integrable function space". The book ends with a list of open problems and an extensive list of references. Each of Chapters 1–10 is supplemented by a section of Exercises and a section of "Notes and Remarks", which outline the origins of many of the results discussed.

Much of the material is very technical, so that a careful reading of the text is no easy undertaking. However the authors have presented their subject well and the English is generally quite acceptable. There are of course a few minor language discrepancies and some typographical errors, but none of these should cause the reader any problems. It is unfortunate that this compendium of high-level research contains the following erroneous assertion on its page 2: ". . . a subset V of a space E is a barrel if and only if V° is bounded in $\sigma(E', E)$ . . . "; it may be corrected as follows: ". . . a subset V of a space E is a barrel if and only if it is the polar of a $\sigma(E', E)$-bounded set . . . '..It is also unfortunate that the authors have not provided an index – with such technical material it is easy to forget definitions but not always easy to locate them again. These are all minor reservations and I can certainly recommend the book to anyone with a research interest in barrelled spaces.

I. TWEDDLE

HERNÁNDEZ, E. and WEISS, G. *A first course on wavelets* (CRC Press, Boca Raton–New York–London–Tokyo, 1996), 512pp., 0 8493 8274 2, $64.95.

This is a very nice book! The content is very interesting and a little out of the ordinary for a book on wavelets. I found it to be written in a style which is easy to read. The authors provide plenty of encouragement to dig deep into the book and seem able to maintain an exposition which continually stirs the reader's interest. I would think any analyst would find things of interest in this book, but researchers in wavelets, harmonic analysis, functional analysis, approximation theory, and signal processing will certainly enjoy large sections of the book.

The authors deal exclusively with wavelets on the real line rather than the more general multivariate wavelets. To my mind this is a wise choice. The overheads involved in dealing with the multivariate case are worth while mainly from the point of view of applications. Otherwise, the added complexity simply serves to obscure the beautiful principles which underlie the theory. In the univariate context an orthonormal wavelet on $\mathbb{R}$ is a function ψ in $L^2(\mathbb{R})$ such that $\{\psi_{j,k} : j, k \in \mathbb{Z}\}$ is an orthonormal basis of $L^2(\mathbb{R})$, where

$$\psi_{j,k}(x) = 2^{(j/2)}\psi(2^j x - k), \qquad j, k \in \mathbb{Z}, x \in \mathbb{R}.$$

In 1910 A. Haar discovered what is now known as the Haar wavelet. It is the function which has value $+1$ on the interval $[0, 0.5)$, -1 on the interval $[0.5, 1)$ and is zero elsewhere. One of the first uses one might think of for a wavelet would be in making an L^2-decomposition of a function or signal into its wavelet series $\sum_{k,j\in\mathbb{Z}} c_{j,k}\psi_{j,k}$. Here each $c_{j,k}$ is the inner product of the function with $\psi_{j,k}$. At this point the fact that the Haar wavelet has compact support is seen to be a very nice feature—it greatly simplifies the computation of these inner products. A nasty feature (at least to a signal processing person) is that ψ is discontinuous. The construction of compactly supported smooth wavelets was a long time coming and represented a major contribution to the field. It is now common to use the multiresolution analysis idea of Mallat to carry out this construction. The details of this programme are the meat of the first four chapters of the book.

The book then moves on to consider some applications of wavelets. These are applications in analysis. A major theme is the idea of an atomic decomposition. Through this the possibility that $\{\psi_{j,k} : j, k \in \mathbb{Z}\}$ forms an unconditional basis for $H^1(\mathbb{R})$ and $L^p(\mathbb{R})$ is investigated. Also these spaces are characterised in terms of conditions on the coefficients in an appropriate wavelets expansion. To give the reader a flavour of these results some ingredients are the Shannon sampling theorem, Calderon's identities, Littlewood–Paley functions and the Hardy–Littlewood maximal function. This section of the book finishes with a treatment of Sobolev spaces, Lipschitz spaces and the Zygmund class. These chapters have a substantial amount of non-wavelet material and a true disciple of wavelets might want to skip them at a first reading. However, to do so would be to miss some very interesting mathematics.

The discussion then returns to wavelets with a very nice chapter dealing with the characterisation of wavelets. Suppose we pick a function ψ off the $L^2(\mathbb{R})$ heap. Can we tell easily that ψ is a wavelet? Furthermore, not all wavelets are generated by a multiresolution analysis. Can we tell further whether our function ψ is obtained from a multiresolution analysis?

The book concludes by devoting the remaining chapters to some computational considerations. The first comes from the fact that symmetry for a wavelet (or generalised linear phase for the corresponding filter) is a very rare occurrence—basically the Haar wavelet is the only one with the required symmetry property. This has led to the spline wavelet industry, where orthonormality is relaxed. The central notion here is that of a "frame". Finally, the practitioner will not want to compute with the continuum $\mathbb{R}$ but rather with some discrete set, so a chapter on discrete transforms is provided. These considerations lead the authors to the very useful decomposition and reconstruction algorithms, which are the centrepiece of the wavelet approach to signal processing.

Although I class this book as a great read, there is a genuine uncertainty in my mind about the experience the book will provide to the reader with little background in advanced analysis. The authors do not hesitate to utilise advanced technology as and when it suits them. They admit in their advice to readers that "distributions, maximal functions, vector-valued inequalities, etc." will make an appearance. The reader is advised not to become discouraged and perhaps to skip these sections initially. I find it difficult to judge whether a reader without a background in such matters will become overwhelmed and discouraged at times. The hardest material in my opinion comes in the middle chapters on function spaces. I found this material to be quite fascinating, but, if I were also meeting BMO and $H^1(\mathbb{R})$ for the first time, I might well not be able to appreciate the intrinsic importance of these chapters. Then the added burden of struggling to master concepts which are not in the mainstream of wavelet theory could easily discourage me. These middle chapters also have the effect of forming a split in the book. The level of difficulty of the later chapters on computational considerations is more in keeping with that of the earlier chapters.

I do not want to end this review on a downbeat note, since overall I like this book very much. Let me conclude by complimenting the authors on their writing style. They seem to be expert at succinctly motivating the material. For example, the Shannon sampling theorem (ironically not due to Shannon at all!) is motivated very nicely from the wavelet series. The section on function spaces will introduce the interested reader to many of the important concepts in a way which I found to be compelling. As I have already remarked, an important ingredient of the $H^1(\mathbb{R})$ theory is the idea of an atomic decomposition. As the authors point out, this idea can be used to define $H^1(\mathbb{R})$ and this would have provided a swift path to the wavelet results the authors are pursuing. However, they commendably give the "usual" definition of this space and take some time to explain how the atomic decomposition characterises the space. This exemplifies their commitment to explaining the material from a pedagogical viewpoint. The reader is also continually helped by contextual comments. Sometimes these serve to remind the reader of what has already been achieved, sometimes they illuminate a concept. Overall, this is a well-written book with some fascinating mathematics, explained in a way which continually engages the interest of the reader.

W. LIGHT