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MAXIMIZATION OF AVAILABILITY OF 1-OUT-OF-2:*G* REPAIRABLE DEPENDENT SYSTEM

B. H. JOSHI AND
A. D. DHARMADHIKARI,* *University of Poona*

Abstract

The IFR property of the stochastic process governing a one-component system supported by an inactive standby and a repair facility when the lifetime of one component and the repair time of the other component are dependent, is established. We solve the problem of selecting repair rates to maximize the steady-state availability for given component failure rates when a lower bound for the MTBF and upper bounds for the steady-state expected number of repairs of the components per unit time and expected number of failures of the system per unit time are given.

1. Introduction

Extensive reviews of two-component repairable system models have been presented by Lie et al. (1977) and Yearout et al. (1986). However, the literature assumes that the lifetimes and repair times of the components which take place simultaneously are statistically independent of each other. There exist situations, as illustrated in Section 3, where such dependency must be considered.

In this letter we record results for Model 3 of Barlow and Proschan (1975), p. 202, with the following modifications. We denote the number of operative components at time t by $\{X(t),\, t \geqq 0\}$, a stochastic process with state space $\{0, 1, 2\}$. We assume that $P[X(0) = 2] = 1$. Initially the failure rate of the online component is λ_i. Upon failure of this component, it is taken for repair instantly, the repair rate is μ and the standby component is switched online and operates with failure rate λ $(< \lambda_i)$. The justification for assuming $\lambda < \lambda_i$ is that 'since no spare is available the component is not utilized to its full capacity'. Further, if the system enters state 2 due to the completion of a repair then the online component continues to operate with changed failure rate λ_i. This is justifiable, since the failure of this component will no longer cause the failure of the system. On the other hand, if the system enters state 0 due to the failure of the online component, the repair rate of the component under repair is increased to μ_1 in order to minimize the expected downtime of the system. The use of the Freund (1961) bivariate exponential distribution for the joint distribution of the lifetimes and the repair times occurring simultaneously seems appropriate.

Further, in Section 3, we give the values of (μ, μ_1) which maximize the steady-state availability when the specific restrictions are imposed on MTBF, the expected number of repairs of the components per unit time and the expected number of failures of the system per unit time.

2. Analysis

Following the definition of an IFR stochastic process (see Ross (1979)), we state the following result.

Theorem. The stochastic process $\{X(t),\, t \geqq 0\}$ underlying the model discussed in Section 1 is IFR.

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* Postal address: Department of Statistics, University of Poona, Pune 411007, India.

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Performance measures of the system are listed below. These are obtained by inverting appropriate Laplace transforms.

1. *Reliability*, $R(t)$, the probability of failure-free operation of the system in $(0, t]$ is

$$R(t)=\left\{\frac{L_1L_2}{(L_2-L_1)}\right\}\left[\left(\frac{1}{L_1}\right)\exp(-L_1t)-\left(\frac{1}{L_2}\right)\exp(-L_2t)\right]$$

where

$$L_1=\frac{(C_1-C_3)}{2},\qquad C_1=\lambda_1+C_2,\qquad C_2=\lambda+\mu,$$

$$C_3=(C_1^2-4C_4)^{\frac{1}{2}},\qquad C_4=\lambda\lambda_1,\qquad L_2=\frac{(C_i+C_3)}{2}.$$

The corresponding hazard rate increases and is bounded above by L_1.

Remark. The MTBF of the system is

(1) $$\mathrm{MTBF}=\int_0^\infty R(t)\,dt=\left(\frac{\lambda+\mu+\lambda_1)}{\lambda\lambda_1}\right..$$

2. *Availability*, $AV(t)$, the probability that the system is operative at time t is

$$AV(t)=\frac{C_5}{C_6}+K_1\exp(-L_3t)+K_2\exp(-L_4t),$$

where

$$C_5=\mu\mu_1+\lambda_1\mu_1,\qquad C_6=C_4+C_5,\qquad K_1=\frac{-C_4}{L_3L_5},\qquad K_2=\frac{C_4}{L_4L_5},$$

$$L_3=\frac{(C_7-L_5)}{2},\qquad L_4=\frac{(C_7+L_5)}{2},\qquad C_7=\mu_1+C_1,\qquad L_5=(C_7^2-4C_6)^{\frac{1}{2}}.$$

The steady-state availability is

(2) $$AV=\lim_{t\to\infty}(AV(t))=\frac{C_5}{C_6}.$$

3. $M(t)$, *the expected number of repairs* of the components in $(0, t]$ is

$$M(t)=\frac{\mu\lambda_1}{C_6}+\frac{C_8t}{C_6}+K_3\exp(-L_3t)+K_4\exp(-L_4t),$$

where

$$C_8=\lambda_1\mu_1C_2,\qquad K_3=\frac{-\mu\lambda_1}{L_3L_5}+\frac{C_8}{2L_3^2L_5},\qquad K_4=\frac{\mu\lambda_1}{L_4L_5}-\frac{C_8}{2L_4^2L_5}.$$

The steady-state expected number of repairs of the components per unit time is

(3) $$M=\lim_{t\to\infty}[M(t)/t]=\frac{C_8}{C_6}.$$

4. $N(t)$, the *expected number of failures* of the system in $(0, t]$ is

$$N(t)=\frac{C_9t}{C_6}+K_5\exp(-L_3t)+K_6\exp(-L_4t),$$

where

$$C_9=\mu_1C_4,\qquad K_5=\frac{C_9}{2L_3^2L_5},\qquad K_6=\frac{-C_9}{2L_4^2L_5}.$$

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The steady-state expected number of failures of the system per unit time is

$$N = \lim_{t\to\infty} [N(t)/t] = \frac{C_9}{C_6}. \tag{4}$$

3. Optimization problem

For the system discussed above, suppose AV is to be maximized for given component specifications, viz. failure rates λ, λ_1, the lower bound for MTBF, and the upper bounds for M, N. This problem can be mathematically formulated as follows:

$$\max AV(\mu, \mu_1) = \frac{(\mu\mu_1 + \lambda_1\mu_1)}{(\lambda\lambda_1 + \mu\mu_1 + \lambda_1\mu_1)},$$

subject to

(MTBF) $$\left[\frac{(\lambda + \mu + \lambda_1)}{\lambda\lambda_1}\right] > a_1,$$

(M) $$\left[\frac{\lambda_1\mu_1(\lambda + \mu)}{(\lambda\lambda_1 + \mu\mu_1 + \lambda_1\mu_1)}\right] \leqq a_2,$$

(N) $$\left[\frac{\lambda\lambda_1\mu_1}{(\lambda\lambda_1 + \mu\mu_1 + \lambda_1\mu_1)}\right] \leqq a_3,$$

(5) $$\mu_1 > \mu > 0,$$

where a_1, a_2, $a_3(< a_2)$, λ and λ_1 are externally supplied positive reals. Let

$$C = a_1\lambda\lambda_1 - \lambda - \lambda_1 < \mu : f(\mu) = \frac{\lambda\lambda_1 a_2}{\{(\lambda - a_2)\lambda_1 + (\lambda_1 - a_2)\mu\}} \geqq \mu_1$$

and

$$g(\mu) = \frac{\lambda\lambda_1 a_3}{\{(\lambda - a_3)\lambda_1 - \mu a_3\}} \geqq \mu_1.$$

The problem (5) may be reformulated as

(6) $$\max Z(\mu, \mu_1) = \mu_1(\mu + \lambda_1) \text{: subject to: } \mu > C \text{: } \mu_1 - f(\mu) \leqq 0 \text{:}$$
$$\mu_1 - g(\mu) \leqq 0 \text{: } \mu_1 - \mu > 0 \text{: } \mu > 0.$$

The feasible region formed from constraints in (6) is a bounded set in $\mathbb{R}^2_+$. Further, as Z increases the convex curves corresponding to the objective function shift in the upward direction. Therefore the diagrammatic representation of the feasible region given by the constraints in (6) can be used successfully to get the optimum solution (μ^*, μ_1^*).

Note that at $\Lambda = \lambda(a_2 - a_3)/a_3$, $f(\Lambda) = g(\Lambda)$. Now, if $\Lambda \in (0, C)$ then $(\mu^*, \mu_1^*) = (C, f(C))$, if $\Lambda > f(\Lambda)$ then $(\mu^*, \mu_1^*) = (\Lambda_1, g(\Lambda_1))$ where Λ_1 is such that $\Lambda_1 < \Lambda$ and $\Lambda_1 = g(\Lambda_1)$, if $\Lambda \in (C, f(\Lambda))$ then $(\mu^*, \mu_1^*) = (\Lambda, f(\Lambda))$.

Remark. '$f(C) < C$' or '$g(C) < C$' causes $\mu_1 < \mu$ and hence the situation of an infeasible solution to the problem arises. For finite λ, λ_1, a_1, a_2, a_3 $(< a_2)$ if $\min\{f(C), g(C)\} > C$, then the problem (6) has a finite optimal solution.

Illustration. In the western part of India there are local power generating stations (LPGS), in addition to regional power generating stations (RPGS). Generally, an LPGS has two generators of which one functions at a time and the second is a standby. When the online generator fails, the standby generator starts operating; however, this time the generator is not used to its full capacity, since the other one is non-operative. Now two activities, production of power by the online generator and repair of the failed generator, take place.

If the repair ends first, the power production is increased. On the other hand, in case of failure of the online generator before completion of the repair of the other generator the actions taken are;

(a) repair rate of the generator already under repair is increased, and

(b) 'power' is borrowed from an RPGS during the downtime of both the generators located at the LPGS.

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