From Crab Pulsar to Magnetar?

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Abstract. We review the evolution of the Crab pulsar's rotational history during the past 35 years. Representing 3.7% of the pulsar's age, it is possible to estimate the likely development of the magnetic field and characteristic age much better than previously. The increasing magnetic field of this pulsar and of other young pulsars, most dramatically the Vela pulsar, raises the interesting possibility that these objects might evolve into magnetars. We discuss the observational case for such a proposal, but note that the origin of these field enhancements may be associated with glitch activity. However, it is understood no better than the origin of the magnetar fields, but may be one and the same.

1. Neutron Star Spin-Down

It is generally accepted that normal pulsars are born in the supernovae of massive stars, and result in neutron stars with short periods of rotation and magnetic fields of about 10¹² G, as deduced from the rate of slow-down.

Neutron star rotation is usually modeled as a rotating magnetic dipole which loses energy in the form of electromagnetic radiation at the rotation frequency. The slow-down rate of the pulsar is then given by the spin-down equation:

$$\dot{\Omega} = -\frac{2m^2 \sin^2 \alpha}{3c^3 I} \Omega^3 \propto -\Omega^n, \tag{1}$$

where n is the braking index and takes a value of 3 in the simple dipole model. Since the dipolar magnetic moment is related to the magnetic field B at the pulsar magnetic equator by $m = BR^3$, Equation (1) allows an estimate of B:

$$B = -\sqrt{\frac{3c^3I}{8\pi^2R^6\sin^2\alpha}P\dot{P}} = 3.2 \times 10^{19}\sqrt{P\dot{P}} \text{ gauss},$$
 (2)

where $P = 1/\nu = 2\pi/\Omega$.

Differentiation of Equation (1) shows that a direct determination of n is obtainable if the second differential can be measured. In terms of angular velocity Ω or frequency $\nu = \Omega/2\pi$:

$$n = \frac{\Omega \ddot{\Omega}}{\dot{\Omega}^2} = \frac{\nu \ddot{\nu}}{\dot{\nu}^2}.$$

$$257$$
(3)

The characteristic age of a pulsar is calculated by integrating the spin-down equation, assuming that the spin-period at birth was very short and that n = 3:

$$\tau = -\frac{1}{(n-1)} \frac{\Omega}{\dot{\Omega}} = \frac{1}{(n-1)} \frac{P}{\dot{P}} = \frac{1}{2} \frac{P}{\dot{P}}.$$
 (4)

For a given value of n, it is easy to show that the values of B and τ , calculated according to Equations (2) and (4), vary with time according to:

$$\frac{\mathrm{d}\tau}{\mathrm{dt}} = \frac{1}{2}(n-1), \quad \frac{\mathrm{d}B}{\mathrm{dt}} = \frac{B}{\tau}(3-n). \tag{5}$$

Significant values of n have been found for the Crab Pulsar ($n=2.515\pm0.005$; Lyne, Pritchard & Smith 1993), for PSR B0540-69 ($n=2.01\pm0.02$; Boyd et al. 1995), for PSR J1119-6127 ($n=3.0\pm0.1$; Camilo et al. 2000) and for PSR B1509-58 ($n=2.8\pm0.2$; Kaspi et al. 1994). Two of these values are significantly less than the expected value of 3. The difficulty in extending this analysis to many other pulsars is that a value of Ω from a short run of data may be dominated by the effects of glitches or of timing noise. Indeed, most of these values are derived from data taken between glitches, and do not reflect the true long-term evolution of the rotation, including the effects of any glitches. The first attempt to describe the long-term slow-down in the presence of a large glitch activity was conducted by Lyne et al. (1996) who determined a value of $n=1.4\pm0.2$ for the Vela pulsar, B0833-45.

Inspection of Equation (5) shows two things. First, the young pulsars which have n < 3 have increasing magnetic fields and they are apparently aging more slowly than the passage of time. We now explore the possibility that young pulsars evolve into magnetars. We do this, firstly by revisiting the slow-down of the Crab pulsar (B0531+21), assessing the slow-down of two other glitching pulsars and then by studying their motion on the $P - \dot{P}$ diagram.

PSR B0531+21. The variation in frequency of this pulsar shows not only the values of frequency and its first two derivatives which

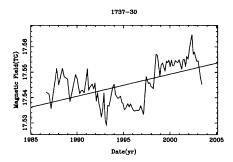


Figure 1. The evolution of magnetic field in PSR B1737-30.

gives the value of n above, but also a highly significant negative value of third derivative. This corresponds to a decreasing value of second derivative and hence a decreasing value of n (Equation [3]). The fact that n=2.5 implies that the magnetic field is increasing and that the characteristic age is increasing at less than one year per year. Moreover the fact that n is decreasing implies an accelerating rate of increase in magnetic field and decreasing rate of age increase.

PSR B1737-30. This pulsar shows multiple, but somewhat smaller, glitches than the Vela pulsar. In this case, over the nearly 20 years since its discovery,

this pulsar's magnetic field is increasing steadily with time (Fig. 1) and its characteristic age is decreasing. The effective value of n is about 0.

PSR J0631+1036. This pulsar also shows multiple glitches but with a rather small amount of recovery after the glitches. The observed long term evolution of $\dot{\nu}$ shows that it is actually increasing with time. This corresponds to not only increasing magnetic fields with time but a characteristic age which is decreasing. The effective value of n in this case is substantially less than 0.

2. Motion in the $P - \dot{P}$ Diagram

We now study the direction of motion of these seven pulsars in the $P-\dot{P}$ diagram. These are shown by the arrows in Figure 2. The three youngest of the pulsars show little glitch activity and have braking indices not very different from 3. However, and it will be noticed that those which are somewhat older are moving towards increasing magnetic field and some of them namely J0631+1036, B0833-45 and B1737-30 are moving towards the top right hand corner of the diagram which is occupied by the magnetars. Additionally, as we have noted, for the Crab, the field is increasing at an increasing rate. All these display substantial glitch activity.

We can ask how fast these latter pulsars might move across the diagram. This depends of course upon the position in the diagram and the value of n. Let us take an example of a pulsar with n=1. Equation (3) shows that this pulsar has a constant value of τ and it also follows that, in this case, it takes a time Δt to change magnetic field from B_i to B_f given by $\Delta t = \tau \ln(B_f/B_i)$. Thus, a decade increase in magnetic field would take only only 2.3τ and to move from 10^{12} G to the 10^{14} G of a magnetar in only $\sim 5\tau$. For a pulsar like Vela (B0833–45), with $\tau \sim 10$ kyr, this is only 50 kyr. If this were to occur, it would not be surprising if a supernova remnant were still seen surrounding the magnetar. Of course, having arrived in the magnetar region, the object would only seem to have a small age of ~ 10 kyr.

There is now a reasonable continuum of pulsars between these young pulsars and the magnetars (e.g., McLaughlin et al. 2003), providing circumstantial evidence that there may be an evolutionary linkage between the two populations. If the hypothesis is correct, the increasing magnetic field must somehow cause the X-ray luminosity to increase, perhaps through its subsequent decay.

3. Conclusion

The movement and distribution of pulsars in the $P - \dot{P}$ diagram has long been something of a mystery. Clearly, none the pulsars discussed in this paper are moving towards the main body of normal pulsars. They are either not the progenitors of normal pulsars or they must undergo a period of magnetic field decay at some time in the future. The latter possibility has been recently addressed by Tauris & Konar (2001). The implications of the discussion above do nothing to explain the conundrum, and, if anything, make it worse, since we suggest that at least some will end up in the region of the magnetars, with longer period

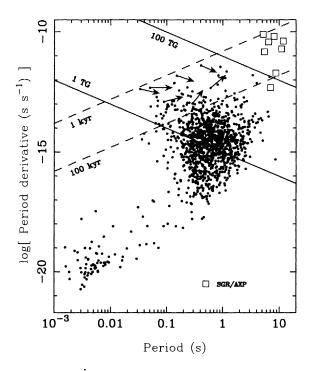


Figure 2. The $P - \dot{P}$ diagram. Normal radio pulsars are shown by filled circles, while the magnetars are shown by open squares. Lines representing characteristic ages of 1 kyr and 100 kyr and equatorial surface magnetic fields of 10^{12} and 10^{14} gauss are shown. For seven young pulsars the observed directions of motion are shown by arrows.

than most normal pulsars. Since these are solitary pulsars, there is no known mechanism which could spin them up to normal pulsar periods.

Why the increase in magnetic field occurs in the way discussed above is not clear, but it may somehow be associated with the large incidence of glitch activity in such pulsars. Whatever the cause, this may be the mechanism by which the enormous magnetic fields of magnetars are created.

References

Boyd, P. T. et al. 1995, ApJ, 448, 365

Camilo, F. et al. 2000, ApJ, 541, 367

Kaspi, V. M. et al. 1994, ApJ, 422, L83

Lyne, A. G., Pritchard, R. S., & Smith F. G. 1993, MNRAS, 265, 1003

Lyne, A. G. et al. 1996, Nature, 381, 497

McLaughlin, M. A. et al. 2003, ApJ, 591, 135

Tauris, T. M., & Konar, S. 2001, A&A, 376, 543