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The Continuous Dependence on the Nonlinearities of Solutions of Fast Diffusion Equations

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Abstract. In this paper, we consider the Cauchy problem

$$\begin{cases} u_t = \Delta(u^m), & x \in \mathbb{R}^N, t > 0, N \geq 3, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N. \end{cases}$$

We will prove that

(i) for $m_c < m, m_0 < 1$, $|u(x,t,m) - u(x,t,m_0)| \to 0$ as $m \to m_0$ uniformly on every compact subset of $\mathbb{R}^N \times \mathbb{R}^+$, where $m_c = \frac{(N-2)_+}{N}$;

(ii) there is a C^* that explicitly depends on m such that

$$\|u(\cdot,\cdot,m) - u(\cdot,\cdot,1)\|_{L^2(\mathbb{R}^N \times \mathbb{R}^+)} \leq C^*|m-1|.$$

1 Introduction

We consider the Cauchy problem

$$\begin{cases} u_t = \Delta(u^m), & \text{in } Q, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N. \end{cases} \tag{1.1}$$

with

$$0 \leq u_0 \leq M, \quad 0 < \int_{\mathbb{R}^N} u_0(x)dx < +\infty, \tag{1.2}$$

where $Q = \mathbb{R}^N \times \mathbb{R}^+$, $N \geq 3$, and

$$m_c < m \leq 1 \tag{1.3}$$

with $m_c = \frac{(N-2)_+}{N}$. Since $N \geq 3$, $m_c = 1 - \frac{2}{N}$.

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In recent years there has been considerable interest in equation (1.1). The equation encompasses for different ranges of m a variety of qualitative properties with a wide scope of applications. For example, the equation is degenerate parabolic as $m > 1$ because the thermal diffusivity $D(u) = u^{m-1}$ vanishes as $u \to 0$. So the problem only has weak solutions (see [6]) in this case. If $m = 1$, the equation is uniformly parabolic, and therefore (1.1) has a unique, global, smooth solution

$$u(x,t,1) = \frac{1}{(2\sqrt{\pi t})^N} \int_{\mathbb{R}^N} u_0(\xi) e^{-\frac{|x-\xi|^2}{4t}} d\xi.$$

But the situation is completely different for $m < 1$, where u^{m-1} blows up as $u \to 0$. It is usually referred to as a singular diffusion equation. It has been proposed in plasma physics and in heat conduction in solid hydrogen (see [5]). Furthermore, the problems (1.1) and (1.2) also have a unique global smooth solution $u(x,t,m)$ for any given $0 < m < 1$ (see [1]) such that

$$u(x,t,m) \in C^\infty(Q) \cap C\big([0,+\infty); L^1(\mathbb{R}^N)\big).$$

As mentioned as above, we can see that the different values of m makes different features of the solutions of (1.1). So we think it is reasonable to divide equation (1.1) into three types:

- if $m > 1$, equation (1.1) is degenerate parabolic;
- if $m = 1$, equation (1.1) is uniformly parabolic;
- if $m < 1$, equation (1.1) is singular parabolic.

Although many authors have studied equation (1.1) (*e.g.*, [4,8,10–12]) for the case of $m > 1$ and $m < 1$, there are only a few results concerning the continuous dependence on the nonlinearities of the equations. In 1981, P. Benilan and M. G. Crandall (see [2]) discussed the continuous dependence on ϕ of solutions of the Cauchy problem of equation

$$\begin{cases} u_t - \Delta\phi(u) = 0, & \text{in } Q, \\ u(x,0) = u_0(x), & x \in \mathbb{R}^N, \end{cases} \tag{1.4}$$

with $u_0 \in L^1(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$. If $\phi_n \colon \mathbb{R} \to \mathbb{R}$ is continuous and nondecreasing for all $n = 1, 2, 3, \dots, \phi_n(0) = 0$, then they obtained the main result (see [2, p. 162]):

$$\|u_n(\cdot,t) - u_\infty(\cdot,t)\|_{L^1(\mathbb{R}^N)} \longrightarrow 0, \quad \text{as } \phi_n \longrightarrow \phi_\infty, \tag{1.5}$$

where u_n are the solutions of the Cauchy problem (1.4). However, as pointed out by [3], the results of [2] are not written in terms of explicit estimates. To study the problem more precisely, B. Cockburn and G. Gripenberg (see [3]), in 1999, extended the result of [2] for the Cauchy problem of degenerate parabolic equations

$$\begin{cases} u_t = \Delta(\phi(u)) + \nabla\cdot(\Phi(u)), \\ u_0(x) = h(x) \end{cases}$$

with the conditions

$$\Phi_j \in C^1(\mathbb{R}, \mathbb{R}^N), \quad \Phi_j(0) = 0, \quad \phi_j(0) = 0, \quad \text{and} \quad \phi_j'(t) > 0,\ t \in \mathbb{R}$$

for $j = 1, 2$. The explicit estimate obtained is

$$\|u_1(\cdot, t) - u_2(\cdot, t)\|_{L^1(\mathbb{R}^N)} \le \|h_1 - h_2\|_{L^1(\mathbb{R}^N)} + \|h_1\|_{TV(\mathbb{R}^N)} \times \Big(t \cdot \sup_{s \in I(h_1)} \|\Phi_1'(s) - \Phi_2'(s)\|_{L^\infty(\mathbb{R}^N)} + 4\sqrt{tN} \sup_{s \in I(h_1)} |\sqrt{\phi_1'(s)} - \sqrt{\phi_2'(s)}| \Big)$$

for any $t > 0$, where

$$I(h) = (-\|h^-\|_\infty, \|h^+\|_\infty), \quad h^+ = \max\{h, 0\}, \quad h^- = -\min\{h, 0\}.$$

To the author's knowledge, there are not many other results on continuous dependence on the nonlinearities of singular parabolic equation up to the date of writing.

To study the approximating character on the nonlinearities of parabolic equations and to give more precise estimates, especially, to give an estimate of solutions between linear and nonlinear equations, this paper discusses the Cauchy problem (1.1) for the parameter $m \in (m_c, 1]$. Owing to the fact that the case of $m \le 1$ is different from the $m > 1$ case, so it is reasonable for us to expect (1.1) to have classical solutions. In fact, Aronson and Bénilan ([1]) proved that problem (1.1) with condition (1.2) has a unique solution $u(x, t, m) \in C^\infty(Q) \bigcap C([0, +\infty); L^1(\mathbb{R}^N))$ for any given $0 < m < 1$ and $N \ge 1$. Moreover, u satisfies the following estimates

$$\Delta v \ge \frac{-k}{t}, \quad (x, t) \in Q, \tag{1.6}$$

$$\frac{-ku}{t} \le u_t \le \frac{u}{(1-m)t}, \quad (x, t) \in Q, \tag{1.7}$$

where $v = \frac{m}{m-1} u^{m-1}$ and $k = (m - m_c)^{-1}$. However, the total mass is not always a constant. In fact, the mass conservation is true only for $m_c < m < 1$, where m_c is defined by (1.3). Clearly, $m_c > 0$ for $N \ge 3$. This shows that some of the mass is lost when $m \in (0, m_c)$ (see [12, pp. 90–94]).

Therefore, we only discuss our problem for $m_c < m \le 1$ for $N \ge 3$ in this paper. As to the other case, for example, if $N = 1$, the result is different (see [9]).

Set

$$\widetilde{A} = \left\{ u(x, t, m); m \in (m_c, 1] \right\}.$$

Then $\widetilde{A}$ is bounded uniformly for all $m \in (m_c, 1]$ in $L^\infty(Q)$. In fact, [7] and the first step of Theorem 1 in [5] proved that $0 < u(x, t, m) \le M$ for $m_c < m < 1$. Certainly, for the classical case of $m = 1$ we also have $0 < u(x, t, 1) \le M$ for all $t > 0$. Therefore,

$$0 < u(x, t, m) \le M, \quad u \in \widetilde{A}. \tag{1.8}$$

Thus, our main result reads as follows.

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Theorem 1.1 *Assume that u_0 satisfies* (1.2) *and m, $m_0 \in (m_c, 1)$. Then for any compact subset Q' of Q,*

$$\lim_{m \longrightarrow m_0} |u(x,t,m) - u(x,t,m_0)| = 0, \quad \text{uniformly on } \overline{Q'}. \tag{1.9}$$

If $m_0 = 1$, then there is a $C^* > 0$ such that

$$\big\| u(\,\cdot\,,\,\cdot\,, m) - u(\,\cdot\,,\,\cdot\,, 1)\big\|_{L^2(Q)} \leq C^* |m-1|, \quad m \in (m^*, 1), \tag{1.10}$$

where

$$m^* = \max\Big\{\frac{2+N}{2N}, m_c\Big\}. \tag{1.11}$$

Remark 1.2 First, if $N < 3$ (for example, $N = 1$ or 2), then (1.11) says that $m^* = \frac{3}{2}$ or 1, so the interval $(m^*, 1)$ is empty, and then (1.10) is not available. Thus we only consider the case of $N \geq 3$ in this paper. Second, the result (1.9) is true for $m_0 \in (m_c, 1)$, so it seems that (1.10) may be true for all $m_0 \in (m_c, 1]$ also. However, (1.10) is made possible by $u(x,t,1) \leq (2\sqrt{\pi})^{-N} \|u_0\|_{L^1(\mathbb{R}^N)} \cdot t^{-\frac{N}{2}}$. From this inequality, we get the explicit decay rate of the function $u(x,t,1)$ on t as t is large. In fact, if we have a similar inequality $u(x,t,m) \leq Ct^{-\alpha}$ for $m_0 \in (m_c, 1)$ with a sufficiently large α, we will get (1.10) for all $m_0 \in (m_c, 1)$ employing the same procedure. Certainly, under the present circumstances, we may also get a similar estimate

$$\big\| u(\,\cdot\,,\,\cdot\,, m) - u(\,\cdot\,,\,\cdot\,, 1)\big\|_{L^2(Q_T)} \leq C_* |m-1|$$

for $m_0 \in (m_c, 1)$. However, this constant C_* depends on T. This is the point.

2 Preliminary Lemmas

Lemma 2.1 *Suppose $u(x,t,m) \in \widetilde{A}$ and $m < 1$. Then*

$$\big|\nabla(u^{\frac{m-1}{2}}(x,t,m))\big| \leq \sqrt{\frac{1-m}{2Nm(m-m_c)t}}\,. \tag{2.1}$$

Proof Let $v = \frac{m}{m-1} u^{m-1}(x,t,m)$, by (1.6) we have

$$v_t = (m-1)v\Delta v + |\nabla v|^2 \geq \frac{-(m-1)k}{t} v + |\nabla v|^2.$$

Employing the right-hand side of (1.7), we have

$$|\nabla v|^2 \leq u^{m-1} \frac{2m}{N(1-m)(m-m_c)} \frac{1}{t}.$$

This yields (2.1) immediately. ∎

Lemma 2.2 *For any $u \in \widetilde{A}$, we have $\int_{\mathbb{R}^N} u dx = \overline{u}_0$ for $t > 0$, where $\overline{u}_0 = \int_{\mathbb{R}^N} u_0 dx$.*

Since $N \geq 3$, as mentioned in the introduction, some of the mass is lost as time grows for $m \leq m_c$, and neighborhoods of infinity is where the mass is lost (see [12], p.90-92). Therefore the result of Lemma 2.2 is true only for all $m > m_c$. We can find the details in the proof of the lemma in [11].

3 The Proofs

We are now in a position to prove our theorem. To do this, we will employ two steps to show the details.

Step 1: Proof of (1.9)

Let Q' be a compact subset of Q, say $Q' = \Omega \times (t_1, t_2)$, and Ω be any bounded domain in $\mathbb{R}^N$, $t_1 > 0$. By (1.7), (1.8), (2.1), and the Arzela–Ascoli theorem, we know that for any $0 < \eta < \frac{1-m_c}{2}$ and $m_0 \in [m_c+\eta, 1-\eta]$, there is a subsequence $u(x, t, m_k)$ and a function $\overline{u}(x, t, m_0) \in C(\overline{Q'})$ such that

$$\lim_{m_k \to m_0} |u(x, t, m_k) - \overline{u}(x, t, m_0)| = 0, \quad \text{uniformly on } \overline{Q'}. \tag{3.1}$$

So we next only need to prove $\overline{u}(x, t, m_0) = u(x, t, m_0)$. In fact, for every $t \in (t_1, t_2)$,

$$\begin{aligned} \|\overline{u}(x, t, m_0) - u(x, t, m_0)\|_{L^1(\Omega)} \le \|\overline{u}(x, t, m_0) - u(x, t, m_k)\|_{L^1(\Omega)} \\ + \|u(x, t, m_k) - u(x, t, m_0)\|_{L^1(\Omega)}. \end{aligned} \tag{3.2}$$

Letting $m \to m_0$, then (3.1) implies the first term of right-hand side of (3.2) converges to zero. As to the second term, by (1.5) we can know that it tends to zero also. Because $u(x, t, m_0)$ and $\overline{u}(x, t, m_0)$ are continuous, we know $\overline{u}(x, t, m_0) = u(x, t, m_0)$ in Ω. And then the arbitrariness of η, Ω, and t yield $\overline{u}(x, t, m_0) = u(x, t, m_0)$ in Q for $\eta \in (m_c, 1)$. Finally, by the uniqueness we know that the total sequence $u(x, t, m)$ converges to $u(x, t, m_0)$ as $m \to m_0$. This means that we can drop k in (3.1). So (3.1) implies (1.9).

Step 2: Proof of (1.10)

Take a function $f \in C_0^\infty(\mathbb{R}^N)$, $0 \le f(x) \le 1$ and

$$f(x) = \begin{cases} 1, & |x| \le 1, \\ 0, & |x| \ge 2, \end{cases}$$

then set $f_n(x) = f(\frac{x}{n})$ for $n > 0$. Clearly, there is a positive constant c such that $|\nabla f_n| \le \frac{c}{n}$.

Let $m \in (m^*, 1)$ and let m^* be defined by (1.11). For every $T > 0$, let

$$H = u^m(x, t, m) - u(x, t, 1), \qquad \psi = \int_T^t H d\tau \quad 0 < t < T.$$

Noticing $[u(x, t, m) - u(x, t, 1)]_t = \Delta H$, multiplying the equation by ψf_n, and then

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integrating by parts on $\mathbb{R}^{\mathbb{N}} \times (0,T)$, we have

$$\int_0^T \int_{\mathbb{R}^N} [u(x,t,m) - u(x,t,1)]\psi_t f_n dxdt = \int_0^T \int_{\mathbb{R}^N} \nabla H \cdot \nabla \psi f_n dxdt + \int_0^T \int_{\mathbb{R}^N} \nabla H \cdot \nabla f_n \psi dxdt. \tag{3.3}$$

To estimate the right-hand side of (3.3), we see that

$$\begin{aligned}
&\left| \int_0^T \int_{\mathbb{R}^N} \nabla H \cdot \nabla f_n \psi dxdt \right| \\
&\quad \le \frac{c}{n} \int_0^T \int_{n \le |x| \le 2n} |\nabla H \cdot \psi| dxdt \\
&\quad \le \frac{c}{n} \int_0^T \left(\int_{n \le |x| \le 2n} |\nabla H|^2 dx \right)^{\frac{1}{2}} \left(\int_{n \le |x| \le 2n} \psi^2 dx \right)^{\frac{1}{2}} dt.
\end{aligned}$$

It follows from (2.1) and $0 < u \le M$ that

$$|\nabla(u^m)|^2 \le \frac{2m}{N(1-m)(m-m_c)t} \cdot u^{1+m} \le M^m \frac{2m}{N(1-m)(m-m_c)t} \cdot u.$$

Since $\int_{\mathbb{R}^N} u dx = \overline{u}_0$, it is easy for us to see $\int_{n \le |x| \le 2n} |\nabla H|^2 dx$ is bounded uniformly with respect to n and

$$\int_{n \le |x| \le 2n} |\nabla H|^2 dx = O(t^{-1}).$$

Similarly, $\int_{n \le |x| \le 2n} \psi^2 dx$ is also bounded uniformly with respect to n when $m > m^*$. Thus we know that

$$\int_0^T \int_{\mathbb{R}^N} \nabla H \cdot \nabla f_n \psi dxdt \longrightarrow 0 \quad \text{as } n \longrightarrow \infty. \tag{3.4}$$

Second, we set $I(T) = \int_0^T \int_{\mathbb{R}^N} \nabla H \cdot \nabla \psi f_n dxdt$. Clearly,

$$\begin{aligned}
I(T) &= \int_0^T \int_{\mathbb{R}^N} \frac{\partial}{\partial t} \nabla \psi \cdot \nabla \psi f_n dxdt = -\frac{1}{2} \int_{\mathbb{R}^N} |\nabla \psi(0)|^2 f_n dx \\
&= -\frac{1}{2} \|\nabla \psi(0) \sqrt{f_n}\|^2_{L^2(\mathbb{R}^N)} dx \le 0.
\end{aligned} \tag{3.5}$$

Combining (3.4) and (3.5) and letting $n \to \infty$ in (3.3), we have

$$\int_0^T \int_{\mathbb{R}^N} [u(x,t,m) - u(x,t,1)]\psi_t dxdt \le 0. \tag{3.6}$$

On the other hand,

$$\begin{aligned}
&\int_0^T \int_{\mathbb{R}^N} \big[u(x,t,m) - u(x,t,1)\big] \psi_t dxdt \\
&= \int_0^T \int_{\mathbb{R}^N} \big[u(x,t,m) - u(x,t,1)\big] \big[u^m(x,t,m) - u(x,t,1)\big] dxdt \\
&= \int_0^T \int_{\mathbb{R}^N} \big[u(x,t,m) - u(x,t,1)\big] \big[u^m(x,t,m) - u^m(x,t,1)\big] dxdt \\
&\quad + \int_0^T \int_{\mathbb{R}^N} \big[u(x,t,m) - u(x,t,1)\big] \big[u^m(x,t,1) - u(x,t,1)\big] dxdt \\
&\overset{\text{def}}{=} I_1 + I_2.
\end{aligned}$$

Hence (3.6) yields

$$I_1 \le |I_2|. \tag{3.7}$$

We first estimate I_1. It is easy to find a $s_0 \in (0,1)$ such that

$$\begin{aligned}
&u^m(x,t,m) - u^m(x,t,1) \\
&= \int_0^1 \frac{d}{ds}\big[su(x,t,m) + (1-s)u(x,t,1)\big]^m ds \\
&= m\big[u(x,t,m) - u(x,t,1)\big] \int_0^1 \big[su(x,t,m) + (1-s)u(x,t,1)\big]^{m-1} ds \\
&= m\big[u(x,t,m) - u(x,t,1)\big] \big[s_0u(x,t,m) + (1-s_0)u(x,t,1)\big]^{m-1}.
\end{aligned}$$

Set $\zeta = s_0u(x,t,m) + (1-s_0)u(x,t,1)$, then

$$u^m(x,t,m) - u^m(x,t,1) = m\zeta^{m-1}\big(u(x,t,m) - u(x,t,1)\big).$$

It follows from $m < 1$ and $0 < u \le M$ that $0 < \zeta \le M$ and therefore,

$$I_1 \ge mM^{m-1} \int_0^T \int_R \big[u(x,t,m) - u(x,t,1)\big]^2 dxdt. \tag{3.8}$$

To estimate I_2, we note that there exists a $\mu \in (m,1)$ such that

$$|I_2| \le |m-1| \int_0^T \int_{\mathbb{R}^N} |u(x,t,m) - u(x,t,1)| \cdot |u^\mu(x,t,1) \ln u(x,t,1)| dxdt.$$

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Thus, combining (3.7) and (3.8) yields

$$\begin{aligned}(3.9)\quad \int_0^T\int_{\mathbb{R}^N}[u(x,t,m)-u(x,t,1)]^2dxdt \\ \le M^{1-m}\Big|\frac{m-1}{m}\Big|\sqrt{\int_0^T\int_{\mathbb{R}^N}[u(x,t,m)-u(x,t,1)]^2dxdt} \\ \times\sqrt{\int_0^T\int_{\mathbb{R}^N}|u^\mu(x,t,1)\ln u(x,t,1)|^2dxdt}.\end{aligned}$$

To estimate the right-hand side of (3.9), we write

$$\begin{aligned}&\int_0^T\int_{\mathbb{R}^N}|u^\mu(x,t,1)\ln u(x,t,1)|^2dxdt\\ &=\int_0^1\int_{\mathbb{R}^N}|u^\mu(x,t,1)\ln u(x,t,1)|^2dxdt+\int_1^T\int_{\mathbb{R}^N}|u^\mu(x,t,1)\ln u(x,t,1)|^2dxdt\\ &\overset{\text{def}}{=} J_1+J_2.\end{aligned}$$

Since $\mu\in(m,1)$, when $m\in(m^*,1)$, then so is μ. Recalling $0<u(x,t,1)\le M$ for $t>0$, we see that there is a $k_1>0$, such that $u^{2\mu-1}(x,t,1)|\ln u(x,t,1)|^2\le k_1$. Thus, it follows from $\int_{\mathbb{R}^N}u(x,t,1)dx=\overline{u}_0$ that

$$(3.10)\qquad J_1\le\int_0^1\int_{\mathbb{R}^N}k_1u(x,t,1)dxdt\le k_1\overline{u}_0.$$

To estimate J_2, we recall $0<u(x,t,1)\le\frac{\overline{u}_0}{(2\sqrt{\pi t})^N}$, and let $q\in(0,2\mu-1-\frac{2}{N})$. Then there is a $k_2>0$ such that $(u(x,t,1))^q(\ln u(x,t,1))^2\le k_2$. Since $2\mu-1-q>0$ and $1-N\mu+\frac{N}{2}+\frac{qN}{2}<0$, we have

$$\begin{aligned}(3.11)\qquad J_2&\le k_2\int_1^T\int_{\mathbb{R}^N}\Big[\frac{\overline{u}_0}{(2\sqrt{\pi t})^N}\Big]^{2\mu-1-q}u(x,t,1)dxdt\\ &\le k_2\overline{u}_0\Big[\frac{\overline{u}_0}{(2\sqrt{\pi})^N}\Big]^{2\mu-1-q}\int_1^T t^{-\frac{N}{2}(2\mu-1-q)}dt\\ &\le\frac{k_2\overline{u}_0}{\mu N-\frac{N}{2}-1-\frac{qN}{2}}\Big[\frac{\overline{u}_0}{(2\sqrt{\pi})^N}\Big]^{2\mu-1-q}.\end{aligned}$$

Finally, using (3.9), (3.10), and (3.11), we have

$$\left\{\int_0^T \int_{\mathbb{R}^N} [u(x,t,m) - u(x,t,1)]^2 dxdt\right\}^{\frac{1}{2}} \leq$$
$$M^{1-m}\left|\frac{m-1}{m}\right| \times \left\{k_1\overline{u}_0 + \frac{k_2\overline{u}_0}{\mu N - \frac{N}{2} - 1 - \frac{qN}{2}}\left[\frac{\overline{u}_0}{(2\sqrt{\pi})^N}\right]^{2\mu-1-q}\right\}.$$

Set

$$C^* = \frac{M^{1-m}}{m} \times \left\{k_1\overline{u}_0 + \frac{k_2\overline{u}_0}{\mu N - \frac{N}{2} - 1 - \frac{qN}{2}}\left[\frac{\overline{u}_0}{(2\sqrt{\pi})^N}\right]^{2\mu-1-q}\right\},$$

then

$$\left[\int_0^T \int_{\mathbb{R}^N} [u(x,t,m) - u(x,t,1)]^2 dxdt\right]^{\frac{1}{2}} \leq C^*|m-1|. \tag{3.12}$$

Since C^* does not depend on T, (3.12) holds for all $T \in (0,\infty)$.

This completes the proof of theorem. ■

References

[1] D. G. Aronson and P. Bénilan, *Régularité des solutions de l'équation des milieux poreux dans* $\mathbb{R}^n$. C. R. Acad. Sci. Paris Sér. A-B **288**(1979), no. 2, A103–A105.

[2] P. Bénilan and M. G. Crandall, *The continuous dependence on* ϕ *of solution of* $u_t - \triangle\phi(u) = 0$. J. Indiana Univ. Math. **30**(1981), no. 2. 161–177. http://dx.doi.org/10.1512/iumj.1981.30.30014

[3] B. Cockburn and G. Gripenberg, *Continuous dependence on the nonlinearities of solutions of degenerate parabolic equations.* J. Differential Equations **151**(1999), no. 2, 231–251. http://dx.doi.org/10.1006/jdeq.1998.3499

[4] C. Ebmeyer, *Regularity in Sobolev spaces for the fast diffusion and the porus medium equation.* J. Math. Anal. Appl. **307**(2005), no. 1, 134–152. http://dx.doi.org/10.1016/j.jmaa.2005.01.009

[5] J. R. Esteban, A. Rodriguez, and J. L.Vázquez, *A nonlinear heat equation with singular diffusivity.* Commun. Partial Differential Equations **13**(1988), no. 8, 985–1039. http://dx.doi.org/10.1080/03605308808820566

[6] B. H. Gilding and L. A. Peletier, *The Cauchy problem for an equation in the theory of infiltration.* Arch. Rational Mech. Anal. **61**(1976), no. 2, 127–140.

[7] M. A. Herrero and M. Pierre, *The Cauchy problem for* $u_t = \triangle u^m$ *when* $0 < m < 1$. Tran. Amer. Math. Soc. **291**(1985), no. 1, 145–158.

[8] K. M. Hui, *Existence of solutions of the very fast diffusion equation.* Nonlinear Anal. **58**(2004), no. 1–2, 75–101. http://dx.doi.org/10.1016/j.na.2004.05.001

[9] J. Pan and L. Gang, *The linear approach for a nonlinear infiltration equation.* European J. Appl. Math. **17**(2006), no. 6, 665–675. http://dx.doi.org/10.1017/S095679250700681X

[10] J. L. Vázquez, *Symmetrization and mass comparison for degenerate nonlinear parobolic and related elliptic equations.* Adv. Nonlinear Stud. **5**(2005), no. 1, 87–131.

[11] ———, *An introduction to the mathematical theory of the porous medium equation.* In: Shape optimization and free boundaries (Montreal, PQ, 1990), NATO Adv. Sci. Inst. Ser. C Math. Phys. Sci., Kluwer Acad. Publ., Dordrecht, 1992, pp. 347–389.

[12] ———, *Smoothing and decay estimates for nonlinear diffusion equations.* Equations of porous medium type. Oxford Lecture Series in Mathematics and its Applications, 33, Oxford University Press, Oxford, 2006.

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