatory enforcement and investor preferences.

Equilibrium dynamics and transitions: As investor preferences for environmental initiatives strengthen (higher γ_I), markets naturally tend to shift away from greenhush equilibria. However, this transition does not necessarily lead directly to truthful disclosure. In markets with weak regulatory enforcement (low K) or inexpensive self-declaration (low $L_{\rm Uncert}$), increasing investor pressure may simply transform greenhush into greenwash.

Policy implications: The results suggest that increasing penalties for false environmental claims (K) can effectively create separating equilibria where truthful disclosure prevails. Setting K sufficiently high such that $K>p-\min(L_B,L_{\rm Uncert})$ ensures that at least one separating equilibrium exists, improving both welfare and environmental outcomes. Increasing the cost of false labelling (L_B) also serves this purpose, but without a corresponding increase in $L_{\rm Uncert}$, it may simply shift the incentive to uncertified greenwashing.

Thirdly, the model highlights potential complementarity between direct regulatory approaches and market-based mechanisms. Policies that simultaneously strengthen certification standards and encourage pro-environmental investor sentiment create a 'pincer movement' that both reduces greenhush and deters greenwash. This suggests that comprehensive policy packages may be more effective than isolated interventions targeting single parameters.

Fourthly, the context-specificity of optimal interventions becomes apparent. In markets with historically low trust or skeptical consumers (low q), approaches that enhance certification credibility may be most effective. Conversely, in markets with optimistic consumer beliefs but indifferent investors, policies that strengthen ESG investment incentives may yield greater improvements.

Limitations and future research: Several limitations of the model suggest directions for future research. Firstly, the framework treats firm types as exogenous, focusing on communication strategies rather than the initial decision to adopt green production technologies. A natural extension would be to endogenize the production choice, exploring how communication strategies influence investment in environmental technologies and vice versa.

Secondly, we assume a representative consumer and a single investor with homogeneous preferences. Future work could extend this to heterogeneous consumer segments with varying environmental preferences and multiple investor types,

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allowing for more nuanced exploration of market segmentation strategies and targeted communication.

Thirdly, real markets exhibit dynamic evolution of stakeholder preferences, regulatory environments and certification technologies. Developing dynamic models that capture these evolutionary processes would enhance understanding of long-term market trajectories and sustainability transitions.

Finally, empirical testing of the model's predictions represents an important avenue for future research. Examining how changes in certification standards, investor preferences and penalty regimes influence the prevalence of greenhush versus greenwash would provide valuable validation and refinement of the theoretical framework developed here.

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Appendix. Proofs

(Proof of proposition 1) Case 1: Certification equilibrium ($s_G={
m Cert}, s_B={
m None}$) On the equilibrium path: $\mu_{
m Cert}=1>\bar{\mu}$, $\mu_{
m None}=0<\bar{\mu}$. We examine two cases for the off-path belief $\mu_{
m Uncert}$.

Case 1.1: $\mu_{\text{Uncert}} < \bar{\mu}$

For the green firm: Equilibrium payoff: $\pi^G_{\text{Cert}} = p - C - (a - \gamma_I) - L_G$ Payoff from deviation to None: $\pi^G_{\text{None}}(\text{dev}) = -C - a$ Payoff from deviation to Uncert: $\pi^G_{\text{Uncert}}(\text{dev}) = -C - (a - \gamma_I) - L_{\text{Uncert}}$.

For the green firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{Cert}}^G &\geq \pi_{\text{None}}^G(\text{dev}) \\ p - C - (a - \gamma_I) - L_G &\geq -C - a \\ p &\geq L_G - \gamma_I. \end{split}$$

This holds by assumption since $p > L_G$.

$$\begin{split} \pi_{\text{Cert}}^G &\geq \pi_{\text{Uncert}}^G(\text{dev}) \\ p - C - (a - \gamma_I) - L_G &\geq -C - (a - \gamma_I) - L_{\text{Uncert}} \\ p - L_G &\geq -L_{\text{Uncert}} \\ p &\geq L_G - L_{\text{Uncert}} \end{split}$$

This holds by assumption since $p > L_G$.

For the brown firm: Equilibrium payoff: $\pi^B_{None} = -C$ Payoff from deviation to Cert: $\pi^B_{Cert}(\text{dev}) = (p-K)-C-L_B$ Payoff from deviation to Uncert: $\pi^B_{Uncert}(\text{dev}) = -C-L_{Uncert}$. For the brown firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{None}}^{B} &\geq \pi_{\text{Cert}}^{B}(\text{dev}) \\ -C &\geq (p-K)-C-L_{B} \\ p &\leq L_{B}+K \end{split}$$

This is the brown firm's binding condition.

$$\begin{split} \pi_{\text{None}}^{\textit{B}} &\geq \pi_{\text{Uncert}}^{\textit{B}}(\text{dev}) \\ -C &\geq -C - L_{\text{Uncert}} \\ 0 &\geq -L_{\text{Uncert}} \\ L_{\text{Uncert}} &\geq 0 \end{split}$$

This holds by assumption. Case 1.1 is possible.

Case 1.2: $\mu_{\text{Uncert}} \geq \bar{\mu}$

For the green firm: Equilibrium payoff: $\pi_{\mathrm{Cert}}^G = p - C - (a - \gamma_I) - L_G$ Payoff from deviation to None: $\pi_{\mathrm{None}}^G(\mathrm{dev}) = -C - a$ Payoff from deviation to Uncert: $\pi_{\mathrm{Uncert}}^G(\mathrm{dev}) = p - C - (a - \gamma_I) - L_{\mathrm{Uncert}}$. For the green firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{Cert}}^G &\geq \pi_{\text{None}}^G(\text{dev}) \\ p - C - (a - \gamma_I) - L_G &\geq -C - a \\ p &\geq L_G - \gamma_I \end{split}$$

This holds by assumption since $p > L_G$.

$$\pi_{\mathsf{Cert}}^G \geq \pi_{\mathsf{Uncert}}^G(\mathsf{dev}) \ p - C - (a - \gamma_I) - L_G \geq p - C - (a - \gamma_I) - L_{\mathsf{Uncert}} \ L_{\mathsf{Uncert}} \geq L_G$$

This does not hold by assumption, so case 1.2 is impossible.

Case 2: Self-declaration equilibrium ($s_G = \text{Uncert}, s_B = \text{None}$)

On the equilibrium path: $\mu_{\text{Uncert}}=1>\bar{\mu}$, $\mu_{\text{None}}=0<\bar{\mu}$. Since $\mu_{\text{Cert}}>\mu_{\text{Uncert}}$, it must be the case that $\mu_{\text{Cert}}=1$.

For the green firm: Equilibrium payoff: $\pi^G_{\text{Uncert}} = p - C - (a - \gamma_I) - L_{\text{Uncert}}$ Payoff from deviation to None: $\pi^G_{\text{None}}(\text{dev}) = -C - a$ Payoff from deviation to Cert: $\pi^G_{\text{Cert}}(\text{dev}) = p - C - (a - \gamma_I) - L_{\text{Cert}}$.

For the green firm to maintain the equilibrium strategy:

$$\pi_{ ext{Uncert}}^G \geq \pi_{ ext{None}}^G(ext{dev})
onumber \ p - C - (a - \gamma_I) - L_{ ext{Uncert}} \geq -C - a
onumber \ p \geq L_{ ext{Uncert}} - \gamma_I$$

This holds by assumption since $p > L_G$.

$$\pi^G_{ ext{Uncert}} \geq \pi^G_{ ext{Cert}}(ext{dev}) \ p - C - (a - \gamma_I) - L_{ ext{Uncert}} \geq p - C - (a - \gamma_I) - L_G \ L_G \geq L_{ ext{Uncert}}$$

This holds by assumption.

For the brown firm: Equilibrium payoff: $\pi^B_{\mathrm{None}} = -C$ Payoff from deviation to Uncert: $\pi^B_{\mathrm{Uncert}}(\mathrm{dev}) = (p-K) - C - L_{\mathrm{Uncert}}$ Payoff from deviation to Cert: $\pi^B_{\mathrm{Cert}}(\mathrm{dev}) = (p-K) - C - L_B$.

For the brown firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{None}}^{\textit{B}} &\geq \pi_{\text{Uncert}}^{\textit{B}}(\text{dev}) \\ -C &\geq (p-K) - C - L_{\text{Uncert}} \\ p &\leq L_{\text{Uncert}} + K \end{split}$$

This is the binding condition for the brown firm

$$\pi_{\mathrm{None}}^{\mathcal{B}} \geq \pi_{\mathrm{Cert}}^{\mathcal{B}}(\mathrm{dev})$$
 $-C \geq (p-K) - C - L_{\mathrm{Cert}}$
 $p \leq L_{\mathrm{Cert}} + K$

This is implied by the former condition and is not binding.

(**Proof of proposition 2**) In a pooling PBE, both firm types choose the same signal $s^* \in \{\text{Cert, Uncert, None}\}$. By Bayes' rule, the consumer's posterior belief upon observing s^* equals the prior belief: $\mu_{s^*} = q$. The consumer purchases if and only if $\mu_{s^*} = q \ge \bar{\mu}$.

Case 1: Pooling on certification ($s_G = s_B = \text{Cert}$) On the equilibrium path: $\mu_{\text{Cert}} = q$.

Case 1.1: $\mu_{\mathrm{None}} \leq \mu_{\mathrm{Uncert}} < \bar{\mu} \leq q$ (purchase occurs on path, not off-path) Green firm:

(1) Equilibrium payoff: $\pi_{\text{Cert}}^G = p - C - (a - \gamma_I) - L_G$

- (2) Deviation payoff from choosing Uncert: $\pi_{\text{Uncert}}^{G}(\text{dev}) = -C (a \gamma_I) L_{\text{Uncert}}$
- (3) Deviation payoff from choosing None: $\pi_{\text{None}}^G(\text{dev}) = -C a$

For the green firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{Cert}}^G &\geq \pi_{\text{Uncert}}^G(\text{dev}) \\ p - C - (a - \gamma_I) - L_G &\geq -C - (a - \gamma_I) - L_{\text{Uncert}} \\ p - L_G &\geq -L_{\text{Uncert}} \\ p &\geq L_G - L_{\text{Uncert}} \end{split}$$

This holds by assumption since $p > L_G$.

$$\pi_{\mathsf{Cert}}^G \geq \pi_{\mathsf{None}}^G(\mathsf{dev})$$
 $p - C - (a - \gamma_I) - L_G \geq -C - a$
 $p > L_G - \gamma_I$

This also holds by assumption.

Brown firm:

(1) Equilibrium payoff: $\pi_{Cert}^B = (p - K) - C - L_B$

(2) Deviation payoff from choosing Uncert: $\pi^{B}_{\text{Uncert}}(\text{dev}) = -C - L_{\text{Uncert}}$

(3) Deviation payoff from choosing None: $\pi_{\text{None}}^{B}(\text{dev}) = -C$

For the brown firm to maintain the equilibrium strategy:

$$\pi^{B}_{ ext{Cert}} \geq \pi^{B}_{ ext{Uncert}}(ext{dev}) \ (p-K) - C - L_{B} \geq -C - L_{ ext{Uncert}} \ p \geq L_{B} + K - L_{ ext{Uncert}}$$

and

$$\begin{split} \pi_{\mathrm{Cert}}^{B} &\geq \pi_{\mathrm{None}}^{B}(\mathrm{dev})\\ (p-K) - C - L_{B} &\geq -C\\ p &\geq L_{B} + K \end{split}$$

Clearly the latter condition is the binding one.

Case 1.2: $\mu_{\rm None} < \bar{\mu} < \mu_{\rm Uncert} \le q$ or $\bar{\mu} < \mu_{\rm None} \le \mu_{\rm Uncert} \le q$ (purchase occurs on path and off-path)

Green firm:

(1) Equilibrium payoff: $\pi_{Cert}^G = p - C - (a - \gamma_I) - L_G$

(2) Deviation payoff from choosing Uncert: $\pi_{\text{Uncert}}^{G}(\text{dev}) = p - C - (a - \gamma_I) - L_{\text{Uncert}}$

(3) Deviation payoff from choosing None: $\pi_{\text{None}}^G(\text{dev}) = -C - a$

For the green firm to maintain the equilibrium strategy:

(1) Cert vs Uncert: Need $\pi_{\text{Cert}}^G \ge \pi_{\text{Uncert}}^G(\text{dev})$:

$$p-C-(a-\gamma_I)-L_G \geq p-C-(a-\gamma_I)-L_{\mathrm{Uncert}}$$

$$L_{\mathrm{Uncert}} \geq L_G$$

This does not hold by assumption.

Case 1.3: $\mu_{\mathrm{None}} \leq \mu_{\mathrm{Uncert}} < q < \bar{\mu}$ (purchase does not occur on path or off-path) Brown firm:

(1) Equilibrium payoff: $\pi_{Cert}^B = -C - L_B$

(2) Deviation payoff from choosing None: $\pi_{\text{None}}^{B}(\text{dev}) = -C$

The brown firm always prefers to deviate since $-C > -C - L_B$. Therefore, when $q < \bar{\mu}$, the brown type has an incentive to deviate, making pooling on Cert impossible regardless of the green firm's incentives.

Only case 1.1 is viable.

Case 2: Pooling on self-declaration ($s_G = s_B = \text{Uncert}$)

On the equilibrium path: $\mu_{\text{Uncert}} = q$

Case 2.1: $\mu_{\mathrm{None}} < \bar{\mu} \le q < \mu_{\mathrm{Cert}}$ (purchase occurs on path and off-path) Green firm:

 $\begin{array}{ll} \text{(1)} & \text{Equilibrium payoff: } \pi^G_{\text{Uncert}} = p - C - (a - \gamma_I) - L_{\text{Uncert}} \\ \text{(2)} & \text{Deviation payoff from choosing Cert: } \pi^G_{\text{Cert}}(\text{dev}) = p - C - (a - \gamma_I) - L_G \\ \text{(3)} & \text{Deviation payoff from choosing None: } \pi^G_{\text{None}}(\text{dev}) = -C - a \end{array}$

For the green firm to maintain the equilibrium strategy:

(1) Uncert vs Cert: Need $\pi_{\text{Uncert}}^G \ge \pi_{\text{Cert}}^G(\text{dev})$:

$$p - C - (a - \gamma_I) - L_{ ext{Uncert}} \geq p - C - (a - \gamma_I) - L_G$$
 $L_G \geq L_{ ext{Incert}}$

This holds by assumption.

(2) Uncert vs None: Need $\pi_{\text{Uncert}}^G \ge \pi_{\text{None}}^G(\text{dev})$:

$$p - C - (a - \gamma_I) - L_{\text{Uncert}} \ge -C - a$$

$$p \ge L_{\text{Uncert}} - \gamma_I$$

This holds by assumption.

Brown firm:

- $\begin{array}{ll} \text{(1)} & \text{Equilibrium payoff: } \pi^B_{\text{Uncert}} = (p-K) C L_{\text{Uncert}} \\ \text{(2)} & \text{Deviation payoff from choosing Cert: } \pi^B_{\text{Cert}}(\text{dev}) = (p-K) C L_B \\ \text{(3)} & \text{Deviation payoff from choosing None: } \pi^B_{\text{None}}(\text{dev}) = -C \end{array}$

For the brown firm to maintain the equilibrium strategy:

(1) Uncert vs Cert: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{Cert}}^{B}(\text{dev})$:

$$(p-K) - C - L_{\text{Uncert}} \ge (p-K) - C - L_{B}$$
 $L_{B} \ge L_{\text{Uncert}}$

This holds by assumption.

(2) Uncert vs None: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{None}}^{B}(\text{dev})$:

$$\begin{aligned} (p-K) - C - L_{\text{Uncert}} \geq -C \\ p \geq L_{\text{Uncert}} + K \end{aligned}$$

This is the binding condition for the brown firm.

Case 2.2: $\bar{\mu} \leq \mu_{\mathrm{None}} \leq q < \mu_{\mathrm{Cert}}$ (purchase occurs on path and off-path) Brown firm:

- $\begin{array}{ll} \text{(1)} & \text{Equilibrium payoff: } \pi^B_{\text{Uncert}} = (p-K) C L_{\text{Uncert}} \\ \text{(2)} & \text{Deviation payoff from choosing Cert: } \pi^B_{\text{Cert}}(\text{dev}) = (p-K) C L_B \\ \text{(3)} & \text{Deviation payoff from choosing None: } \pi^B_{\text{None}}(\text{dev}) = p C \end{array}$

For the brown firm to maintain the equilibrium strategy:

(1) Uncert vs Cert: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{Cert}}^{B}(\text{dev})$:

$$(p-K) - C - L_{ ext{Uncert}} \ge (p-K) - C - L_B$$
 $L_B > L_{ ext{Uncert}}$

This holds by assumption.

(2) Uncert vs None: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{None}}^{B}(\text{dev})$:

$$(p-K) - C - L_{\text{Uncert}} \ge p - C$$

 $0 \ge L_{\text{Uncert}} + K$

This cannot hold by assumption, so case 2.2 is impossible.

Case 2.3: $\mu_{\mathrm{None}} < q < \bar{\mu} < \mu_{\mathrm{Cert}}$ or (purchase does not occur on path, occurs off-path) Green firm:

- $\begin{array}{ll} \text{(1)} & \text{Equilibrium payoff: } \pi_{\text{Uncert}}^G = -C (a \gamma_I) L_{\text{Uncert}} \\ \text{(2)} & \text{Deviation payoff from choosing Cert: } \pi_{\text{Cert}}^G(\text{dev}) = p C (a \gamma_I) L_G \\ \text{(3)} & \text{Deviation payoff from choosing None: } \pi_{\text{None}}^G(\text{dev}) = -C a \end{array}$

For the green firm to maintain the equilibrium strategy:

(1) Uncert vs Cert: Need $\pi_{\text{Uncert}}^G \ge \pi_{\text{Cert}}^G(\text{dev})$:

$$\begin{aligned} -C - (a - \gamma_I) - L_{\text{Uncert}} \geq p - C - (a - \gamma_I) - L_G \\ L_G - L_{\text{Uncert}} \geq p \end{aligned}$$

This is violated since $p > L_G$.

Case 2.4: $\mu_{\mathrm{None}} < q < \mu_{\mathrm{Cert}} < \bar{\mu}$ (purchase does not occur on path or off-path) Brown firm:

- (1) Equilibrium payoff: $\pi^B_{\text{Uncert}} = -C L_{\text{Uncert}}$ (since $\theta_{\text{Uncert}} = 0$, no penalty) (2) Deviation payoff from choosing Cert: $\pi^B_{\text{Cert}}(\text{dev}) = -C L_B$
- (3) Deviation payoff from choosing None: $\pi_{\text{None}}^{B}(\text{dev}) = -C$

For the brown firm to maintain the equilibrium strategy:

(1) Uncert vs Cert: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{Cert}}^{B}(\text{dev})$:

$$-C - L_{\text{Uncert}} \ge -C - L_{B}$$

$$L_{B} \ge L_{\text{Uncert}}$$

This holds by assumption.

(2) Uncert vs None: Need $\pi_{\text{Uncert}}^{B} \geq \pi_{\text{None}}^{B}(\text{dev})$:

$$\begin{aligned} -C - L_{\text{Uncert}} \geq -C \\ 0 \geq L_{\text{Uncert}} \end{aligned}$$

Since we assume $L_{\rm Uncert} > 0$, this condition is violated.

Only case 2.1 is viable.

Case 3: Pooling on no claim ($s_G = s_B = \text{None}$)

- (1) On the equilibrium path: $\mu_{\text{None}} = q$. Purchase occurs ($\theta_{\text{None}} = 1$) if and only if $q \geq \bar{\mu}$.
- (2) Off the equilibrium path: $\mu_{Cert} > q$ and $\mu_{Uncert} > q$

Case 3.1: $\bar{\mu} \leq q < \mu_{\text{Uncert}}$ (purchase occurs on path and off-path) Green firm:

- (1) Equilibrium payoff: $\pi_{\text{None}}^G = p C a$
- (2) Deviation payoff from choosing Uncert: $\pi_{\text{Uncert}}^{G}(\text{dev}) = p C (a \gamma_I) L_{\text{Uncert}}$

For the green firm to maintain the equilibrium strategy:

$$egin{aligned} \pi_{ ext{None}}^G &\geq \pi_{ ext{Uncert}}^G(ext{dev}) \ p - C - a &\geq p - C - (a - \gamma_I) - L_{ ext{Uncert}} \ 0 &\geq \gamma_I - L_{ ext{Uncert}} \ L_{ ext{Uncert}} &\geq \gamma_I \end{aligned}$$

Brown firm:

(1) Equilibrium payoff: $\pi_{\text{None}}^B = p - C$

(2) Deviation payoff from choosing Uncert: $\pi_{\text{Uncert}}^{B}(\text{dev}) = (p - K) - C - L_{\text{Uncert}}$

For the brown firm to maintain the equilibrium strategy:

$$\pi_{ ext{None}}^{B} \geq \pi_{ ext{Uncert}}^{B}(ext{dev})$$
 $p-C \geq (p-K)-C-L_{ ext{Uncert}}$ $K+L_{ ext{Uncert}} \geq 0$

This holds since K > 0 and $L_{\text{Uncert}} \ge 0$.

Case 3.2: $q < \bar{\mu} < \mu_{\mathrm{Uncert}}$ (purchase does not occur on path, does occur off-path) Green firm:

(1) Equilibrium payoff: $\pi^G_{\mathrm{None}} = -C - a$ (2) Deviation payoff: $\pi^G_{\mathrm{Uncert}}(\mathrm{dev}) = p - C - (a - \gamma_I) - L_{\mathrm{Uncert}}$

For the green firm to maintain the equilibrium strategy:

$$\begin{split} \pi_{\text{None}}^{\text{G}} &\geq \pi_{\text{Uncert}}^{\text{G}}(\text{dev}) \\ -C - a &\geq p - C - (a - \gamma_I) - L_{\text{Uncert}} \\ L_{\text{Uncert}} - \gamma_I &\geq p \end{split}$$

This does not hold since $p > L_{Uncert}$, so case 3.2 is impossible.

Case 3.3: If $q < \mu_{\text{Uncert}}, \mu_{\text{Cert}} < \bar{\mu}$ (purchase does not occur on or off-path) Green firm:

(1) Equilibrium payoff: $\pi^G_{\mathrm{None}} = -C - a$ (2) Deviation payoff: $\pi^G_{\mathrm{Uncert}}(\mathrm{dev}) = -C - (a - \gamma_I) - L_{\mathrm{Uncert}}$

For the green firm to maintain the equilibrium strategy:

$$egin{aligned} \pi_{ ext{None}}^G &\geq \pi_{ ext{Uncert}}^G(ext{dev}) \ -C-a &\geq -C-(a-\gamma_I)-L_{ ext{Uncert}} \ L_{ ext{Uncert}} &\geq \gamma_I \end{aligned}$$

Brown firm:

(1) Equilibrium payoff: $\pi^B_{
m None} = -C$ (2) Deviation payoff: $\pi^B_{
m Uncert}({
m dev}) = -C - L_{
m Uncert}$

For the brown firm to maintain the equilibrium strategy:

$$\pi_{ ext{None}}^{B} \geq \pi_{ ext{Uncert}}^{B}(ext{dev}) \ -C \geq -C - L_{ ext{Uncert}} \ L_{ ext{Uncert}} \geq 0$$

This always holds, so only the green firm condition is binding.

Case 3.4: If $q < \mu_{\rm Uncert} < \bar{\mu} < \mu_{\rm Cert}$ (purchase does not occur on path, occurs off-path for certification)

Green firm:

- $\begin{array}{ll} \text{(1)} & \text{Equilibrium payoff: } \pi^G_{\text{None}} = -C a \\ \text{(2)} & \text{Deviation payoff: } \pi^G_{\text{Uncert}}(\text{dev}) = -C (a \gamma_I) L_{\text{Uncert}} \\ \text{(3)} & \text{Deviation payoff: } \pi^G_{\text{Cert}}(\text{dev}) = p C (a \gamma_I) L_{\text{Cert}} \end{array}$

For the green firm to maintain the equilibrium strategy:

$$egin{aligned} \pi_{ ext{None}}^G & \geq \pi_{ ext{Uncert}}^G(ext{dev}) \ -C - a & \geq -C - (a - \gamma_I) - L_{ ext{Uncert}} \ L_{ ext{Uncert}} - \gamma_I & \geq 0 \end{aligned}$$
 $egin{aligned} \pi_{ ext{None}}^G & \geq \pi_{ ext{Cert}}^G(ext{dev}) \ -C - a & \geq p - C - (a - \gamma_I) - L_{ ext{Cert}} \ L_{ ext{Cert}} - \gamma_I & \geq p \end{aligned}$

The latter condition cannot hold, since $p > L_{Cert}$.

(Proof of theorem 1) We analyse when different equilibria can simultaneously satisfy their respective existence conditions from propositions 1 and 2.

Part 1: Separating equilibria coexistence

Separating and corresponding pooling equilibria:

For TC and CGW, we note that TC requires $p - K \le L_B$ from proposition 1, while CGW requires $p-K \ge L_B$ from proposition 2. These inequalities are mutually exclusive except at the boundary point where $p-K=L_B$. Similarly, TSD requires $p-K \leq L_{\text{Uncert}}$ while SGW requires $p-K \geq L_{\text{Uncert}}$. These conditions can only be simultaneously satisfied at the knife-edge case where $p - K = L_{Uncert}$. TC and SGW coexistence:

The TC equilibrium exists when $p - K \le L_B$, and the SGW equilibrium exists when $p - K \ge L_{\text{Uncert}}$. Since $L_B > L_{Uncert}$, there exists a non-empty interval of prices where both conditions can be simultaneously satisfied. Specifically, these equilibria coexist when $L_{\text{Uncert}} \leq p - K \leq L_B$. TSD and CGW non-coexistence:

For TSD, proposition 1 requires $p - K \le L_{\text{Uncert}}$, while for CGW, proposition 2 requires $p - K \ge L_B$. Given our assumption that $L_B > L_{Uncert}$, these conditions define disjoint sets on the real line. Therefore, TSD and CGW cannot coexist under any parameter configuration.

Part 2: Greenhush coexistence

We now examine when the Greenhush equilibrium can coexist with other equilibria. Since the greenhush equilibria require $\mu_{\text{None}} = q$, and the separating equilibria require $\mu_{\text{None}} = 0$, and q > 0, the two types of equilibria are incompatible. Similarly, since $\mu_{\mathrm{None}} < q$ in the greenwash equilibria, these are not compatible with greenhush.

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