

Interfacial arrest in buoyant viscoplastic injections

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We study buoyant miscible injections of dense viscoplastic fluids into lighter Newtonian fluids in inclined closed-end pipes, at the high-Péclet-number regime. We integrate experiments involving camera imaging and ultrasound Doppler velocimetry, and computational fluid dynamics simulations, to provide a detailed analysis of interfacial dynamics, flow phases/regimes, velocity field, yielded and unyielded zones, and interfacial arrest mechanisms. The flow dynamics is governed by Reynolds (Re), Froude (Fr) and Bingham (B) numbers, the viscosity ratio (M), inclination angle (β), or their combinations, such as $\chi \equiv 2Re/Fr^2$. As the interface evolves, our results reveal a transition from an inertial-dominated phase, characterised by linear front advancement at the injection velocity, to a viscoplastic-dominated phase, marked by deceleration and eventual interfacial arrest governed by the yield stress. The critical transition length between these phases ($\mathcal{L} \approx 1.26Fr^{0.14}$) is determined by a balance between inertial and buoyant stresses. Experimental findings confirm buoyancy-driven slumping in our flows, consistent with the theoretical yield number criterion ($Y \equiv B/\chi$), with maximum interfacial arrest lengths scaling as $L_s \sim 1/Y$. These results also classify arrested and unhalted interfacial flow regimes on a plane involving $\chi \cos(\beta)/B$ and Y . Furthermore, we demonstrate that the interfacial arrest mechanism arises from interactions between buoyancy, rheology and geometry, as diminishing shear stresses promote unyielded zone expansion near the interface, progressively encompassing the viscoplastic layer and halting flow when stresses fall below the yield stress.

Key words: plastic materials, non-Newtonian flows, low-Reynolds-number flows

1. Introduction

Viscoplastic fluids, as a subclass of non-Newtonian fluids, behave like solids under stresses below a critical yield stress, but flow once this threshold is surpassed (Balmforth, Frigaard & Ovarlez 2014; Louvet, Bonn & Kellay 2014; Thompson, Sica & de Souza Mendes 2018; Noma *et al.* 2021). Typically modelled via the Bingham and Herschel–Bulkley frameworks (Liu, Balmforth & Hormozi 2019), these fluids are widely employed in diverse industrial applications, including food processing (e.g. ketchup, chocolate) (Balmforth *et al.* 2014), pulp and paper production (Villalba *et al.* 2023), cleaning (Fernandes *et al.* 2022a), three-dimensional (3-D) printing (Lohse 2022; van der Kolk, Tieman & Jalaal 2023), healthcare (e.g. toothpaste) (Coussot 2014), biomedical engineering (Garcia-Gonzalez, Garzon-Hernandez & Arias 2018), and oil and gas operations (Nelson & Guillot 2006; Eslami, Akbari & Taghavi 2022). The yield stress of viscoplastic fluids stabilises injection processes (Chaparian, Owens & McKinley 2022) but also leads to a complex interplay between flow structure and rheological state, resulting in solid-like zones where stresses are insufficient to induce flow (Freydier, Chambon & Naaïm 2017). Unlike Newtonian fluids, the motion of viscoplastic fluids may cease under the dominance of yield stress (Dubash *et al.* 2009), with confined geometries further complicating flow due to the interactions between rheology, fluid dynamics, geometry and operational conditions. Accurate prediction of interface dynamics and yielding/unyielding behaviour is therefore crucial for understanding these complex flows (Bonn *et al.* 2017; Frigaard 2019). In this context, our study investigates the interfacial motion arrest (stoppage) in buoyant injections of a dense viscoplastic fluid into an inclined, closed-end pipe pre-filled with a quiescent, lighter Newtonian fluid. Through an integrated experimental and numerical approach, we aim to elucidate the underlying flow dynamics, the flow phases/regimes and the interfacial arrest mechanism.

Buoyant miscible displacement and injection flows involving Newtonian and viscoplastic fluids have been extensively studied across various geometries, including circular pipes (Akbari & Taghavi 2020; Charabin & Frigaard 2024), porous media (Singh *et al.* 2019), inclined channels (Sahu *et al.* 2009) and Hele-Shaw cells (Brlin, Roisman & Tropea 2020; l'Estimé *et al.* 2022; Talon, Goyal & Meiburg 2013; John *et al.* 2013; Divoux *et al.* 2020). For Newtonian fluids, previous studies have identified various flow regimes, such as viscous- and inertial-dominated flow regimes (Séon *et al.* 2005), while characterising buoyant miscible injection flows in near-horizontal pipes/channels under varying Reynolds and Froude numbers and inclinations (Taghavi *et al.* 2012b). In addition, Matson & Hogg (2012) analysed viscous exchange flows between fluids of differing densities in horizontal channels with rectangular and circular cross-sections, demonstrating the impact of viscosity contrast on stratified flow dynamics. Stratified displacement patterns – stable, wavy, cyclic, and inertial – have also been documented, with sharp, well-defined interfaces in the slumping regime (Etrati & Frigaard 2018). For viscoplastic fluids, previous research has revealed how yield stress impacts displacement processes and stationary residual wall layer formation (Taghavi *et al.* 2009), with subsequent studies identifying centre-type and slump-type regimes largely independent of pipe inclination, imposed flow or yield stress magnitude (Taghavi *et al.* 2012a; Alba *et al.* 2013a). In addition, several flow regimes, including smooth, wavy and corrugated, have been experimentally observed during viscoplastic fluid displacement in horizontal pipes, driven by inertial and viscous forces (Moisés *et al.* 2016).

When a denser viscoplastic fluid is injected at a low rate into a quiescent, lighter Newtonian fluid in an inclined configuration, it initially slumps beneath the lighter one; however, the interface between the viscoplastic and Newtonian fluids may eventually halt

at a specific point due to yield stress dominating gravity (Faramarzi, Akbari & Taghavi 2024). Extensive studies of relevant viscoplastic flows emphasise such interface dynamics, particularly in free-surface thin-film flows (Balmforth *et al.* 2006; Hogg & Matson 2009; Ancey & Cochard 2009; Chambon, Ghemmour & Naaïm 2014; Freydier *et al.* 2017; Bates & Ancey 2017; Liu *et al.* 2019; Valette *et al.* 2021; Hinton & Hogg 2022; Moyers-Gonzalez *et al.* 2023). These studies often involve a dynamically passive Newtonian fluid (e.g. air) with minimal impact on the viscoplastic flow, where a viscoplastic fluid column collapses under gravity and halts, allowing measurements of its final spread or height. Findings reveal that viscoplastic flows reach a static equilibrium as yield stress balances gravitational forces and the streamwise pressure gradient (Balmforth *et al.* 2006; Hogg & Matson 2009). Furthermore, the extent of the spread, critical thickness or final arrest height in uniform layers of viscoplastic fluids is directly linked to yield stress (Liu, Balmforth & Hormozi 2018; Modolo *et al.* 2019; Valette *et al.* 2021; Moyers-Gonzalez *et al.* 2023).

Interfacial viscoplastic flows involving dynamically passive surrounding fluids typically have two main regions: a yielded zone near the substrate (on which the viscoplastic fluid moves), where high shear stresses induce a pronounced velocity gradient, and an unyielded (rigid) upper region, where shear stresses are zero at the free surface, separated by a yield surface (Freydier *et al.* 2017). When layer thickness varies, slight yielding in the upper region creates a pseudo-plug, which kinematically behaves as a rigid body but allows small extensional flow variations (Freydier *et al.* 2017; Liu *et al.* 2019). Provided that the pseudo-plug eventually reaches the substrate, the entire layer becomes rigid (Piau 2005; Liu *et al.* 2019). These flows are often modelled with thin-layer approximations based on lubrication theory, assuming the flow length exceeds layer thickness and approaches a steady, two-layer velocity profile with a pseudo-plug region beneath the surface (Ancey & Cochard 2009). In contrast to free-surface flows that gradually halt, viscoplastic fluid flows in pipes and channels stop abruptly when the pressure gradient is removed, and the inclusion of multi-fluid systems further complicates the analysis by introducing additional challenges in defining flow cessation conditions (Hogg & Matson 2009).

Several studies have applied analytical and semi-analytical methods to investigate interfacial viscoplastic fluid slumping in pipes involving a dynamically active surrounding fluid, typically considering buoyant miscible flows (Frigaard & Ngwa 2004; Taghavi 2018). For viscoplastic fluid flow cessation, prior research has defined a conservative threshold based on geometrical parameters and the balance of buoyant and yield stresses in such systems (Crawshaw & Frigaard 1999; Frigaard & Crawshaw 1999). Beyond this threshold, the viscoplastic fluid flow ceases. Later studies predicted the maximum slumping length below this threshold and characterised the interface upon cessation (Malekmohammadi *et al.* 2010). For viscoplastic fluids in near-horizontal inclinations, the flow also halts once buoyant forces are dominated by yield stress, creating static conditions (Frigaard & Ngwa 2004).

The formation and behaviour of unyielded layers in viscoplastic flows have been extensively studied across various flow configurations (Swain *et al.* 2015; Roustaei *et al.* 2016; Valette *et al.* 2021; Taylor-West & Hogg 2022; Taylor-West & Hogg 2023; Joulaei, Rahmani & Taghavi 2024; Ribinskas *et al.* 2024). Computational studies, such as Roustaei & Frigaard (2013), showed that in channel flows, small wall amplitudes maintain intact unyielded layers that expand in narrower regions and contract in wider ones, while larger amplitudes cause plug breaking (Frigaard & Ryan 2004). In displacement flows, experimental studies have shown persistent flattened interfacial fronts, i.e. indicative of plug-like flow below the yield stress (Gabard & Hulin 2003); analogous behaviour has been observed in exchange flows between viscoplastic materials and Newtonian oils

(Varges *et al.* 2018). Numerical simulations have also shown that increasing the Bingham number leads to a larger unyielded region and reduces interfacial instabilities (Swain *et al.* 2015).

Within the class of miscible viscoplastic injection flows, we are interested in those in which buoyancy is a significant force in driving the fluid motion. Such flows fall into the category of high-Péclet-number flow ($Pe = \hat{D}\hat{V}_0/\hat{D}_m \gg 1$, where $\hat{D}$ is the pipe diameter, $\hat{D}_m$ is the molecular diffusivity and $\hat{V}_0$ is the mean injection velocity), indicating negligible mixing over the relevant experimental time scales. From a modelling perspective, a kinematic equation rather than a diffusion equation can effectively model the fluid interface in such flows. It has long been shown that, in the high-Péclet regime, these flows approach their immiscible limit with zero surface tension as long as flow stability is maintained (Chen & Meiburg 1996; Petitjeans & Maxworthy 1996; Govindarajan & Sahu 2014). From an experimental perspective, studies with Newtonian fluids (e.g. water) and viscoplastic fluids (e.g. Carbopol solutions) also show stable, sharp interfaces (Hassanzadeh, Frigaard & Taghavi 2023). These are also considered high-Péclet miscible flows with negligible interfacial tension effects, as the surface tension difference between Carbopol solutions and water is typically less than 10 %, regardless of the Carbopol concentration (Boujlel & Coussot 2013; Jørgensen *et al.* 2015; Jalaal, Kemper & Lohse 2019).

Over the past two decades, various studies have explored multiple and multilayer viscoplastic fluid flows with sharp interfaces (Dauck *et al.* 2019; Kowal & Worster 2019; Kowal 2021; Kumar *et al.* 2021; Shah, Pegler & Minchew 2021; Zhu, He & Meiburg 2021; Gyllenberg & Sayag 2022; Leung & Kowal 2022). For instance, Christy & Hinton (2023) developed a depth-integrated model for gravity-driven two-layer flows of generalised Newtonian fluids, applicable to one-dimensional (1-D) and some two-dimensional (2-D) topographies, particularly in shallow, inertia-less cases. The model links volume flux directly to fluid rheology by expressing the flux in terms of integrals over stress, which are determined by the constitutive laws of the fluids, eliminating the need for explicit velocity profiles. Etrati & Frigaard (2018) introduced a model for buoyant miscible flows in inclined pipes, accounting for inertia and buoyancy in a two-layer flow, predicting backflow and instability for various viscosity ratios. Similarly, Taghavi (2018) applied a lubrication approximation model to analyse buoyant displacement flows in nearly horizontal channels, incorporating parameters such as buoyancy number, viscosity ratio and non-Newtonian properties to describe interface propagation.

The industrial motivation for this study on buoyant viscoplastic injections arises from plug and abandonment (P and A) processes in oil and gas wells, i.e. an area expected to grow significantly in the coming decades (Trudel *et al.* 2019; Hassanzadeh *et al.* 2023; Hassanzadeh & Taghavi 2024). These processes protect underground water aquifers and the atmosphere from oil and gas migration (Khalifeh & Saasen 2020). For instance, in the widely used dump bailing method, heavy viscoplastic cement slurry is injected eccentrically into a long, closed-end casing (pipe) to displace *in situ* light fluids (e.g. water) and seal the wellbore (Nelson & Guillot 2006). The cement slurry (a complex viscoplastic fluid) interacts with wellbore fluids, affecting cement placement (Nelson & Guillot 2006; Khalifeh & Saasen 2020; Dahi Taleghani & Santos 2023). Relevant to this, research on the injection of heavy viscoplastic fluids into a closed-end pipe shows distinct flow regimes, such as viscoplastic separation, coiling, mixing and slumping (Akbari & Taghavi 2022a; Faramarzi *et al.* 2024; Kazemi *et al.* 2024). Among these, stable slumping with minimal mixing may be preferred for near-horizontal cementing (Taghavi *et al.* 2009; Sarmadi, Renteria & Frigaard 2021). However, due to the yield

	Flow type (method)	Geometry	Fluids involved	References
Free surface flows	Dam-break (numerical)	2-D rectangle, 3-D cylinder, 3-D prismatic columns	Viscoplastic fluid/air	Valette <i>et al.</i> (2021); Moyers-Gonzalez <i>et al.</i> (2023)
		Axisymmetric cylinder	Viscoplastic fluid/Newtonian fluid (air/water)	Liu <i>et al.</i> (2018)
	Surge (numerical, analytical)	Inclined plane	Viscoplastic fluid/air	Liu <i>et al.</i> (2019); Hinton & Hogg (2022)
		Inclined plane	Viscoplastic fluid/air	Freydier <i>et al.</i> (2017)
Confined geometry flows	Exchange flow (analytical or experimental)	Slot (parallel plates) and pipe	Heavy/light Bingham fluids	Frigaard & Ngwa (2004)
		Horizontal closed pipe	Heavier xanthan solution/lighter viscoplastic fluid	Malekmohammadi <i>et al.</i> (2010)
	Imposed flow (experimental, numerical)	Closed-end pipe	Heavy viscoplastic fluid/light Newtonian fluid	Current study

Table 1. Summary of previous studies on interfacial arrest in viscoplastic flows, highlighting the distinct focus of this study.

stress, the stable slumping of a viscoplastic fluid may be transient, leading to the arrest of the viscoplastic fluid interface evolution, which complicates achieving the optimal cementing length. In this context, this study fundamentally investigates the interfacial motion arrest mechanisms in buoyant viscoplastic injection flows through experiments and computational fluid dynamics (CFD) simulations. Table 1 contextualises the present study within the broader body of work on interfacial arrest in viscoplastic flows, which have primarily addressed free-surface scenarios (e.g. dam-break and surge flows) or confined exchange flows involving displacement of a lighter fluid by a denser viscoplastic one. In contrast, our buoyant injection of a miscible, low-yield-stress viscoplastic fluid into a closed-end pipe at near-horizontal inclinations, where a stable slumping regime (Faramarzi *et al.* 2024) dominates, as examined here, constitutes a novel configuration. Focusing on this flow, we examine the factors such as the interface characteristics, flow regimes, front dynamics, yielded/unyielded zones, interfacial arrest length, velocity field, etc. From a practical perspective, our findings can also impact viscoplastic fluid placements across multiple industries, including oil and gas well cementing, 3-D printing, food production and cosmetics.

The paper is structured as follows. Section 2 presents the research methodology, with § 2.1 describing the experimental set-up, including procedures, fluid preparation and rheological characterisation; § 2.2 explains the CFD set-up. Section 3 provides our results, covering general observations (§ 3.1), flow, interface and front dynamics (§ 3.2), and the interfacial arrest mechanism with a focus on velocity fields and yielded/unyielded zones (§ 3.3). Finally, § 4 summarises our key findings and contributions.

2. Methodology

This section outlines our methodologies for investigating buoyant miscible viscoplastic injection flows in an inclined closed-end pipe through experimental analysis and CFD

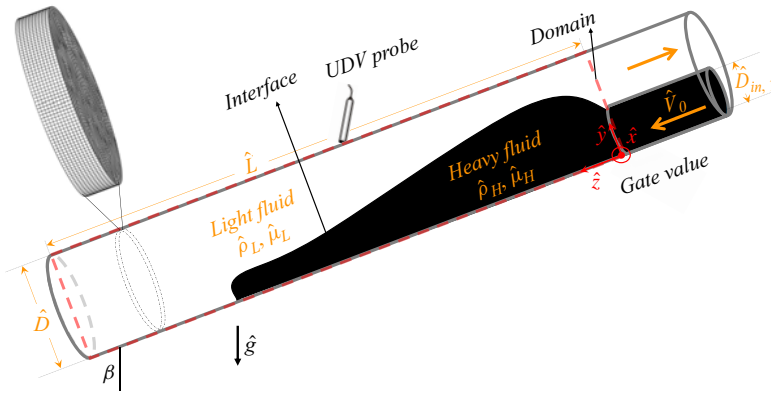


Figure 1. Schematic of the experimental set-up (§ 2.1) for buoyant miscible injection of a heavy viscoplastic fluid into a closed-end inclined pipe filled with a lighter Newtonian fluid. The heavy fluid slumps beneath the lighter fluid, forming a stable interface, within the highlighted flow domain of interest (red). A gate valve separates the fluids prior to the start of an experiment. Velocity profiles are measured using UDV. The same schematic also applies to CFD simulations (§ 2.2), with the upper section depicting a representative meshed domain segment.

simulations. Section 2.1 details the experimental set-up, including fluid preparation, apparatus configuration and visualisation techniques mimicking industrial cementing conditions. Section 2.2 describes complementary CFD simulations in OpenFOAM using the finite volume method. Together, these approaches analyse interfacial front dynamics, velocity field and yielded/unyielded zones, providing insights into factors governing interfacial arrest in our viscoplastic fluid flow.

2.1. Experiments

2.1.1. Experimental set-up

Inspired by oil and gas well cementing processes, our experiments use a scaled-down apparatus with two eccentrically arranged transparent pipes: the outer pipe (inner diameter 3.81 cm) simulates the wellbore, and the inner pipe (inner diameter 1.27 cm, outer diameter 2.54 cm) simulates the cement slurry injector. A precision gear pump (Ismatec 405A) controls the viscoplastic fluid injection rate, and the set-up is mounted on a rotating frame to adjust inclinations. A gate valve at the inner pipe's end isolates the injected fluid before starting the experiment. Figure 1 presents a schematic of the experimental set-up. In this figure, the dashed lines indicate the study domain, located downstream of the injection inlet and confined to the outer pipe. The z -axis is aligned along the pipe's axis, the y -axis is perpendicular to the pipe and represents height, and the x -axis completes the right-handed coordinate system, pointing horizontally across the pipe and is perpendicular to the y - z plane.

2.1.2. Fluid preparation

The *in situ* fluid is deionised water (Newtonian), while the injected fluid is a non-Newtonian viscoplastic Carbopol gel. Preparation begins by adding 0.8 g L^{-1} of Fountain Pen India black ink to deionised water for visualisation. Sugar and Carbopol powder are then added to create solutions with specific densities and yield stresses, monitored using a high-precision density meter (Anton Paar DMA 35). The solution is mixed at 360 rpm for 1 hour, left to equilibrate for 24 hours to remove air bubbles and neutralised to $\text{pH} = 7$ with sodium hydroxide (0.29 g NaOH per 1 g Carbopol) to form a gel. Finally, centrifugation

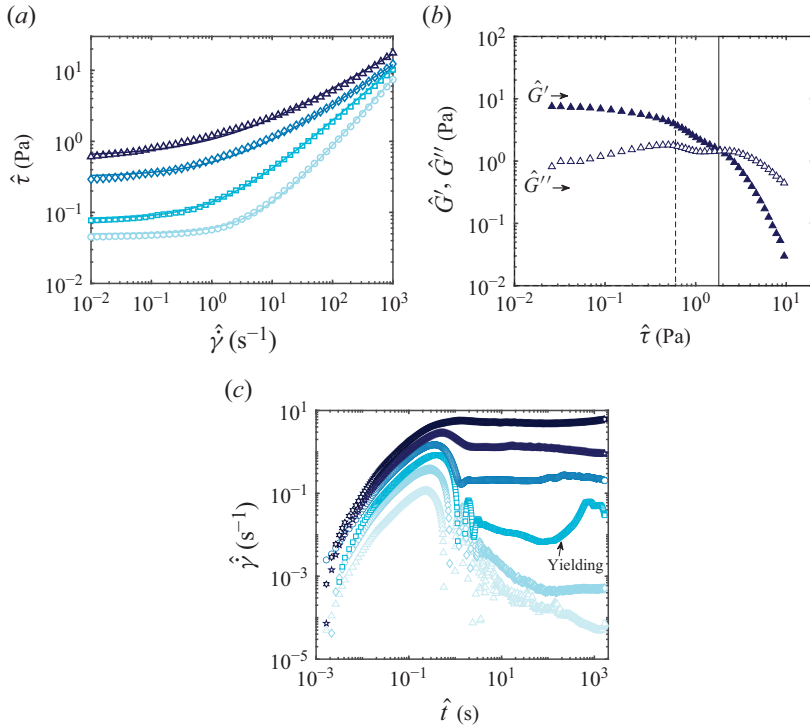


Figure 2. (a) Flow curves for samples II (○), III (□), VII (◇), and IX (△), with colour-matched Herschel–Bulkley fits (see (2.1) and table 2), where darker shades indicate higher $\hat{\tau}_y$. (b,c) Results for sample IX. (b) Oscillation amplitude sweep showing storage modulus ($\hat{G}'$, ▲) and loss modulus ($\hat{G}''$, △) as functions of shear stress ($\hat{\tau}$). Dashed and solid lines mark $\hat{\tau}_y$ from Herschel–Bulkley fits (panel a) and the cross-over point. (c) Creep tests with stress values 0.2 (△), 0.4 (◇), 0.6 (□), 1 (○), 1.5 (☆) and 2 (✱) Pa. Symbol darkness indicates stress levels, showing shear rate ($\hat{\gamma}$) over time, with flow transition at $\hat{\tau}_y \approx 0.6$ Pa.

at 360 rpm for 1 hour ensures homogeneity. All stirring steps were carefully performed using moderate rotational speed and a centred blade to prevent turbulent mixing and air entrainment into the solution.

2.1.3. Rheological characterisation

The rheological properties of the Carbopol solutions are measured using a rheometer (DHR-3, TA Instruments) with a parallel-plate geometry (40 mm diameter, 1 mm gap). To prevent wall slip, 400-grit sandpaper is attached to the plates (Ozkan *et al.* 2012; Habibi *et al.* 2016). Figure 2(a) presents steady-state flow curves for selected Carbopol mixtures from controlled shear rate ($\hat{\gamma}$) tests across $10^{-2} < \hat{\gamma} < 10^3$ s⁻¹. The markers show the rheometry data, with the solid lines representing the fitted Herschel–Bulkley model:

$$\begin{cases} \hat{\gamma} = 0, & |\hat{\tau}| \leq \hat{\tau}_y, \\ \hat{\tau} = \hat{\tau}_y + \hat{\kappa} \hat{\gamma}^n, & |\hat{\tau}| > \hat{\tau}_y, \end{cases} \quad (2.1)$$

where $\hat{\tau}$ represents the shear stress, $\hat{\kappa}$ is the consistency index, n denotes the power-law index and $\hat{\tau}_y$ is the yield stress. Hence, the characteristic viscosity of the viscoplastic fluid ($\hat{\mu}_H$) is expressed as

$$\hat{\mu}_H = \hat{\tau}_y \hat{\gamma}_c^{-1} + \hat{\kappa} \hat{\gamma}_c^{n-1}, \quad (2.2)$$

Sample	$\hat{\tau}_y$ (Pa)	$\hat{\kappa}$ (Pa s ^{<i>n</i>})	<i>n</i>	$\hat{\rho}_H$ (kg m ⁻³)	Carbopol powder concentration (wt/wt %)	Sugar concentration (wt/wt %)
I	0.04	0.11	0.70	1004	0.035	1.66
II	0.05	0.01	0.96	1004	0.036	1.66
III	0.08	0.06	0.75	999	0.040	0.33
IV	0.14	0.07	0.68	1003	0.042	1.43
V	0.22	0.11	0.65	1002	0.043	1.16
VI	0.28	0.14	0.63	1018	0.044	5.32
VII	0.32	0.19	0.60	1008	0.046	2.72
VIII	0.40	0.21	0.59	1018	0.046	5.32
IX	0.62	0.45	0.52	1018	0.050	5.32

Table 2. Properties of Carbopol solutions used: $\hat{\tau}_y$ (yield stress), $\hat{\kappa}$ (consistency index), *n* (power-law index), $\hat{\rho}_H$ (density), Carbopol powder concentration (wt/wt %) and sugar concentration (wt/wt %).

where $\hat{\gamma}_c = \hat{V}_0 / \hat{D}$, with $\hat{V}_0$ as the average injection velocity and $\hat{D}$ the inner diameter of the outer pipe. The Herschel–Bulkley model curve in [figure 2\(a\)](#) matches the rheometry data, with darker colours indicating higher yield stresses. Beyond the critical stress, the flow curves display shear-thinning behaviour, while at low shear rates, the shear stress approaches the yield stress ($\hat{\tau}_y$) (Frigaard 2019; Ovarlez & Hormozi 2019). [Table 2](#) summarises the rheological parameters and densities of the Carbopol solutions, i.e. Samples I–IX.

As shown in [figure 2\(b\)](#) for Sample IX, oscillatory rheometry tests at 1 Hz with stress ranging from 10^{-2} to 10^1 Pa assess the viscoelastic properties in the linear viscoelastic limit of Carbopol solutions, measuring the storage modulus ($\hat{G}'$) and loss modulus ($\hat{G}''$) for elastic and viscous behaviours, respectively. At low stress levels, solid-like behaviour is observed as $\hat{G}'$ exceeds $\hat{G}''$ (Ovarlez & Hormozi 2019; Griebler & Rogers 2022; Hassanzadeh *et al.* 2023). As the stress increases, viscous behaviour dominates, with $\hat{G}''$ surpassing $\hat{G}'$, indicating flow. The vertical line in [figure 2\(b\)](#) marks the critical cross-over stress point at 1.4 Pa for Sample IX (Ovarlez & Hormozi 2019; Griebler & Rogers 2022), which differs from the yield stress obtained in [figure 2\(a\)](#) from the Herschel–Bulkley model (marked as a vertical dashed line); this is consistent with literature findings in viscoplastic rheology (Fernandes *et al.* 2017; Jalaal *et al.* 2019).

Carbopol solutions may exhibit elasticity and thixotropy depending on concentration and preparation method (Dinkgreve *et al.* 2018; Iceri *et al.* 2023). However, as shown in [table 2](#) and [figure 2\(b\)](#), and consistent with prior studies (Piau 2007; Dinkgreve *et al.* 2018; Isukwem *et al.* 2024; Ribinskas *et al.* 2024; Taylor-West & Hogg 2024), such effects are negligible at low concentrations (<0.15 wt./wt.%), as used in our experiments. Also, the elastic time scale, estimated as $\lambda \approx (\hat{\kappa} / \hat{G}')^{1/n}$ (Luu & Forterre 2009), is $O(10^{-1})$ s – much shorter than the experimental time scale $O(10^2)$ s – further indicating negligible elastic effects. Consequently, elasticity and thixotropy are not included in our analysis, as they may not be expected to significantly influence the results.

[Figure 2\(c\)](#) presents an example of our creep test results. As shown, the initial transient shear stress oscillations due to inertia quickly dampen, resulting in a stable creep response (Lidon, Villa & Manneville 2017). For stresses below the yield stress ($\hat{\tau}_y = 0.62$ Pa), i.e. at $\hat{\tau} = 0.2$ and 0.4 Pa, the shear rate decreases to near zero, indicating no flow. For stresses above the yield stress ($1\text{--}2$ Pa), the shear rate stabilises, signalling a flow, with the transition occurring around $\hat{\tau} \approx 0.6$ Pa, consistent with the Herschel–Bulkley model and earlier rheological findings (see [figure 2a](#) and [table 2](#)).

Parameter	Name	Range or value
$\hat{D}$	Inner diameter of outer pipe	3.81 (cm)
$\hat{D}_{in,1}$	Inner diameter of inner pipe	1.27 (cm)
$\hat{D}_{in,2}$	Outer diameter of inner pipe	2.54 (cm)
$\hat{\rho}_H$	Heavy fluid density	999–1018 (kg m ⁻³)
$\hat{\rho}_L$	Light fluid density	998 (kg m ⁻³)
$\hat{\mu}_L$	Light fluid viscosity	0.001 (Pa s)
$\hat{\mu}_H$	Heavy fluid viscosity	0.04–5.1 (Pa s)
$\hat{L}$	Length of flow domain	46 (cm)
$\hat{V}_0$	Injection velocity	3–85 (mm s ⁻¹)
$\hat{g}$	Gravitational acceleration	9.81 (m s ⁻²)
$\hat{D}_m$	Molecular diffusivity	~10 ⁻⁹ (m ² s ⁻¹)
$\hat{\tau}_y$	Heavy fluid yield stress	0.04–0.62 (Pa)
$\hat{\kappa}$	Heavy fluid consistency index	0.01–0.45 (Pa s ⁿ)

Table 3. Dimensional parameters and their ranges in the experiments, with the circumflex symbol (̂) indicating dimensional quantities, throughout the manuscript.

2.1.4. Imaging, calibration and velocity measurement

The experiment preparation begins with rigorous cleaning and meticulous filling to prevent bubble formation, followed by introducing deionised water into the outer pipe and the heavier viscoplastic solution into the inner pipe. The pump is set to a predefined flow rate, and the gate valve at the inner pipe's end is opened. A digital camera (Basler acA2040, 4.2 MP, 90 fps) records the injection flow dynamics. Continuous injection displaces the lighter fluid eventually into the annular section, exiting through the outlet (see figure 1). The flow rate is measured by recording the exit volume and time, with verification from pump data and a flow meter (FTB-421, Omega, ±3 % accuracy).

For imaging, light-emitting diodes placed behind the pipes and separated by diffusive panels provide uniform lighting. Light absorption calibrations are performed following standard procedures (Séon *et al.* 2006). Before each experiment, reference images of the pipe with transparent and dark fluids are captured to calculate normalised concentration fields using the Beer–Lambert law (Séon *et al.* 2006; Taghavi, Mollaabbasi & St-Hilaire 2017). An in-house image processing algorithm then analyses the experimental images to extract the flow features.

Ultrasound Doppler velocimetry (UDV) is used to acquire local axial velocity profiles. For UDV measurements, copolyamide particles (60 µm diameter) are added to both the *in situ* and injected fluids at a concentration of 0.2 g L⁻¹ and mixed for 4 hours at 400 rpm. The UDV probe is positioned on the outer pipe at various locations below the gate valve, as shown in figure 1, and set at a 75° angle to optimise the signal-to-noise ratio (Akbari & Taghavi 2023).

2.1.5. Study scope and dimensionless parameters

Tables 3 and 4 respectively list the dimensional and dimensionless parameters affecting the flow dynamics and their ranges in this study. The dimensionless forms are obtained using $\hat{V}_0$ as the characteristic velocity scale, $\hat{D}$ as the characteristic length and $\hat{\mu}_H \hat{V}_0 / \hat{D}$ as the characteristic stress. The density difference or the sum of the two fluid densities is used as the density scale, while the effective viscosity of the viscoplastic fluid, $\hat{\mu}_H$, serves as the viscosity scale.

Parameter	Name	Definition	Range
δ	Aspect ratio	$\hat{L}/\hat{D}$	12.07
DR_1	Diameter ratio 1	$\hat{D}_{in,1}/\hat{D}$	0.34
DR_2	Diameter ratio 2	$\hat{D}_{in,2}/\hat{D}$	0.67
Re	Reynolds number	$\frac{(\hat{\rho}_H + \hat{\rho}_L)\hat{V}_0\hat{D}}{2\hat{\mu}_H}$	0.04–60
At	Atwood number	$\frac{\Delta\hat{\rho}}{\hat{\rho}_H + \hat{\rho}_L}$	0.0005–0.01
Fr	Froude number	$\frac{\hat{V}_0}{\sqrt{At\hat{g}\hat{D}}}$	0.08–4.4
B	Bingham number	$\frac{\hat{\tau}_y\hat{D}}{\hat{\mu}_H\hat{V}_0}$	0.2–0.97
Pe	Péclet number	$\frac{\hat{D}\hat{V}_0}{\hat{D}_m}$	10^5 – 10^7
M	Viscosity ratio	$\frac{\hat{\mu}_L}{\hat{\mu}_H}$	0.0002–0.025
n	Power-law index	–	0.52–0.96
β	Inclination angle	From vertical	45° – 90°

Table 4. Ranges of dimensionless parameters in our experiments. Note that $\Delta\hat{\rho} = \hat{\rho}_H - \hat{\rho}_L$.

We assume the outer pipe diameter as the relevant length scale, since displacement and flow regimes develop within the outer pipe. This choice aligns with prior studies using similar confined injection configurations (Akbari & Taghavi 2022a; Ghazal & Karimfazli 2022) and core-annular flows (Housz *et al.* 2017; Sahu 2021; Sun *et al.* 2022), where the domain transverse dimensions serve as the characteristic length scale. In contrast, studies involving unconfined media, such as jet or injection flows in large tanks, typically use the injection inlet diameter as the dominant scale (Milton-McGurk *et al.* 2021, 2022; Talluru *et al.* 2021).

Considering the industrial relevance of our scaled-down set-up, certain geometry-related dimensionless parameters – diameter ratios (DR_1 , DR_2) and aspect ratio (δ) – are held constant and, therefore, their effects are ignored. However, prior studies have shown that varying these parameters may influence the flow dynamics. For example, Akbari & Taghavi (2022a) demonstrated that increasing inner pipe wall thickness (DR_2) alters pressure drop, shifting their flow regime from breakup to coiling or buckling. Similarly, Ghazal & Karimfazli (2022) found that their injector-to-channel width ratio (DR) significantly affects flow and mixing in viscoplastic–Newtonian fluid systems, with optimal mixing suppression observed for $0.3 \lesssim DR \lesssim 0.5$.

In this study, the Atwood number is small ($At \ll 1$), indicating small density variations, and it is therefore neglected, while the Froude number (Fr) is retained to capture buoyancy effects. In other words, the experiments use small density differences (partly for convenience (transparent fluids) and partly to reduce parametric complexity), ensuring the validity of the Boussinesq approximation, where density differences influence buoyancy via Fr but not through differential fluid accelerations, which are negligible. As shown in table 4, the experiments span a significant Fr range (0.08–4.4), comparable to industrial

conditions (0.1–30) (Akbari & Taghavi 2021), confirming that the selected density contrasts yield dimensionless numbers relevant to practical applications. A high Péclet number ($Pe \gg 1$) indicates negligible molecular diffusion within the experimental time frame. The power-law index (n) may be less relevant as a separate parameter, as it is also included in defining the viscosity ratio:

$$M = \frac{\hat{\mu}_L}{\hat{\mu}_H} = \frac{\hat{\mu}_L}{\hat{\tau}_y \hat{\gamma}_c^{-1} + \hat{\kappa} \hat{\gamma}_c^{n-1}}. \quad (2.3)$$

Note that, while the classical Bingham number is typically defined as the ratio of yield stress to the viscous term and ranges from 0 to ∞ , this study adopts a normalised form – commonly referred to as the plastic number in the literature (Figueiredo *et al.* 2024; Siqueira *et al.* 2024). Following this convention, we conveniently use the term Bingham number (given in table 4) to present the plastic number, consistent with prior studies on injection and displacement flows involving viscoplastic fluids (Akbari & Taghavi 2022a; Kazemi *et al.* 2024; Faramarzi *et al.* 2024).

Based on the considerations outlined, the flow behaviour in this study may be characterised by five dimensionless numbers: Reynolds number (Re), Froude number (Fr), Bingham number (B), viscosity ratio (M) and inclination angle (β), along with combinations of these parameters. For example, balancing the characteristic buoyant stress and the characteristic viscous stress due to the imposed flow yields a dimensionless buoyancy number (χ), further describing the flow dynamics:

$$(\hat{\rho}_H - \hat{\rho}_L) \hat{g} \hat{D} \sim \hat{\mu}_H \frac{\hat{V}_0}{\hat{D}} \Rightarrow \chi = \frac{(\hat{\rho}_H - \hat{\rho}_L) \hat{g} \hat{D}^2}{\hat{\mu}_H \hat{V}_0}, \quad (2.4)$$

which integrates Re and Fr as

$$\chi \equiv \frac{2Re}{Fr^2}. \quad (2.5)$$

Before proceeding to the next sections, note that throughout the manuscript, unless otherwise stated, lengths, velocities and time are normalised by scaling with $\hat{D}$, $\hat{V}_0$ and $\hat{D}/\hat{V}_0$, respectively, to generalise the findings.

2.2. CFD simulations

We use 3-D numerical simulations to complement our experiments. Consistent with § 2.1.5, to make the governing equations, parameters and variables dimensionless, we use $\hat{V}_0$ as the velocity scale, pipe diameter $\hat{D}$ as the length scale, $\hat{D}/\hat{V}_0$ as the time scale, and $\hat{\mu}_H \hat{V}_0/\hat{D}$ to scale pressure and stresses. From a CFD perspective, a natural formulation includes the motion equations coupled to the concentration–diffusion equation:

$$Re [1 - \varphi(c) At] \left(\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \boldsymbol{\tau} - \frac{1}{Fr^2} \varphi(c) \mathbf{e}_g, \quad (2.6)$$

$$\nabla \cdot \mathbf{u} = 0, \quad (2.7)$$

$$\frac{\partial c}{\partial t} + \mathbf{u} \cdot \nabla c = \frac{1}{Pe} \nabla^2 c, \quad (2.8)$$

where the phase change between pure heavy and light fluids is modelled via a scalar concentration, c , with the function $\varphi = 1 - 2c$ interpolating between 1 and -1 for $c \in [0, 1]$; here, $c = 1$ corresponds to the pure heavy viscoplastic fluid and $c = 0$ corresponds to the pure light Newtonian fluid. Moreover, $\mathbf{u}$ denotes the velocity field,

τ the deviatoric stress and p the pressure. Note that we have already subtracted the static pressure gradient of the heavy fluid from the pressure gradient term before scaling. Here, $\mathbf{e}_g$ is the normalised gravity vector in Cartesian coordinates, i.e. $\mathbf{e}_g = (0, \sin(\beta), \cos(\beta))$.

With large Pe , diffusion is initially confined to thin interfacial layers of size $\sim Pe^{-1/2}$, remaining sharp over reasonable time scales if there is no instability or mixing. These flows resemble immiscible fluids (Sahu & Vanka 2011; Redapangu, Sahu & Vanka 2012) at infinite capillary number, modelled by taking $Pe \rightarrow \infty$ and neglecting the diffusive term in (2.8).

The constitutive equation for the light Newtonian fluid is given by

$$\tau_{L,ij} = M \dot{\gamma}_{ij}(\mathbf{u}), \quad (2.9)$$

and the heavy viscoplastic fluid follows the Herschel–Bulkley constitutive law:

$$\dot{\gamma}(\mathbf{u}) = 0 \Leftrightarrow \tau_H(\mathbf{u}) \leq B, \\ \tau_{H,ij}(\mathbf{u}) = \left[(1 - B) \dot{\gamma}^{n-1}(\mathbf{u}) + \frac{B}{\dot{\gamma}(\mathbf{u})} \right] \dot{\gamma}_{ij}(\mathbf{u}) \Leftrightarrow \tau_H(\mathbf{u}) > B, \quad (2.10)$$

where the strain rate tensor has the following components:

$$\dot{\gamma}_{ij}(\mathbf{u}) = \left\{ \nabla \mathbf{u} + (\nabla \mathbf{u})^T \right\}_{ij}, \quad (2.11)$$

and the second invariants, $\dot{\gamma}(\mathbf{u})$ and $\tau_H(\mathbf{u})$, are defined by

$$\dot{\gamma}(\mathbf{u}) = \left[\frac{1}{2} \sum_{i,j=1}^3 [\dot{\gamma}_{ij}(\mathbf{u})]^2 \right]^{1/2}, \quad \tau_H(\mathbf{u}) = \left[\frac{1}{2} \sum_{i,j=1}^3 [\tau_{H,ij}(\mathbf{u})]^2 \right]^{1/2}. \quad (2.12)$$

To model the viscoplastic rheology for the Herschel–Bulkley fluid, we employ the Papanastasiou regularisation method (Papanastasiou 1987), widely employed due to its relatively simple implementation in standard numerical codes (Mitsoulis 2010; Fernandes *et al.* 2022b; Taglieri São & de Freitas Maciel 2023), presented as

$$\mu_{H,g} = \frac{\hat{\mu}_{H,g}}{\hat{\mu}_H} = (1 - B) \dot{\gamma}^{n-1} + \frac{B (1 - \exp(-\mathcal{R}\dot{\gamma}))}{\dot{\gamma}}, \quad (2.13)$$

where $\mu_{H,g}$ is the dimensionless general viscosity, based on the Papanastasiou model, and $\mathcal{R}$ refers to the dimensionless regularisation parameter in our study. This regularisation model approximates unyielded regions as areas of very high viscosity, with viscosity dropping sharply near the yield surface. Increasing $\mathcal{R}$ sharpens this transition, closely approximating the true Herschel–Bulkley model. Larger $\mathcal{R}$, combined with a fine mesh, improves simulation accuracy but increases computational cost (Dimakopoulos, Pavlidis & Tsamopoulos 2013). Testing $\mathcal{R}$ values from 10^1 to 10^5 revealed appropriate convergence in velocities and yield surface locations, with $\mathcal{R} = 10^4$ accurately capturing our viscoplastic flow features.

The regularisation of yield stress fluids complicates precise yield surface detection, as unyielded regions behave as highly viscous fluids rather than rigid solids; to address this, we applied a standard criterion defining the critical strain rate $\dot{\gamma}_c$, i.e. the solution to the following nonlinear equation (Frigaard & Nouar 2005; Putz, Frigaard & Martinez 2009; Joulaei *et al.* 2024):

$$(1 - B) \dot{\gamma}_c^n = B \exp(-\mathcal{R}\dot{\gamma}_c), \quad (2.14)$$

where $\dot{\gamma}_c$ denotes the critical strain rate magnitude, below which regions are classified as unyielded.

Various models can describe the relationship between the concentration and the physical properties of the fluid, such as density and viscosity. In this work, the dimensionless mixture density ($\rho_{mixture}$) and viscosity ($\mu_{mixture}$) are modelled as (Akbari & Taghavi 2023)

$$\rho_{mixture} = cAt + \frac{1 + At}{2}, \quad (2.15)$$

$$\mu_{mixture} = (1 - c)M + c. \quad (2.16)$$

Given the large Pe number and significant viscosity difference, the fluid pseudo-interface is thin, making the choice of mixing rule negligible for our flow system behaviour. To ensure this, the simulation results with the linear interpolation (2.15 and 2.16) were validated through additional simulations with nonlinear mixing rules, e.g. in the following form:

$$\mu_{mixture} = \mu_{H,g} \exp \left[c \log \left(\frac{M}{\mu_{H,g}} \right) \right]. \quad (2.17)$$

The results confirmed no observable differences in the flow behaviour from the choice of the mixing rule, validating the robustness of our initial hypothesis.

Our CFD simulations consider scenarios identical to our experiments, allowing us to define the initial and boundary conditions. For instance, the entire domain is initially filled with a light fluid (concentration $c = 0$), and an imaginary inlet concentration of $c = 1$ is imposed for $t > 0$. The boundary conditions include a uniform velocity at the inlet and a pressure outlet condition at the domain boundaries, while the pipe walls and the end surface are treated as solid, stationary boundaries with no-slip and no-penetration conditions.

The governing equations are discretised and solved using the finite-volume method, implemented in OpenFOAM. An optimised twoLiquidMixingFoam solver with the volume-of-fluid approach (Marić *et al.* 2020) is used, as it effectively captures concentration distribution and fluid velocities and has been successfully applied to miscible injection flows with non-Newtonian fluids (Akbari & Taghavi 2023). This solver, i.e. an incompressible multiphase model with buoyancy effects, is ideal for fluids with contrasting densities and viscosities, leading to complex flows.

Buoyancy effects in the flow arise from gravity acting on a variable-density field and include both the gravitational component along the pipe axis and the corresponding pressure gradient. In OpenFOAM, the pressure gradient term ($-\nabla p$) in the momentum equation (see 2.6) is handled via pressure-velocity coupling through the PIMPLE algorithm. The gravitational body force is explicitly included as a source term, implemented in a variable-density framework where the density varies with a scalar concentration field c .

Mesh resolutions ranging from 245 717 to 2 130 031 cells were tested to assess accuracy. A mesh with 1 535 503 cells (with a segment depicted in figure 1) was ultimately selected as the optimal balance between accuracy and computational efficiency, resolving flow characteristics without excessive cost. Further refinement yielded minimal improvement, as our flow was well resolved at this resolution. The chosen mesh also provided stable, accurate results in regions with high gradients, such as interfaces and boundary layers, without requiring finer refinement. All simulations were conducted with reasonable convergence criteria, reducing velocity residuals below 10^{-5} at each step, ensuring stable and reliable numerical solutions.

Gradients were computed using the Gauss linear scheme, with divergence terms handled by Gauss linearUpwind grad(U) for velocity flux and Gauss VanLeer for phase fraction,

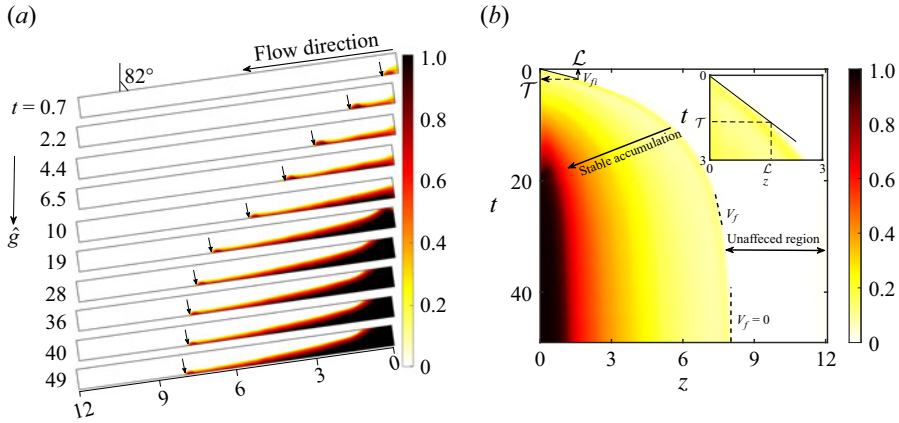


Figure 3. Experimental results. (a) Snapshots from a typical experiment in the interfacial arrest regime, with $Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$ and $Fr = 0.73$, showing dimensionless times, with arrows indicating the position of the interfacial front. Here and in all snapshot figures, throughout the manuscript, a dimensionless field of view of 1×12 is shown. (b) Spatiotemporal diagram for the experiment in panel (a), with solid black line marking the inertial-dominated phase with constant front velocity, V_f (inverse slope of black dashed line). Vertical dashed line indicates $V_f = 0$ (interfacial motion arrest). The inset details the inertial-dominated phase, highlighting the critical transition length ($\mathcal{L}$) and time ($\mathcal{T}$). Colourbars in both panels show viscoplastic fluid concentration.

maintaining sharp interfaces. The GAMG method (Lyu, Izadi & Taghavi 2020) efficiently solved the discretised equations, using a multigrid approach for speed and memory efficiency. The PIMPLE algorithm, combining PISO (Constant *et al.* 2017) and SIMPLE (Ferziger, Perić & Street 2019) methods, iteratively corrected velocity and pressure fields, enhancing convergence, accuracy and numerical robustness, while enabling larger time steps with a high Courant number (Deng *et al.* 2022). Temporal discretisation used the explicit Euler method, with the time step adjusted to keep the Courant number below 0.5. All simulations were run in parallel on the Compute Canada HPC cluster (allowance of 1033 CPU cores, 2 GB memory per core), with a computational domain of $1\,535\,503$ grid cells; each case required 12–15 hours to complete.

3. Results and discussion

Supported by experimental results and CFD simulations, this section analyses the interfacial dynamics of our flow.

3.1. General observations

Figure 3(a) shows snapshots of a typical viscoplastic fluid injection flow experiment. Upon injection, the heavier viscoplastic fluid forms a jet with considerable inertial effects. The heavy viscoplastic fluid then slumps beneath the light ambient Newtonian fluid, initially maintaining a near-constant velocity. Beyond a certain point, the interfacial front velocity decreases, causing the viscoplastic fluid to accumulate near the inlet as the injection continues. Meanwhile, the interfacial front motion continuously slows down and eventually stops, with the interface becoming stationary. This corresponds to a complete interfacial arrest. Under continuous injection, once the interfacial front is arrested, the injected viscoplastic fluid redirects towards the outlet. Each experiment spans $O(10^2)$ seconds to confirm flow cessation and stabilisation.

Figure 3(b) presents the spatiotemporal representation of the normalised depth-averaged concentration field for the viscoplastic fluid in the experiment corresponding to figure 3(a). The colourmap illustrates the mean concentration, $\bar{c}(z, t)$, averaged over the pipe diameter, as a function of the spatial position, z , and time, t . Here,

$$\bar{c}(y, z, t) = \int_{-1/2}^{+1/2} c(x, y, z, t) dx, \quad (3.1)$$

$$\bar{\bar{c}}(z, t) = \int_0^1 \bar{c}(y, z, t) dy, \quad (3.2)$$

where the integration limits $-1/2$ and $+1/2$ correspond to the pipe's half-width. The function $c(x, y, z, t)$ symbolises the true concentration field, while $\bar{c}(y, z, t)$ refers to the concentration captured in side-view experimental images. In general, the spatiotemporal map in figure 3(b) provides insights into the time-dependent flow dynamics, interfacial behaviours and front dynamics. For example, the inverse of the slope of the boundary between the coloured and white regions (identified by the black dashed lines) represents the dimensionless interfacial front velocity, V_f . As time progresses and the interface develops, two interfacial flow phases emerge: at short times, inertia from the imposed flow dominates, causing the interfacial front to advance at a velocity nearly equal to the mean imposed flow velocity ($V_f \approx 1$ in dimensionless form), defining the inertial-dominated phase. Later, the flow inertia diminishes, and the balance between viscoplasticity (encompassing effective viscosity and yield stress) and longitudinal buoyancy governs the forward motion of the interface, marking the viscoplastic-dominated phase. The solid black line in figure 3(b) and its inset marks the initial inertial-dominated phase ($z \lesssim 2$, $t \lesssim 2$), which transitions to a viscoplastic-dominated phase as inertial stresses diminish. In the latter phase, buoyancy drives the interface, but the front velocity slows down while the interface shape changes, indicating effective viscous stresses balancing buoyant stresses and causing the deceleration, i.e. a signature of viscous exchange flows (Séon *et al.* 2007; Taghavi *et al.* 2012b). As indicated by the black arrow, the depth-averaged concentration behind the front increases as V_f drops below the injection velocity, resulting in viscoplastic fluid accumulation. The vertical dashed lines at longer times exhibit the interfacial front arrest and stationary condition ($V_f = 0$), suggesting a complete overcoming of all stresses by the yield stress. The unaffected region shown in white is indicative of a pure light fluid beyond the arrested interfacial front. The inset in figure 3(b) provides a detailed view of the inertial-dominated phase and the transition to the viscoplastic-dominated phase, highlighting the critical transition length ($\mathcal{L}$) and time ($\mathcal{T}$).

Figure 4(a) compares the time-dependent interfacial front position from the experiment (lines) with CFD simulation results (symbols) for three different cases. The CFD results exhibit reasonable agreement with the experimental results, validating the observed experimental phases, including the inertial- and viscoplastic-dominated phases. During the initial phase ($z \lesssim 2$), i.e. the inertial-dominated phase, both CFD simulations and experiments show similar front velocities ($V_f \approx 1$), as the flow is dominated by the imposed flow inertia. This observation is consistent with previous studies on buoyant miscible injections of Newtonian (Akbari & Taghavi 2021) and viscoplastic fluids (Faramarzi *et al.* 2024; Akbari & Taghavi 2022a), across various flow regimes that can generally occur in such injection flows, including mixing, separation, and stable and unstable slumping (Faramarzi *et al.* 2024). Figure 4(b) provides simulation snapshots of the concentration profile, interface and front development, which align well with experimental data, albeit with some discrepancy in interface height, likely due to uncertainties in experimental conditions. While the experiment provides side-view concentration data, the CFD simulations reveal

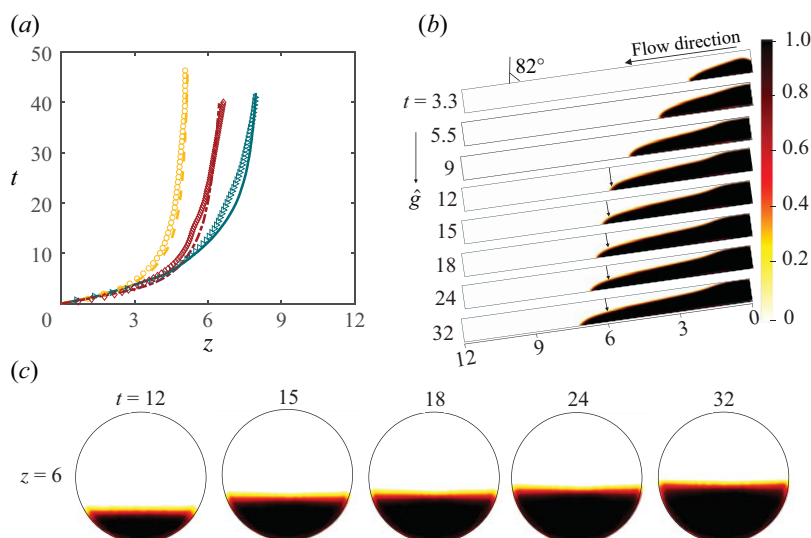


Figure 4. CFD and experimental results. (a) Time-dependent interfacial front position from CFD simulations (symbols) versus experiments (lines) in the pipe's centre plane (z - y plane at $x = 0$): ($\circ$, - -) for $Re = 1.3$, $M = 0.001$, $B = 0.74$, $\beta = 88^\circ$, $Fr = 0.55$; ($\diamond$, -) for $Re = 1$, $M = 0.0009$, $B = 0.75$, $\beta = 82^\circ$, $Fr = 0.47$; ($\triangleright$, -) for $Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$, $Fr = 0.73$. (b) CFD snapshots of concentration profiles, at the pipe centre plane (i.e. z - y plane at $x = 0$), for $Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$ and $Fr = 0.73$. (c) Cross-sectional concentration contours in the x - y plane at $z = 6$ at different times, marked by arrows in panel (b). Colourbar shows heavy viscoplastic fluid concentration.

cross-sectional variations, as shown in figure 4(c), with contours of the heavy viscoplastic fluid. These results show a horizontal interface where the dense viscoplastic fluid slumps stably beneath the lighter fluid and progressively forms an unyielded zone as stress falls below the yield threshold. Note that although we focus on stable flows with negligible secondary flows, at ranges of the Péclet number considered ($Pe = 10^5$ – 10^7), thin mixing layers naturally develop at the interface, as observed experimentally (figure 3a) and consistently captured in CFD simulations (figure 4b).

Before proceeding, note that, from a CFD perspective, the analysis of concentration and velocity profiles (omitted for brevity) reveals that, despite the no-slip condition enforcing zero velocity at the wall, at longer times, the concentration near the ‘triple contact point’ approaches that of the pure viscoplastic fluid. The near-wall concentration becomes non-zero immediately after the viscoplastic front arrival, indicating no persistent Newtonian fluid computational ‘cell’ along the wall. This behaviour results from: (i) density-driven instability, where the initially unstable configuration – heavier viscoplastic fluid overlying the lighter Newtonian phase near the wall – rapidly dominates; and (ii) fluid miscibility, where diffusion and interfacial mixing, enhanced by the mesh and cell size near the wall, prevent the persistence of an unmixed Newtonian layer. Consequently, no permanent Newtonian pocket remains at the lower wall at long times.

3.2. Flow, interface and front dynamics

This section analyses the flow, interface and front dynamics, highlighting distinct flow phases and regimes during the viscoplastic fluid injection. Figure 5(a) presents the temporal evolution of the interfacial front position, illustrating the inertial-dominated, viscous–buoyant and yield-stress-dominated phases. The latter two regimes can be collectively described as the viscoplastic-dominated phase, during which the effects of

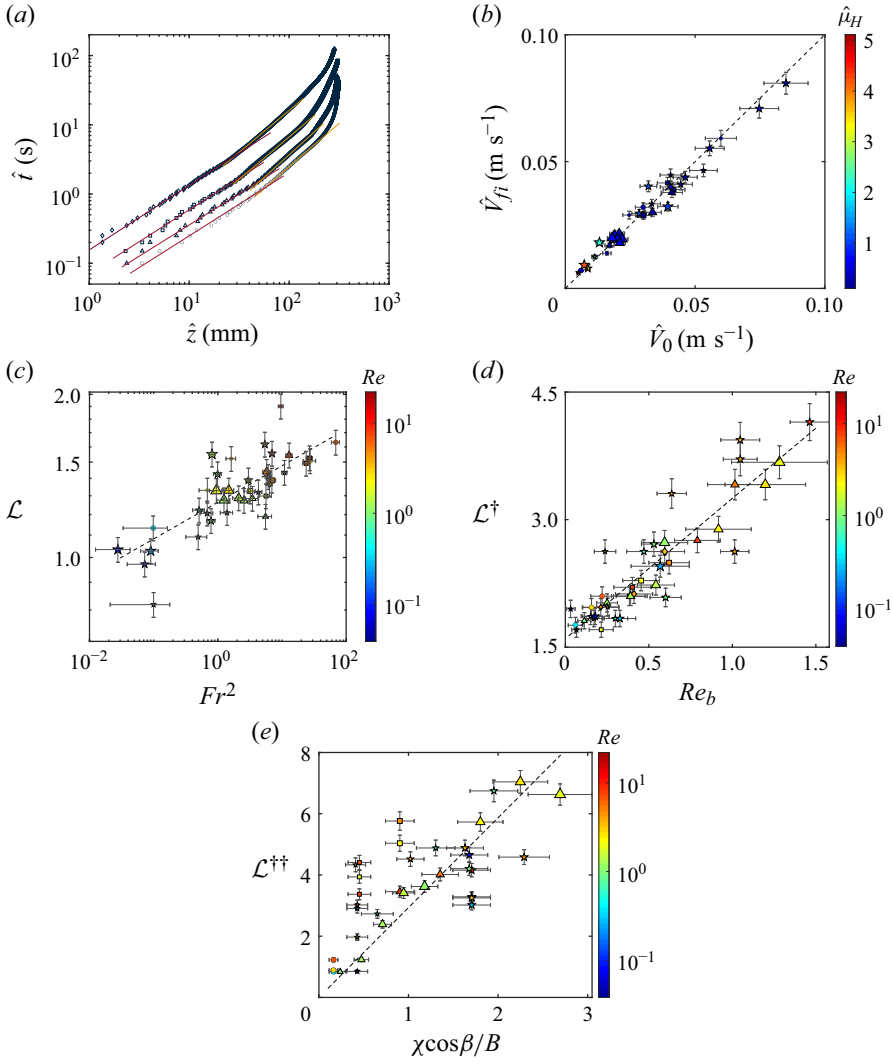


Figure 5. Experimental results. (a) Variation of interfacial front distance from the gate valve over time in log-log coordinates. Symbols represent experiments with: $\circ$ ($Re = 3.4$, $M = 0.0016$, $B = 0.67$, $\beta = 82^\circ$, $Fr = 0.91$), $\square$ ($Re = 0.24$, $M = 0.0005$, $B = 0.83$, $\beta = 75^\circ$, $Fr = 0.22$), $\diamond$ ($Re = 0.1$, $M = 0.0003$, $B = 0.75$, $\beta = 82^\circ$, $Fr = 0.14$) and $\triangle$ ($Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$, $Fr = 0.73$). Red lines fit early-time data with slope ~ 1 in the inertial-dominated phase and brown lines fit the initial stage of the viscoplastic-dominated phase. (b) Dimensional interfacial front velocity in the inertial-dominant phase ($\hat{V}_{fi}$) versus $\hat{V}_0$; dashed lines indicate 1 : 1 relationship. (c) Critical length, $\mathcal{L}$ (transition from an inertial-dominated phase to a viscoplastic-dominated phase) versus Fr^2 . (d) Power-law trend length $\mathcal{L}^+$ versus Re_b , with the dashed line marking a linear fit. (e) Sharp decay length ($\mathcal{L}^{++}$) versus $\chi \cos \beta / B$, with the dashed line marking a linear fit. Marker sizes indicate $\cos \beta$ in panel (b–e); colours in panel (b) show $\hat{\mu}_H$ magnitude, while in the rest, they represent Re . Symbols indicate $\Delta \hat{\rho}$: circles (1 kg m⁻³); squares (4 kg m⁻³); triangles (5 kg m⁻³); diamonds (10 kg m⁻³); pentagrams (20 kg m⁻³).

viscous and yield stresses increasingly control the front dynamics. In the initial inertial-dominated phase, the front advances at a constant velocity that matches the injection speed, resulting in a linear trend with a slope of approximately one in dimensionless coordinates. This behaviour is consistently observed across both CFD simulations and experiments, where the early-time data collapse in the dimensionless t – z space (figure 4a).

The flow then transitions into a viscous–buoyant regime, where the front velocity decays following a power-law trend with exponents ranging from 0.76 to 0.9. This regime reflects a balance between longitudinal buoyant stresses and viscous resistance and appears largely independent of the yield stress. The power-law behaviour manifests as a straight-line segment on a log–log plot, as shown by the brown lines in [figure 5\(a\)](#). Finally, the front enters a yield-stress-dominated phase, where the velocity decays more rapidly, deviating from the earlier power-law behaviour. This regime ultimately leads to complete interfacial arrest. The increasing effective viscous stress and diminishing longitudinal buoyant stress (due to interface flattening) result in progressive slowing until the yield stress fully dominates the flow.

[Figure 5\(b\)](#) presents the constant interfacial front velocity (dimensional) in the inertial-dominated phase, $\hat{V}_{fi}$, plotted against the mean imposed flow, $\hat{V}_0$, confirming that $\hat{V}_{fi}$ closely matches $\hat{V}_0$. Minor discrepancies arise from measurement errors in $\hat{V}_0$ and $\hat{V}_{fi}$. Our results identify a critical development length from the injection origin ($\mathcal{L}$), i.e. the effective jet length, marking the transition from an inertial-dominated phase to a viscoplastic-dominated phase. Our findings suggest that $\mathcal{L}$ may be governed by a dimensionless group balancing the imposed inertial stress with the resisting buoyant stress, expressed as

$$\hat{\rho}\hat{V}_0^2 \sim \Delta\hat{\rho}\hat{g}\hat{D} \quad \Rightarrow \quad \frac{\hat{\rho}\hat{V}_0^2}{\Delta\hat{\rho}\hat{g}\hat{D}} \propto Fr^2, \quad (3.3)$$

where $\hat{\rho} = (\hat{\rho}_H + \hat{\rho}_L)/2$. Equation (3.3) shows consistency with the results presented in [figure 5\(c\)](#), demonstrating that $\mathcal{L}$ increases with increasing imposed inertial stress (i.e. increasing Re , as indicated by the colourbar) or decreasing buoyant stress. A correlation can accordingly be suggested for $\mathcal{L}$ as

$$\mathcal{L} \approx 1.26Fr^{0.14}, \quad (3.4)$$

which, considering $V_f \approx 1$ in the inertial-dominated phase, results in a critical transition time $\mathcal{T} \approx \mathcal{L}$. [Figure 5\(c\)](#) also demonstrates that the critical length exhibits minimal dependence on β . Furthermore, for all inclination angles, cases with larger Fr correspond to greater $\mathcal{L}$.

Our results suggest that the power-law trend length, $\mathcal{L}^\dagger$, observed after the inertial phase (brown lines in [figure 5\(a\)](#)), is governed by a balance between characteristic longitudinal buoyant stresses and the corresponding viscous stresses:

$$\Delta\hat{\rho}\hat{g}\hat{D} \cos \beta \sim \frac{\hat{\mu}_H \hat{V}_i}{\hat{D}} \equiv \frac{\hat{\mu}_H \left(\frac{\Delta\hat{\rho}}{\hat{\rho}} \hat{g}\hat{D} \right)^{1/2}}{\hat{D}} \quad \Rightarrow \quad Re_b = \frac{\hat{\rho}^{1/2} (\Delta\hat{\rho})^{1/2} \hat{g}^{1/2} \hat{D}^{3/2} \cos \beta}{\hat{\mu}_H}, \quad (3.5)$$

where $\hat{V}_i = ((\Delta\hat{\rho}/\hat{\rho})\hat{g}\hat{D})^{1/2}$ is the characteristic buoyancy-induced inertial velocity and Re_b is the corresponding Reynolds number. This balance between longitudinal buoyancy forces and viscous resistance governs the gradual deceleration of the front in this intermediate regime. [Figure 5\(d\)](#) shows that $\mathcal{L}^\dagger$ increases nearly linearly with Re_b , highlighting the balance between buoyancy and the corresponding viscous stresses in this region. Therefore, $\mathcal{L}^\dagger$ characterises the extent of the viscous–buoyant regime prior to the onset of yield-stress control.

Moreover, our results suggest that the sharp decay length, $\mathcal{L}^{\dagger\dagger}$, may be estimated, albeit very crudely, by balancing the characteristic longitudinal buoyant stress and yield stress:

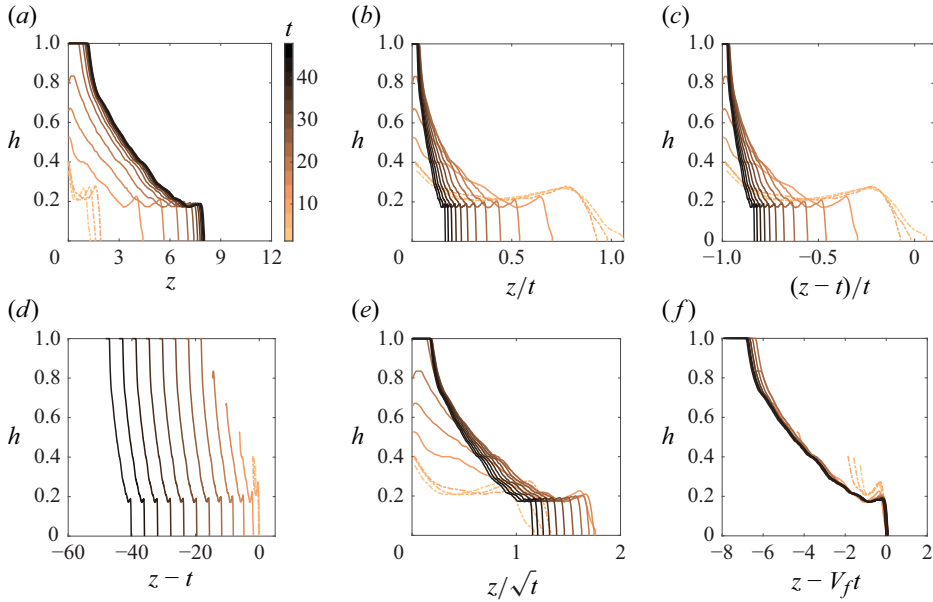


Figure 6. Experimental results. (a) Time-dependent variation in interface height (h) versus z . (b–e) Effect of similarity parameters on interface height profiles: (b) z/t ; (c) $(z-t)/t$; (d) $z-t$; (e) $z/\sqrt{t}$ and (f) $z-V_f t$ (i.e. $z-z_f(t)$, where z_f is the front position). Across all panels, the experimental parameters correspond to figure 3. The colourbars represent t , with darker colours indicating longer times. Dash-dotted and solid line profiles represent the inertial- and viscoplastic-dominated phases, respectively.

$$\Delta \hat{\rho} \hat{g} \hat{D} \cos \beta \sim \hat{\tau}_y \Rightarrow \frac{\Delta \hat{\rho} \hat{g} \hat{D} \cos \beta}{\hat{\tau}_y} \propto \frac{Re \cos(\beta)}{Fr^2 B} \equiv \frac{\chi \cos(\beta)}{B}. \quad (3.6)$$

Figure 5(e) shows that $\mathcal{L}^{\dagger\dagger}$ increases with axial buoyancy ($\chi \cos \beta$) and decreases with yield stress (B), indicating that $\chi \cos(\beta)/B$ is a relevant governing dimensionless group, although deviations from linearity suggest the proposed form is not fully adequate.

Analysing interface height profiles (h) provides insights into interfacial front dynamics. Figure 6(a) illustrates the evolution of h throughout a typical experiment, spanning the inertial-dominated phase to the viscoplastic-dominated phase. During the inertial-dominated phase ($z \lesssim 2$), the first three profiles (identified by dash-dotted lines) maintain similar shapes with consistent spacing at predefined time intervals (Δt). Upon entering the viscoplastic-dominated phase (identified by solid lines), the interface height increases rapidly before slowing, as reflected by the decreasing profile spacing. This stage corresponds to reduced front velocity as inertial effects wane and effective viscous and buoyant stresses govern the flow. Longitudinal buoyancy stresses, influenced by density differences, geometric inclination and interface slope, drive the flow, while increasing effective viscosity impedes motion, progressively reducing the shear rate. The interfacial motion experiences significant deceleration as the interface nears arrest. The interface evolution finally halts when shear stresses within the viscoplastic layer fall below the yield stress. As discussed in figures 3 and 5, this deceleration and arrest result from decreasing shear rates, increasing effective viscosity, and, as will be seen, expanding unyielded regions within the viscoplastic fluid layer. These factors collectively slow the interface until it halts, as visually seen in figures 5(a) and 6(a).

The interface height profiles in [figure 6\(a\)](#) also reveal outward and upward spreading of the viscoplastic fluid as time progresses. A small inertial bump appears at the interfacial front from the onset, similar to those reported in previous studies, e.g. [Taghavi *et al.* \(2012b\)](#), which examined buoyant miscible displacement flows with Newtonian fluids. This inertial bump is a flow feature driven by the balance of buoyancy and local inertia, likely unrelated to the viscoplastic nature of the flow.

We now analyse whether the interface height profiles in both the inertial- and viscoplastic-dominated phases exhibit self-similarity, indicating the dominant physical flow mechanism and simplifying the interfacial flow into a universal model. Several moving reference frames were evaluated: z/t ([figure 6b](#)); $(z - t)/t$ ([figure 6c](#)); $z - t$ ([figure 6d](#)); $z/\sqrt{t}$ ([figure 6e](#)); and $z - V_f t$ ([figure 6f](#)). The dimensionless frame z/t suggests linear propagation, with the early-time interface heights (inertial-dominated phase) ‘almost’ collapsing onto one another, especially at the frontal region, but no self-similarity is observed in the longer-time interface heights (viscoplastic-dominated phase). Similarly, $(z - t)/t$ shows a collapse in the inertial-dominated phase for $(z - t)/t \in [-1, 0]$, with unsteady travelling waves emerging at longer times. The $z - t$ frame confirms uniform interfacial front advancement in the inertial-dominated phase, but no self-similarity in the viscoplastic-dominated phase. The $z/\sqrt{t}$ frame reveals no self-similarity, indicating that a macroscopically diffusive interface does not govern the flow. Only $z - V_f t$ achieves convergence to self-similar curves in both phases, each with distinct master curves. In the inertial-dominated phase, self-similarity is observed roughly in z/t and $(z - t)/t$, and more completely in $z - t$ and $z - V_f t$, which coincide due to $V_f \approx 1$ in this phase. In the viscoplastic-dominated phase, only $z - V_f t$ maintains self-similarity. Since V_f varies with time, this indicates an unsteady travelling wave at the interfacial front.

When a heavier fluid is placed above a lighter one in a closed-end pipe, purely Newtonian viscous fluids and viscoplastic fluids exhibit distinct behaviours. For near-horizontal inclinations with Newtonian fluids, density differences drive the heavier fluid downward, displacing the lighter one. This causes slow slumping, eventually leading to a stratified configuration with the heavier fluid below the lighter one ([Frigaard & Ngwa 2004](#); [Taghavi *et al.* 2009](#)), as purely viscous stresses cannot halt motion. Even highly viscous Newtonian fluids continue moving, albeit slowly ([Etrati & Frigaard 2018](#)). However, with viscoplastic fluids, two key differences may arise. First, due to the yield stress, there are scenarios in which the heavy viscoplastic fluid can remain atop the lighter fluid with a stable interface. Second, even if the viscoplastic fluid slumps beneath the lighter fluid, it may stop at a certain point as the interface elongates, e.g. as observed in our experiments. Thus, as we place the heavy viscoplastic fluid atop the light Newtonian fluid via injection, two critical questions arise: (1) Under what conditions does the interface between the heavy viscoplastic and light Newtonian fluids remain stable or become unstable? (2) What is the maximum slumping extent/length during the interface motion before it is arrested?

Regarding the first question, several foundational experimental and theoretical studies ([Frigaard 1998](#); [Frigaard & Scherzer 1998, 2000](#); [Crawshaw & Frigaard 1999](#); [Frigaard & Crawshaw 1999](#)) have examined the conditions under which viscoplastic fluids initiate slow, viscous slump flows following interface failure, i.e. when the interface becomes unstable. These studies demonstrated that the yield stress can inhibit fluid movement, preventing further motion following an initial disturbance. Through dimensional analysis, they concluded that any criterion defining when the flow becomes stationary must depend solely on the inclination angle, β , and the yield number, which is the ratio of the yield stress to the buoyancy stress:

$$Y = \frac{\hat{\tau}_y}{\Delta \hat{\rho} \hat{g} \hat{D}} \equiv \frac{B}{\chi}, \quad (3.7)$$

where Y represents the yield number. Both experimental and analytical studies have characterised the yield number ranges governing the initiation and sustained flow of viscoplastic fluids in quiescent states across various flow configurations. Pertinent to this study's flow configuration, Frigaard & Crawshaw (1999) and Crawshaw & Frigaard (1999) studied interface stability in viscous exchange flows between miscible Bingham fluids with differing densities and rheological properties under density-unstable conditions in inclined, closed-end pipes. Using motion equations, and applying no-slip boundary condition and velocity and stress continuity at the interface, they developed a lubrication-type two-layer stratified flow model to establish a conservative boundary between flow and no-flow conditions:

$$Y_m = \left([\tau_{y,e}(\phi) \cos(\beta)]^2 + [\tau_{y,n}(\phi) \sin(\beta)]^2 \right)^{\frac{1}{2}}, \quad (3.8)$$

where Y_m is the modified theoretical yield number, $\tau_{y,e}$ is the tangential component of deviatoric stress (off-diagonal components), $\tau_{y,n}$ is the normal component of deviatoric stress (diagonal components), ϕ is defined in radians and varies between $-\pi/4$ and $\pi/4$, and β is the deviation from the vertical. A flow configuration where only one fluid has a yield stress corresponds to $\phi = \pm\pi/4$, $\tau_{y,e} = 0.3043$ and $\tau_{y,n} = 0.25$. Applying (3.8) to this scenario produces the solid line in figure 7(a), which delineates flow and no-flow conditions. Projecting our experimental data onto this Y - β plane shows that, at various inclination angles, our results fall below the theoretical yield number separating flow and no-flow regions. Thus, our study remains below the theoretical condition, allowing the injected viscoplastic fluid to flow naturally within the lighter fluid, initiating buoyancy-driven interfacial slumping.

Regarding the above mentioned second question, Frigaard & Ngwa (2004) and Malekmohammadi *et al.* (2010) studied exchange flows of two viscoplastic fluids in near-horizontal pipes with closed ends, using scaling from thin-film lubrication theory. They identified the minimal surface inclination needed to initiate flow, through applying no-slip conditions at walls, stress and velocity continuity across the interface, and negligible surface tension, assuming miscibility on a short time scale. Their results showed that buoyancy-driven slumping continues until a maximum static slump length is reached, where the shear stresses from density differences, interface slope and inclination balance the yield stresses. In our flow configuration, we adapt the analysis of Frigaard & Ngwa (2004), considering the simplification that only one fluid is viscoplastic (see Appendix A). The maximum static or upper-bound slump length for our density-unstable conditions, as per Frigaard & Ngwa (2004), can be transformed and expressed as

$$L_s = \frac{1}{Y} \int_0^1 \frac{\beta_0(h)}{\beta_c(h) - \alpha \beta_0(h)} dh, \quad (3.9)$$

where α is determined by writing a balance between longitudinal buoyant stress and the yield stress of the heavy fluid, i.e. (3.6):

$$\alpha \equiv \frac{Re \cos(\beta)}{Fr^2 B} \equiv \frac{\chi \cos(\beta)}{B}, \quad (3.10)$$

and the geometrical functions $\beta_0(h)$ and $\beta_c(h)$, which depend solely on h , are given in Appendix A.

Figure 7(b) compares the theoretical maximum static length – i.e. the axial length over which the interfacial front evolves before it stops (when the interface is arrested) –

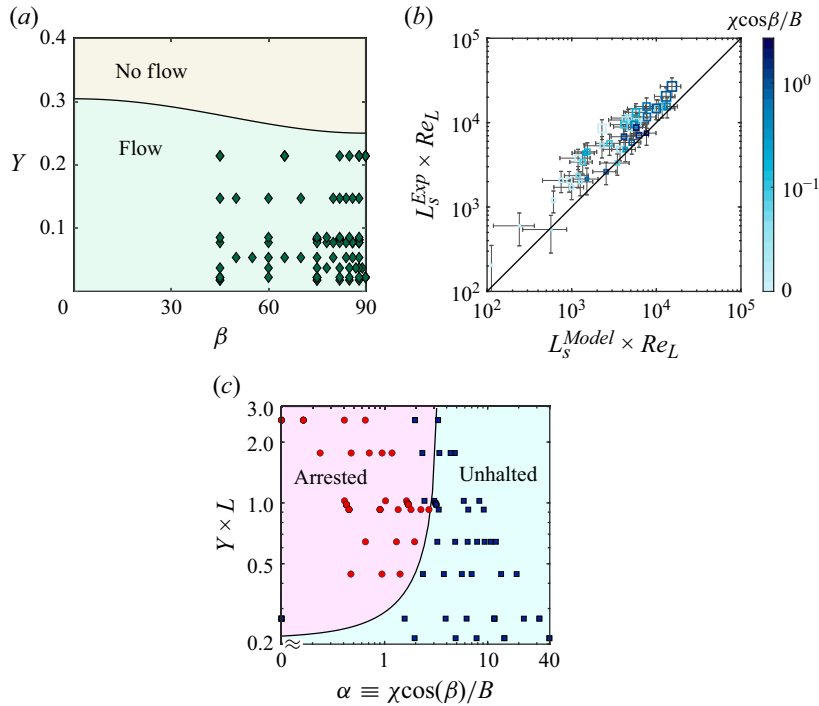


Figure 7. Experimental and theoretical results. (a) Experimental results on the Y – β plane. The dividing line marks the theoretical yield number for angles (0° – 90°) in a density-unstable configuration based on (3.8). (b) Experimental arrest length ($L_s^{Exp} \times Re_L$) versus model-determined arrest length ($L_s^{Model} \times Re_L$), with marker size and colour representing Re_L and $\chi \cos(\beta)/B$, respectively. Solid line is the 1:1 line. (c) Model-based classification (3.9) of the arrested and unhalted regimes on the $Y \times L$ and $\alpha \equiv \chi \cos(\beta)/B$ plane. In panel (c), pink and cyan areas represent the arrested and unhalted interfacial flow regimes, with red circles and blue squares indicating experimental data for each regime. Experiments with $\chi \cos(\beta)/B = 0$ are shown on the left of panel (c).

with corresponding experimental data. The data are adjusted by subtracting the inertial-dominated length ($\mathcal{L}$) to match the inertialess theoretical model. Both axes are scaled by $Re_L = \hat{\rho}_L \hat{V}_0 \hat{D} / \hat{\mu}_L$ to enhance the visualisation of data distribution. As shown, our experimental static lengths follow the model, but generally exceed the model's predictions.

Despite the model's simplifying assumptions and the challenges associated with viscoplastic fluid experiments, its predictions show reasonable agreement with experimental results, although some discrepancies are observed, particularly where experimental values exceed model predictions. One likely cause is the yield stress value used in the analysis, derived from Herschel–Bulkley model fitting, which differs slightly from values obtained by other methods (see figure 2 and table 2, and compare figures 2a and 2b). This reflects the nuanced yielding behaviour of viscoplastic fluids and suggests that steady-state shear rheometry may not always provide the most appropriate yield stress for different flow conditions. This discrepancy influences the value of L_s^{Model} in figure 7(b). Other experimental parameters, such as density, inclination and length, may also have minor effects on L_s^{Model} . Another source of discrepancy arises from fundamental differences between the simplified model and the richer dynamics of the experimental flow, where several effects are omitted. For example, the model neglects the potential weak influence of injection velocity on the measured L_s^{Exp} , which may contribute to deviations

observed in [figure 7\(b\)](#) at higher Re_L values, where the gap between experimental data and model predictions exceeds that at lower Re_L , as seen in the log-log plot.

Note that, while our analysis builds on the framework of Frigaard & Ngwa (2004), our configuration involves continuous injection, where the finite volume emerges dynamically rather than being prescribed. This distinction is crucial: the final volume and front position arise from the interplay of inertia, buoyancy and viscoplasticity. Thus, the finite-volume model is used here as an idealised upper bound for viscoplastic arrest, not to fully capture the evolving dynamics. In fact, these differences can help explain the discrepancies between experimental interfacial arrest lengths and theoretical predictions (as shown in [figure 7b](#)).

Following standard methods (Saltelli, Tarantola & Campolongo 2000; Sobol 2001; Ebtehaj *et al.* 2015; Reed *et al.* 2022), we conducted a sensitivity analysis revealing that model–experiment deviations increase significantly with rising density contrast, decreasing yield stress and steeper inclinations, indicating reduced accuracy when the Boussinesq approximation or yield-dominated assumptions are violated. Exceeding the small-density-difference regime or using very low yield stress leads to a well over 10-fold increase in error, while higher yield stress improves accuracy by a factor of ~ 3 . Shifting inclination from near-horizontal to moderately steep causes a ~ 5 -fold increase in error, consistent with the growing influence of transverse pressure gradients and secondary flows. Injection velocity, although not explicitly included in the model, also correlates with deviations, suggesting residual inertial effects in the viscoplastic-dominated regime. These trends demonstrate that model accuracy is governed primarily by density contrast and yield stress, with injection velocity acting as a secondary but important factor. The model remains quantitatively reliable only within a narrow regime where inertia is negligible, density differences are small and flow remains stably stratified. This aligns with our previous findings (Faramarzi *et al.* 2024) that the stable slumping regime is bounded by $\chi \cos \beta / B \lesssim 40$ and $Re/M < 2800$. Furthermore, as shown in [figure 7\(a\)](#), very high yield stress may induce a no-flow condition at small $\chi \cos \beta / B$. Thus, the model is valid only in flowable regimes, characterised by stable stratification and the absence of secondary flow, with strong agreement for $0.1 \lesssim \chi \cos \beta / B \lesssim 3$ and $Re/M \lesssim 1000$.

Let us now classify the flow regimes based on whether the interface motion becomes arrested within the finite length of our pipe. Considering that both (3.9) and our experimental results indicate that varying conditions produce different static lengths, a large number of experiments were conducted. Experiments where the interfacial motion arrest occurs before the front reaches the pipe end are categorised as the ‘arrested’ regime, while those where arrest does not occur before the front reaches the pipe end are classified as the ‘unhalted’ regime. Using (3.9), the theoretical classification of such regimes is achieved by plotting the $Y \times L_s^{Model}$ curve for various $\alpha \equiv \chi \cos(\beta)/B$ values, as shown in [figure 7\(c\)](#), delineating arrested (above the curve) and unhalted (below the curve) regime regions. Our experimental results show that reducing the flow domain length can shift an experiment from the arrested regime to the unhalted regime. Similarly, changes in density difference, yield stress or inclination angle (e.g. an increased inclination shifting a datapoint to the right) can reclassify an experiment from the arrested to the unhalted regime. In general, our experimental results align well with this classification. However, discrepancies arise where some unhalted regime experiments fall within the predicted arrested regime region. This occurs because, as illustrated in [figure 7\(b\)](#), the experimental static lengths are generally greater than those predicted by the model ($L_s^{Exp} > L_s^{Model}$). Depending on the pipe length, this difference may cause the experimentally observed static length to exceed the pipe length, resulting in the classification of a datapoint as belonging

to the unhalted regime, even though the interface in that experiment would be arrested in a slightly longer pipe.

Two limiting cases offer further insight into the flow behaviour. First, for a horizontal pipe ($\beta = \pi/2$) with finite injection velocity ($\hat{V}_0$), experiments can exhibit both ‘unhalted’ and ‘arrested’ regimes. In the model, this condition yields $\alpha \approx 0$, reducing the governing equation to $L_s = 1/Y \int_0^1 \beta_0(h)/\beta_c(h) dh$, indicating that, under flowable conditions, natural slumping occurs due to interface height (Frigaard & Ngwa 2004). Consequently, the model predicts both flow regimes depending on pipe length and other parameters, consistent with experimental observations. Second, in the absence of imposed flow ($\hat{V}_0 = 0$) but at high inclinations, distinct flow configurations emerge, including drainage flows (Akbari, Frigaard & Taghavi 2024) and buoyancy-driven exchange flows (Charabin & Frigaard 2024). In drainage flows, the viscoplastic fluid in the inner pipe drains under gravity into the surrounding Newtonian fluid, with flow onset controlled by density difference, rheology and inclination angle. In exchange flows, vertical pipes with a low-viscosity Newtonian fluid below and a viscoplastic fluid above exhibit flow cessation for large Y , while lower Y values lead to central upward motion and descending viscoplastic displacement, forming regimes such as helical finger, disconnected finger and slug flow. In contrast to these, our study focuses on flowable regimes with non-zero injection velocity, where the viscoplastic fluid penetrates the ambient phase via buoyant and inertial forces.

3.3. Arrest mechanism

To further elucidate the interfacial arrest mechanism, this section examines velocity, shear rate and shear stress fields as well as yield/unyielded zones, using a combination of experimental and CFD results.

Figure 8(a) presents the dimensional axial velocity profiles from UDV measurements, corresponding to the experiment shown in figure 3, within $0 \leq \hat{y} \leq 0.038$ m at $\hat{x} = 0$ and $\hat{z} = 0.06$ m. The profiles darken over time. Positive velocities denote the injection direction, while negative values indicate counterflow, as the light in-place Newtonian fluid moves upward when the heavy viscoplastic fluid slumps beneath it. Near the upper wall, velocities remain near zero due to the no-slip condition, although UDV measurement reflections at the lower wall hinder exact zero-velocity measurements (Alba *et al.* 2013b). However, a no-slip condition for the Carbopol solution, consistent with prior studies (Alba *et al.* 2013a; Alba & Frigaard 2016; Freydier *et al.* 2017; Liu *et al.* 2018, 2019; Ball & Balmforth 2021; Valette *et al.* 2021; Isukwem *et al.* 2024), was validated through additional UDV measurements near the lower pipe wall, where a zero-velocity region was observed (figure 8b), confirming negligible wall slip. As time progresses, figure 8(a) shows that an unyielded zone forms (seen as uniform velocity profiles) in the viscoplastic fluid layer due to its yield stress, consistent with previous findings (Kazemi *et al.* 2024). Both injection and counterflow velocities decrease over time, with the pseudo-unyielded region expanding in the y -direction, propagating towards the lower pipe wall, progressively enveloping the pipe’s entire cross-section and reducing axial velocity to approach zero.

Using experiments, shear stresses can be roughly estimated from the shear rate above the interface within the Newtonian layer (but not within the viscoplastic layer) due to the constant viscosity of the Newtonian layer, as seen in figure 8(c). This also allows a rough estimate of the shear stresses at the interface. Figure 8(d) shows that these interfacial shear stresses progressively decrease over time as the interface height grows, until $\hat{h} \approx 0.03$ m (i.e. $h \approx 3/4$ in dimensionless form), indicating heightened susceptibility to unyielded zone formation in this region. In addition, the interfacial shear stresses remain small overall and typically below the viscoplastic fluid’s yield stress (considering the shear stress

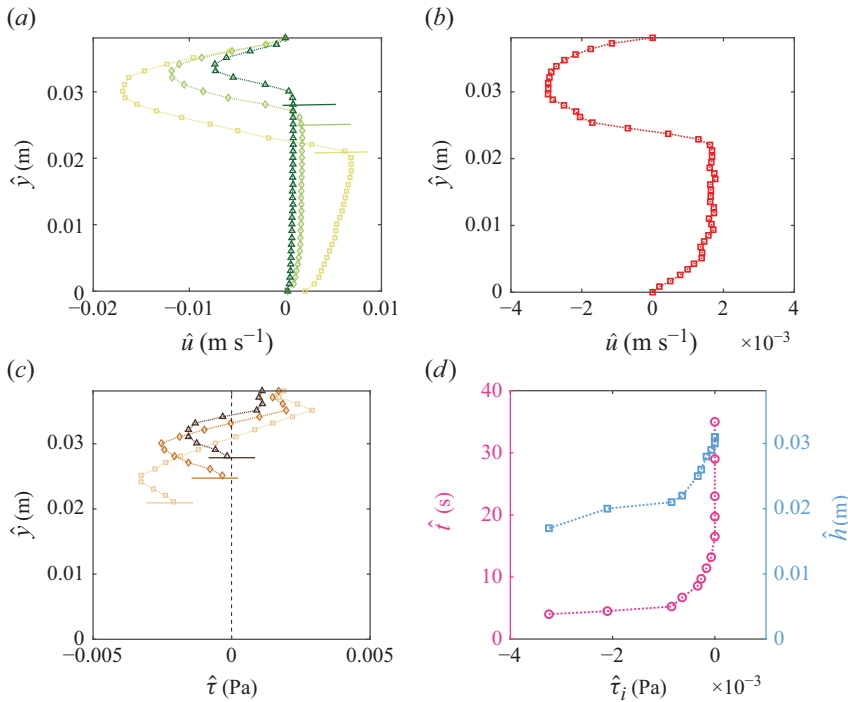


Figure 8. Experimental results (dimensional) at $\hat{x} = 0$ and $\hat{z} = 0.06$ (m). (a) Axial velocity profiles versus $\hat{y}$, determined by UDV for the experiment shown in figure 3. Colour intensity represents time [5.2, 8.6, 11.4] (s), with darker curves indicating longer times. (b) Axial velocity profile versus $\hat{y}$ from additional UDV measurements (UDV probe at the lower pipe wall), taken at $\hat{x} = 0$, $\hat{z} = 0.12$ m and $\hat{t} = 19$ s for $Re = 2.2$, $M = 0.003$, $B = 0.75$, $\beta = 82^\circ$ and $Fr = 0.68$. (c) Approximated axial shear stress ($\hat{\tau}$) within the light, Newtonian layer. (d) Shear stress at the interface ($\hat{\tau}_i$), as a function of time (pink curve with circular markers corresponding to the left axis) and $\hat{h}$ (cyan curve with square markers corresponding to the right axis). In panels (a) and (b), colour-matched solid lines on the curves indicate the interface position between the two fluids ($\hat{h}$).

continuity at the interface), suggesting that the unyielded zones begin to develop in the proximity of the interface, most likely expanding from the neighbourhood of $h \approx 3/4$. This hypothesis will be further explored by our CFD simulation results.

The CFD simulations provide detailed insights into the velocity field across the entire flow domain, capturing the advancing interfacial front, regions beyond the front, and both inertial- and viscoplastic-dominated phases, along with insights into the interfacial arrest phenomena. Figure 9(a) shows axial velocity vectors in the pipe centre plane (i.e. the y - z plane at $x = 0$). The velocity vectors primarily align in the z -direction, indicating stable flow, especially far from the injection point and the interfacial front. Near the inlet, velocities are much higher, consistent with the injection velocity, and decrease as the front advances. This suggests that unyielded zones would be less pronounced in the inertial-dominated phase than in the viscoplastic-dominated phase. Near the front, counterflow interactions may induce instabilities, suggesting a small yielded region, which diminishes further back, as shown by velocimetry studies (Andreini, Epely-Chauvin & Ancy 2012; Freydier *et al.* 2017; Akbari & Taghavi 2022a,b; Faramarzi *et al.* 2024). No velocity vectors appear beyond the front, showing that the light, ambient Newtonian fluid remains stationary until it is displaced by the interface as the viscoplastic fluid moves, as also confirmed by UDV measurements (omitted for brevity).

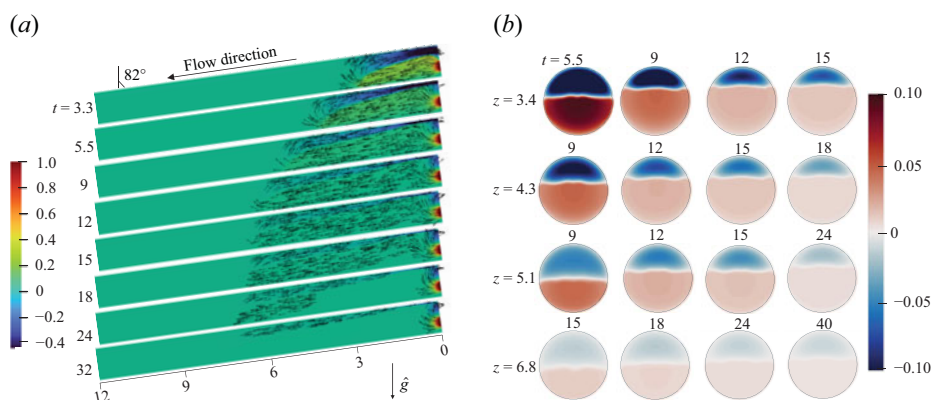


Figure 9. CFD results. (a) Velocity vectors in the z -direction and (b) cross-sectional z -direction velocity profiles at various times and locations for $Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$ and $Fr = 0.73$. The colourbar shows velocity magnitude, with positive values for forward flow and negative for counterflow.

Figure 9(a) shows velocity deviations near the inlet and near-zero velocities downstream at late times, indicating unyielded plug formation following interfacial arrest (as later confirmed in figure 10). Continued injection leads to flow redirection due to mass conservation, with reversal near $z = 0$ observed in both simulations and experiments (figures 3 and 4). This reversal marks a deviation from hydrostatic conditions, with vertical motion implying non-negligible vertical accelerations and non-hydrostatic dynamics during the transition to arrest. Spatially, this localised behaviour occurs in the region where the inertial-dominated regime was initially observed. Also, UDV measurements near the inlet, taken after complete cross-sectional filling with the viscoplastic fluid, confirmed late-time flow reversal (figure omitted for brevity). Similar behaviour was reported by Akbari & Taghavi (2021) for buoyant miscible Newtonian fluids, where UDV profiles 5 cm above the gate valve showed negative axial velocities towards the outlet.

Figure 9(b) illustrates velocity profiles within the cross-section, across multiple time intervals and axial positions, with the axial position increasing from top to bottom and time progressing from left to right. The profiles display lower velocities near the pipe walls due to the no-slip condition and reveal a significant velocity reduction over time at a given location or at extended times for a specific axial position.

Figure 10(a) illustrates the evolution of unyielded zones (black) on the pipe's centre plane (y - z plane at $x = 0$), initially forming near the interface (red) where shear stresses are lowest, as also confirmed by UDV measurements (figure 8). These zones expand towards the pipe centre and eventually towards the pipe's lower wall over time. Near the inlet, where the inertial-dominated phase appears at early times, unyielded regions are minimal. However, moving away from the inlet and as the flow transitions to the viscoplastic-dominated phase, unyielded zones form and progressively grow, increasing the effective viscosity of the viscoplastic layer. This growth causes the interfacial front to decelerate and eventually halt as unyielded regions encompass the entire viscoplastic layer.

An intriguing aspect of the unyielded zone formation onset is observed at $t = 5.5$, where unyielded zones initially develop in regions with higher interface heights; this is influenced by the height variations along the pipe length (z). Over time, the unyielded zones expand to regions with lower interface heights, becoming fully established. Meanwhile, in regions with higher interface heights, the unyielded zones extend towards the pipe centre, but small yielded zones remain near the lower wall, where shear stresses are relatively large. These observations indicate that the interface height influences unyielded zone formation,

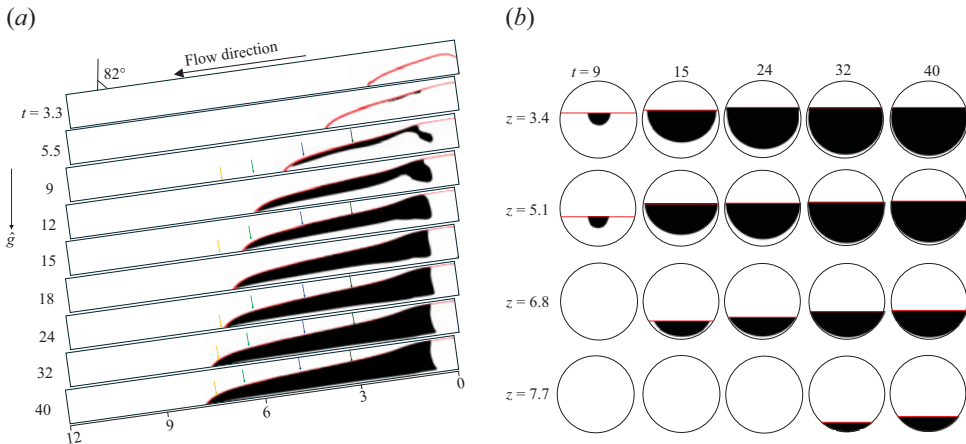


Figure 10. CFD results. (a) Snapshots of numerically derived yield surfaces, with the red line indicating the interface, in the y - z plane at $x = 0$ for $Re = 1.8$, $M = 0.001$, $B = 0.56$, $\beta = 82^\circ$ and $Fr = 0.73$. (b) Cross-sectional yield surfaces in the x - y plane at various times and positions ($z = 3.4, 5.1, 6.8$ and 7.7), marked in panel (a) by arrows (dark green, blue, green and orange, respectively).

with higher heights being initially more susceptible, but lower-height regions ultimately exhibiting a higher unyielded-to-yielded ratio.

Figure 10(b) illustrates the evolution of unyielded areas in cross-sectional views at various times and locations, showing that these zones initially form at the interface (at $x \approx 0$), where the shear stress is minimal and farthest from the pipe walls, where it is maximal. Over time, and with increasing distance from the injection point, the proportion of unyielded regions grows, reflecting the rising influence of solid-like behaviour and effective viscosity in the viscoplastic fluid. In both the y - z (figure 10a) and x - y (figure 10b) planes, the unyielded zones expand continuously without breakage, indicating stratified flow stability within the viscoplastic fluid layer.

Before concluding the manuscript, let us summarise the interpretation of our observations in this section. The arrest mechanism in our buoyant viscoplastic injections arises from the interplay between buoyancy-driven motion, rheological behaviour and geometrical constraints within the inclined pipe. As the flow transitions from an inertial-dominated phase to a viscoplastic-dominated phase, inertia becomes less relevant and buoyancy drives the interface forward. Simultaneously, the yield stress gains relative significance, increasing the viscoplastic fluid's effective viscosity and resisting the longitudinal buoyancy component. This leads to interface deceleration as the yield stress surpasses the diminishing shear stresses within the viscoplastic layer. Unyielded zones first emerge near the interface, where shear stresses are minimal and progressively expand towards the pipe centre, walls, confining yielded regions to zones of maximal shear near the walls. This progressive reduction in active flow area eventually arrests the interface once shear stresses throughout the viscoplastic layer drop below the yield stress, yielding static equilibrium.

4. Conclusions

We investigated buoyant injections of dense viscoplastic fluids into lighter Newtonian fluids in inclined closed-end pipes, i.e. a scenario relevant to cementing processes in plug and abandonment operations for oil and gas wells. Our study focused on miscible flows in the high-Péclet-number regime, where molecular diffusion is negligible. Our approach

integrated experimental analysis and CFD simulations. The experiments involved injecting heavy Carbopol solutions into deionised water under the Boussinesq approximation, with interfacial flow dynamics recorded by a camera and axial velocity profiles captured using ultrasound Doppler velocimetry. The experiments were supported by CFD simulations conducted in OpenFOAM. This combined methodology enabled a detailed examination of interfacial dynamics, flow phases and regimes, velocity field, yielded and unyielded zones, and interfacial arrest mechanisms. Our findings demonstrated that, in general, the flow dynamics is governed by Reynolds (Re), Froude (Fr) and Bingham (B) numbers, as well as the viscosity ratio (M) and inclination angle (β), along with combined parameters, such as $\chi \equiv 2Re/Fr^2$.

Our results demonstrated that, as the interface evolves in time and space, the flow transitions from an inertial-dominated phase, where the interfacial front advances linearly at the injection velocity, to a viscoplastic-dominated regime comprising two stages: an intermediate viscous–buoyant phase, characterised by a power-law front deceleration due to the interplay of buoyant and effective viscous stresses, and a late yield-stress-dominated phase, marked by front deceleration and eventual interfacial arrest. The experimental interface height profiles revealed rapid outward and upward spreading during the early stage of the viscoplastic-dominated phase, followed by slower dynamics as unyielded zones expand in the viscoplastic fluid, resulting in front deceleration. The critical transition length between the inertial- and viscoplastic-dominated phases was determined by a balance of inertial and buoyant stresses, expressed as $\mathcal{L} \approx 1.26Fr^{0.14}$.

Regarding interface stability, our experimental results confirmed that viscoplastic fluids initiate buoyancy-driven slumping at various inclinations, consistent with the theoretical yield number criterion, $Y \equiv B/\chi$. The maximum slumping extent (i.e. interfacial arrest length) is generally governed by a balance of longitudinal buoyant stress and yield stress, scaling as $L_s \sim 1/Y$. While our results showed consistency between experimental and theoretical interfacial arrest lengths, the experimental values generally exceeded theoretical predictions due to the effects of imposed flow. Using a plane defined by $\chi \cos(\beta)/B$ and $L \times Y$, with L being the pipe length, we classified experimental conditions into ‘arrested’ and ‘unhalted’ regimes, showing good agreement with theoretical models and illustrating how variations in yield stress, inclination and domain length dictate flow cessation.

As the flow transitions from the inertial-dominated phase to the viscoplastic-dominated phase, the interfacial arrest mechanism arises from the interplay of buoyancy-driven motion, rheological behaviour and geometrical constraints within the inclined pipe. Initially, inertia propels the interface forwards; however, as shear stresses diminish, the viscoplastic fluid’s yield stress increasingly dominates, raising effective viscosity and resisting longitudinal buoyancy. Unyielded zones first form near the interface (where shear stresses are minimal) and expand towards the pipe centre and walls, reducing active flow areas and decelerating the interfacial front. They initially emerge at higher interface heights and grow across the viscoplastic layer, driven by shear stress gradients and interface height variations. Also, velocity vectors align within fluid layers but show diminishing gradients near the interface. The interface motion halts when shear stresses throughout the viscoplastic layer fall below the yield stress, establishing static equilibrium.

The findings of this study can serve as a predictive tool for understanding and manipulating the arrest, stoppage and blockage of interfacial viscoplastic fluid flows in pipelines and confined geometries. They also have potential applications in plug and abandonment processes and can be extended to other industrial processes, such as oil transportation (Souas, Safri & Benmounah 2021) and food processing (Metcalf & Lester

2009). In this context, future studies could investigate the influence of geometrical factors, such as diameter ratios and eccentricity, on interfacial viscoplastic fluid flow dynamics.

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Declaration of interests. The authors report no conflict of interest.

Appendix A. Spatial profile functions

Frigaard & Ngwa (2004) presented the following equation for the upper-bound slump length derived for the case where a denser viscoplastic fluid overlays a lighter viscoplastic fluid within a pipe:

$$L_s = \frac{1}{\kappa} \int_0^1 \frac{1}{\tau_{c,Y} \frac{\beta_c(h)}{\beta_0(h)} + \tau_{m,Y} \frac{\beta_m(h)}{\beta_0(h)} + \frac{\tau_{min,Y}}{\beta_0(h)} - \alpha} dh, \quad (A1)$$

where $\tau_{c,Y}$ and $\tau_{m,Y}$ are the scaled yield numbers, which are generally defined as follows:

$$\tau_{k,Y} = \frac{\hat{\tau}_k}{\Delta \hat{\rho} \hat{g} \hat{D} \kappa}, \quad k \in \{c, m\}, \quad (A2)$$

where the subscript c corresponds to the denser viscoplastic fluid, while m represents the surrounding medium (ambient fluid). The parameter κ is defined as

$$\kappa = \frac{\max(\hat{\tau}_{c,Y}, \hat{\tau}_{m,Y})}{\Delta \hat{\rho} \hat{g} \hat{D}}. \quad (A3)$$

For our flow configuration, in which a denser viscoplastic fluid slumps beneath a Newtonian fluid, $\kappa = Y$ (3.7). Given this definition, we have $\tau_{c,Y} = 1$, and since the ambient fluid is Newtonian, we have $\tau_{m,Y} = 0$ and $\tau_{min,Y} = \min(\hat{\tau}_{c,Y}, \hat{\tau}_{m,Y}) / \Delta \hat{\rho} \hat{g} \hat{D} \kappa = 0$. Thus, the final expression is

$$L_s = \frac{1}{Y} \int_0^1 \frac{\beta_0(h)}{\beta_c(h) - \alpha \beta_0(h)} dh. \quad (A4)$$

The geometrical functions (or spatial profile functions) $\beta_0(h)$, $\beta_m(h)$, and $\beta_c(h)$ are as follows:

$$\beta_0(h) \equiv \frac{\alpha_c \alpha_m}{s_i} = \frac{1}{16} \left[\frac{s_c s_m}{s_i} - (1 - 2h)^2 s_i + (1 - 2h)(s_c - s_m) \right], \quad (A5)$$

$$\beta_m(h) \equiv \frac{\alpha_c s_m}{s_i} = \frac{1}{4} \left[\frac{s_c s_m}{s_i} - (1 - 2h) s_m \right], \quad (A6)$$

$$\beta_c(h) \equiv \frac{\alpha_m s_c}{s_i} = \frac{1}{4} \left[\frac{s_c s_m}{s_i} - (1 - 2h) s_c \right], \quad (A7)$$

where the functions α_c , α_m , s_c , s_m and s_i , which depend solely on h , are defined as follows:

$$\alpha_c(h) = 1 - \alpha_m(h) \equiv \frac{1}{\pi} \left(\cos^{-1}(1 - 2h) - 2(1 - 2h)[h(1 - h)]^{0.5} \right), \quad (\text{A8})$$

$$s_c(h) = 4 - s_m(h) \equiv \frac{4}{\pi} \cos^{-1}(1 - 2h), \quad (\text{A9})$$

$$s_i(h) = \frac{8}{\pi} [h(1 - h)]^{0.5}. \quad (\text{A10})$$

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