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ON A RESULT OF S. KOSHITANI

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Let G be a p-solvable group with a p-Sylow subgroup P of order p^a and let $t(G)$ be the nilpotency index of the radical of a group algebra of G over a field of characteristic p. The purpose of this paper is to give an elementary proof of the following result of Koshitani [**1**, Theorem].

Proposition. *Assume that p is odd and P is metacyclic. If $t(G)=a(p-1)+1$ then P is elementary abelian.*

Proof. As in the proof of [**2**, Proposition 1], we may assume that $O_{p'}(G)=1$, $|P|=p^3$, $U=O_p(G)$ is elementary abelian of order p^2 and G/U is a subgroup of $\mathrm{Aut}(U)=GL(2,p)$. If G/U is reducible then we may assume that G/U is a group consisting of upper triangular matrices, which has a normal p-Sylow subgroup. Hence P is normal in G, which is impossible. Thus G acts irreducibly on U. By the Frattini argument, we can see $G=N_G(V)U$ where V is a p'-subgroup such that $O_{p,p'}(G)=UV$. Since $N_U(V)$ is normal in G and U is a minimal normal subgroup of G, we have $N_U(V)=1$ by $O_{p'}(G)=1$. Let $\langle w\rangle$ be a p-Sylow subgroup of $N_G(V)$, which is a p-Sylow subgroup of $GL(2,p)$ and let $\{x,y\}$ be a basis of U such that $\left(\begin{smallmatrix}1 & 0\\ 1 & 1\end{smallmatrix}\right)$ is the matrix of w with respect to this basis. Then we obtain that

$$P=\langle w,x,y \mid w^p=x^p=y^p=1,\ x^w=x,\ y^w=xy,\ x^y=x\rangle.$$

Since p is odd, it follows from these relations that P is of exponent p, contrary to the fact that P is a metacyclic group of order p^3.

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