*Proceedings of the Edinburgh Mathematical Society* (2002) **45**, 255–256 ©

# BOOK REVIEWS

CHUDINOVICH, I. AND CONSTANDA, C. *Variational and potential methods in the theory of bending of plates with transverse shear deformation* (Monographs and Surveys in Pure and Applied Mathematics, no. 115, Chapman & Hall/CRC, 2000), 248pp., 1 584 88155 0, £46.99.

This well-written book offers a rigorous mathematical study of a wide range of important boundary-value problems for the system of partial differential equations that describe the equilibrium bending of elastic plates with transverse shear deformation. The boundary-value problems studied include interior and exterior Dirichlet and Neumann boundary-value problems, problems with elastic, mixed and combined boundary data, problems for plates with homogeneous inclusions, plates with cracks, and plates on a generalized elastic foundation. In each case the authors provide results on existence, as well as uniqueness and continuous dependence on the data, of the solution in Sobolev spaces, employing both the variational and integral equation methods. Most of these results are based on the authors' own research and are published here in book form for the first time.

The book is divided into seven chapters and one appendix. The first chapter gives a brief discussion of the equilibrium equations for bending of an elastic plate with transverse shear deformation and the formulation of the boundary-value problems, as well as properties (in a classical sense) of the plate potentials and the derivation of boundary integral equations for the problems.

In Chapter 2 the interior and exterior Dirichlet and Neumann boundary-value problems are analysed using the variational method. The equivalent variational formulations are derived in Sobolev spaces (for interior problems) or in weighted Sobolev spaces (for exterior problems), and their solvability as well as the stability in spaces of distributions (weak solution of the problems) then follows from the Lax–Milgram Theorem. In Chapter 3 the (weak) solutions of the interior and exterior Dirichlet and Neumann problems are sought in the form of plate potentials, reducing the problems to integral equations on the boundary of the domain. These equations are solved by means of specially constructed algebras of boundary operators (the Poincaré–Steklov operators), whose mapping properties in Sobolev spaces are studied in detail and which associate with a displacement field on the boundary the corresponding boundary moments and forces. Chapter 4 deals with the transmission problems, including cases of an infinite or a finite plate with a finite inclusion and a multiply connected finite plate. The crack problems for finite or infinite plates are discussed in Chapter 5, while other types of boundary conditions such as elastic, mixed and combined conditions are investigated in Chapter 6. The last chapter considers the interior and exterior Dirichlet and Neumann problems for plates on an elastic foundation. To make the book self-contained some basic material on distribution theory and Sobolev spaces is provided in an appendix.

From the reviewer's point of view this book is most suitable for a reader with a basic knowledge of functional analysis. It is amazing that the authors have managed to cover so many fundamental boundary-value problems and present the variational method and the boundary integral equation method applied side-by-side in a single volume of only about 236 pages. This feature of the book will certainly strengthen understanding of both the model and the methods. The writing style is very clear, the book is self-contained and easy to read, and it should be extremely valuable to researchers interested in applied analysis and mathematical models in elasticity.

BO ZHANG

LEE, J. M. *Introduction to topological manifolds* (Graduate Texts in Mathematics, vol. 202, Springer, 2000), xvii+385 pp., 0 387 95026 5 (softback) £24, 0 387 98759 2 (hardback) £48.

This book provides a very readable introduction to manifolds at the beginning graduate level. It grew out of notes used in the first third of a year-long course taught by the author at the University of Washington; the remaining two thirds focuses on smooth manifolds. Because of this, the book has a very strong geometric flavour, quite uniquely among introductory texts on general and algebraic topology. Its aim is to provide the necessary amount of topological knowledge needed for the further study of manifolds, reflecting the author's belief that manifolds are part of the basic vocabulary of mathematics and need to be part of the education of every student of mathematics. The geometric slant of the book is clearly seen in its contents.

The introductory first chapter discusses some disciplines in which manifolds came to play a prominent role, including classical mechanics, general relativity and quantum field theory. The next three chapters provide a brief and selective introduction to general topology: topological spaces; subspaces, products and quotients; connectedness and compactness. Manifolds are the main examples.

Chapter 5 introduces simplicial complexes and such concepts as orientability and Euler characteristic. Triangulability theorems are also discussed here (with a complete proof for 1-manifolds and a sketch of a proof for surfaces). These are used in Chapter 6 to classify 1-manifolds and compact surfaces.

Chapters 7–10 introduce basic notions of homotopy theory with particular emphasis on the fundamental group. This section includes a brief introduction to group theory (Chapter 9) and the proof and applications of the Seifert–Van Kampen theorem (Chapter 10).

Chapters 10 and 11 are on covering spaces and covering groups. The examples are particularly interesting here: classifying coverings of tori and lens spaces, and proving that surfaces of higher genus are covered by the hyperbolic disk.

Chapter 13, the last, gives a brief but illuminating introduction to homology theory, mainly singular. The essential properties of homology groups proved here include homotopy invariance and the Mayer–Vietoris theorem.

The reviewer found this a well-written and enjoyable book. It is an excellent precursor to the study of differential geometry, but it will also be useful as an introduction to algebraic topology. There is a wealth of examples, exercises and problems, as well as many illustrations emphasizing the geometric intuition behind concepts and proofs. A small number of typographical and minor errors will be found (the errata can be downloaded from the author's home page at the University of Washington).

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