Canad. Math. Bull. Vol. **53** (2), 2010 pp. 223–229
doi:10.4153/CMB-2010-041-1

Density of Polynomial Maps

Chen-Lian Chuang and Tsiu-Kwen Lee

Abstract. Let R be a dense subring of $\mathrm{End}(_DV)$, where V is a left vector space over a division ring D. If $\dim_D V = \infty$, then the range of any nonzero polynomial $f(X_1, \dots, X_m)$ on R is dense in $\mathrm{End}(_DV)$. As an application, let R be a prime ring without nonzero nil one-sided ideals and $0 \neq a \in R$. If $af(x_1, \dots, x_m)^{n(x_i)} = 0$ for all $x_1, \dots, x_m \in R$, where $n(x_i)$ is a positive integer depending on $x_1, \dots, x_m$, then $f(X_1, \dots, X_m)$ is a polynomial identity of R unless R is a finite matrix ring over a finite field.

1 Results

Throughout, V is a left vector space over a division ring D. Let $\mathrm{End}(_DV)$ denote the ring of endomorphisms of $_DV$. For $c \in \mathrm{End}(_DV)$ and a subspace W of $_DV$, let $c\restriction_W$ denote the restriction of c to W. The finite topology of $\mathrm{End}(_DV)$ is obtained by endowing each $c \in \mathrm{End}(_DV)$ with the family of neighborhoods

$$\{x \in \mathrm{End}(_DV) \mid x\restriction_W = c\restriction_W\},$$

where W ranges over all finite-dimensional subspaces of $_DV$. Let F denote the center of D. By a (noncommuting) polynomial over F, we mean an element of the free algebra $F\{X_1, X_2, \dots\}$ over the field F generated by indeterminates $X_1, X_2, \dots$. The range of a polynomial $f(X_1, \dots, X_m) \in F\{X_1, X_2, \dots\}$ on a subring R of $\mathrm{End}(_DV)$ is defined to be

$$\mathcal{R}(f; R) \stackrel{\text{def.}}{=} \{f(x_1, \dots, x_m) \in \mathrm{End}(_DV) \mid x_1, \dots, x_m \in R\}.$$

Let R be a dense subring of $\mathrm{End}(_DV)$. Assume that $\dim_D V = \infty$. Chuang [2, Lemma 1] proved that $\mathcal{R}(f; R)$ is a dense subset of $\mathrm{End}(_DV)$ for the case $f(X_1, X_2) = X_1X_2 - X_2X_1$. Wong extended this to nonzero multilinear polynomials [10, Lemma 2]. Our purpose here is to extend these results to their full generality.

Theorem 1.1 *Let R be a dense subring of $\mathrm{End}(_DV)$ and let $f(X_1, X_2, \dots, X_m)$ be a nonzero polynomial. If $\dim_D V = \infty$, then $\mathcal{R}(f; R)$ is a dense subset of $\mathrm{End}(_DV)$.*

This actually follows from Theorem 1.2, a more detailed and generalized version.

Received by the editors May 19, 2007.
Published electronically April 6, 2010.
The work was supported in part by the National Science Council of Taiwan and National Center for Theoretical Sciences, Taipei Office.
AMS subject classification: **16D60**, 16S50.
Keywords: density, polynomial, endomorphism ring, PI.

Theorem 1.2 *Let R be a dense subring of* $\mathrm{End}({}_DV)$ *and let* $f(X_1, X_2, \ldots, X_m; Y)$ *be a polynomial involving Y nontrivially. Assume that* $\dim {}_DV = \infty$*. Then given* $c_1, c_2, \ldots, c_n \in \mathrm{End}({}_DV)$ *and a finite-dimensional subspace V_0 of ${}_DV$, there exist* $x_1, x_2, \ldots, x_m, y_1, y_2, \ldots, y_n \in R$ *such that*

$$f(x_1, x_2, \ldots, x_m; y_i)\restriction_{V_0} = c_i\restriction_{V_0} \quad \textit{for } i = 1, 2, \ldots, n.$$

Granted this, we can immediately give the proof.

Proof of Theorem 1.1 Since $f(X_1, X_2, \ldots, X_m)$ is nonzero, it must involve nontrivially some X_i, say X_m. Write $f = f(X_1, \ldots, X_{m-1}; X_m)$. Let $c \in \mathrm{End}({}_DV)$ and let V_0 be a finite-dimensional subspace of ${}_DV$. We apply Theorem 1.2 with X_m playing the role of Y. So there exist $x_1, \ldots, x_{m-1}, x_m \in R$ such that

$$f(x_1, \ldots, x_{m-1}; x_m)\restriction_{V_0} = c\restriction_{V_0}.$$

So $\mathcal{R}(f; R)$ intersects nontrivially any neighborhood of $\mathrm{End}({}_DV)$ and is hence dense. ■

As an application to Theorem 1.2 we will prove the following.

Theorem 1.3 *Let R be a prime ring with extended center C and without nonzero nil one-sided ideals. Let $f(X_1, \ldots, X_m)$ be a non-commuting polynomial over C and $0 \neq a \in R$. Suppose that for all $x_1, \ldots, x_m \in R$, there exists an integer $n(x_i) \geq 1$, depending on $x_1, \ldots, x_m$, such that $af(x_1, \ldots, x_m)^{n(x_i)} = 0$. Then $f(x_1, \ldots, x_m) = 0$ for all $x_1, \ldots, x_m \in R$, unless R is a finite matrix ring over a finite field.*

We refer the reader to [6] for the case $f(X) = X$ and to [4, 7] for the case where $f(X_1, \ldots, X_m)$ is a multilinear polynomial. On the other hand, as pointed out in [11], if R is an $n \times n$ matrix ring over a finite field, then by [3, Theorem] for any $1 < k \leq n$ there exists a polynomial $f(X_1, \ldots, X_m)$, not a polynomial identity of R, such that $f(x_1, \ldots, x_m)^k = 0$ for all $x_1, \ldots, x_m \in R$. Theorem 1.3 can be also generalized to one-sided ideals as in [4, 6]. For simplicity, we state the result without proof.

Theorem 1.4 *Let R be a prime ring without nonzero nil one-sided ideals. Let $f(X_1, \ldots, X_m)$ be a non-commuting polynomial over the extended centroid C of R. Given $0 \neq a \in R$ and a one-sided ideal I of R, suppose that for all $x_1, \ldots, x_m \in I$,*

$$af(x_1, \ldots, x_m)^{n(x_i)} = 0 \textit{ for some } n(x_i) \geq 1 \textit{ depending on } x_1, \ldots, x_m.$$

Then the following hold unless C is a finite field and I is generated by an idempotent e in the socle of R:

(i) *If I is a right ideal, then either $aI = 0$ or $f(x_1, \ldots, x_m)I = 0$ for all $x_1, \ldots, x_m \in I$.*
(ii) *If I is a left ideal, then $If(x_1, \ldots, x_m) = 0$ for all $x_1, \ldots, x_m \in I$.*

By [4, Main Theorem], we can drop the exceptional case when $f(X_1, \ldots, X_m)$ is a multilinear polynomial.

https://doi.org/10.4153/CMB-2010-041-1 Published online by Cambridge University Press

2 Proofs

Proof of Theorem 1.2 It suffices to prove the following.

Claim: For any given $a_1, \dots, a_m, b_1, \dots, b_n \in \operatorname{End}({}_DV)$ and for any given $m+n+1$ finite-dimensional subspaces $V_0, V_1, \dots, V_m; U_1, \dots, U_n$ of ${}_DV$ with

$$V_0 \cap (V_1 + \cdots + V_m + U_1 + \cdots + U_n) = 0, \tag{2.1}$$

there exist $x_1, \dots, x_m; y_1, \dots, y_n \in R$ satisfying the following:

(i) $x_1\restriction_{V_1} = a_1\restriction_{V_1}, \dots, x_m\restriction_{V_m} = a_m\restriction_{V_m}$,
(ii) $y_1\restriction_{U_1} = b_1\restriction_{U_1}, \dots, y_n\restriction_{U_n} = b_n\restriction_{U_n}$,
(iii) $f(\vec{x}; y_1)\restriction_{V_0} = c_1\restriction_{V_0}, \dots, f(\vec{x}; y_n)\restriction_{V_0} = c_n\restriction_{V_0}$,

where $f(\vec{x}; y_i) \stackrel{\text{def.}}{=} f(x_1, x_2, \dots, x_m; y_i)$ for $1 \le i \le n$.

Indeed, our theorem follows directly by taking $V_i = 0 = U_j$ for $1 \le i \le m$ and $1 \le j \le n$. Let $\alpha \in F$ be the constant term of $f(\vec{X}; Y)$. Replacing $f(\vec{X}; Y)$ by $f(\vec{X}; Y) - \alpha$ and $c_1, \dots, c_n$ by $c_1 - \alpha, \dots, c_n - \alpha$, respectively, we may assume that $f(\vec{X}; Y)$ has no constant term. Write

$$f(\vec{X}; Y) = X_1 f_1(\vec{X}; Y) + \cdots + X_m f_m(\vec{X}; Y) + Y g(\vec{X}; Y).$$

We proceed by induction on the total degree of $f(X_1, X_2, \dots, X_m; Y)$ and divide our argument into four cases.

Case 1: $g(\vec{X}; Y)$=0. Then some $f_i(\vec{X}; Y)$, say $f_m(\vec{X}; Y)$, must involve Y nontrivially. Since $\dim {}_DV = \infty$, there exists a subspace V_0' of V such that $\dim {}_DV_0' = \dim {}_DV_0$ and such that $V_0' \cap (V_0 + V_1 + \cdots + V_m + U_1 + \cdots + U_n) = 0$. Fix an isomorphism $\sigma\colon V_0 \to V_0'$. By (2.1), we pick $a_m' \in \operatorname{End}({}_DV)$ such that

$$a_m'\restriction_{V_m} = a_m\restriction_{V_m} \quad \text{and} \quad a_m'\restriction_{V_0} = \sigma.$$

For $1 \le i \le m-1$, we also pick $a_i' \in \operatorname{End}({}_DV)$, such that

$$a_i'\restriction_{V_i} = a_i\restriction_{V_i} \quad \text{and} \quad a_i'\restriction_{V_0} = 0.$$

Clearly, $f_m(\vec{X}; Y)$ has smaller degree than $f(\vec{X}; Y)$. By the induction hypothesis, there exist $x_i, y_j \in R$, $1 \le i \le m$ and $1 \le j \le n$, satisfying the following:

- $x_i\restriction_{V_0+V_i} = a_i\restriction'_{V_0+V_i}$ for $i = 1, \dots, m$,
- $y_j\restriction_{U_j} = b_j\restriction_{U_j}$ for $j = 1, \dots, n$,
- $f_m(\vec{x}; y_j)\restriction_{V_0'} = \sigma^{-1} \circ (c_j\restriction_{V_0})$ for $j = 1, \dots, n$.

By our choice of a_i', these x_i also satisfy (i). For $v \in V_0$, we have $vx_i = va_i' = 0$ for $1 \le i \le m-1$ and $vx_m = va_m' = v\sigma$. So for $v \in V_0$,

$$vf(\vec{x}; y_j) = v(x_1 f_1(\vec{x}; y_j) + \cdots + x_m f_m(\vec{x}; y_j)) = vx_m f_m(\vec{x}; y_j) = (v\sigma)(\sigma^{-1} \circ c_j) = vc_j$$

for $1 \le j \le n$. So $f(\vec{x}; y_j)\restriction_{V_0} = c_j\restriction_{V_0}$. This proves (iii).

Case 2: $g(\vec{X}; Y)$ is a nonzero constant, say, $0 \ne \beta \in F$. By (2.1) and the density of R in $\operatorname{End}({}_DV)$ there exist $x_i, y_j \in R$ satisfying the following:

https://doi.org/10.4153/CMB-2010-041-1 Published online by Cambridge University Press

- $x_i\restriction_{V_i} = a_i\restriction_{V_i}$ and $x_i\restriction_{V_0} = 0$ for $1 \le i \le m$.
- $y_j\restriction_{U_j} = b_j\restriction_{U_j}$ and $y_j\restriction_{V_0} = \beta^{-1}c_j\restriction_{V_0}$ for $1 \le j \le n$.

Trivially, (i) and (ii) hold. For $v \in V_0$ we have $vx_i = 0$ for $1 \le i \le m$ and $vy_j = \beta^{-1}vc_j$ for $1 \le j \le n$. So we have

$$\begin{aligned} vf(\vec{x}; y_j) &= v\big(x_1 f_1(\vec{x}; y_j) + \cdots + x_m f_m(\vec{x}; y_j) + y_j g(\vec{x}; y_j)\big) \\ &= vy_j g(\vec{x}; y_j) = (\beta^{-1}v)c_j\beta = vc_j. \end{aligned}$$

So (iii) also holds, as claimed.

Case 3: $g(\vec{X}; Y)$ involves Y nontrivially. By (2.1), we pick a_i', $1 \le i \le m$, such that

$$a_i'\restriction_{V_i} = a_i\restriction_{V_i} \quad \text{and} \quad a_i'\restriction_{V_0} = 0.$$

Since $\dim_D V = \infty$, there exists a subspace V_0' of V such that $\dim_D V_0' = \dim_D V_0$ and such that $V_0' \cap (V_0 + V_1 + \cdots + V_m + U_1 + \cdots + U_n) = 0$. Fix an isomorphism $\sigma\colon V_0 \to V_0'$. By (2.1) again, we pick b_j', $1 \le j \le n$, such that

$$b_j'\restriction_{U_j} = b_j\restriction_{U_j} \quad \text{and} \quad b_j'\restriction_{V_0} = \sigma.$$

Clearly, g has smaller degree than f. Note that $\sigma^{-1} \circ (c_j\restriction_{V_0})$ is defined on V_0', which is disjoint from $V_0 + \sum_{i=1}^m V_i + \sum_{j=1}^n U_j$. By the induction hypothesis, there exist $x_i, y_j \in R$ such that

- $x_i\restriction_{V_0+V_i} = a_i\restriction'_{V_0+V_i}$ for $i = 1, \ldots, m$.
- $y_j\restriction_{V_0+U_j} = b_j\restriction'_{V_0+U_j}$ for $j = 1, \ldots, n$.
- $g(\vec{x}; y_j)\restriction_{V_0'} = \sigma^{-1} \circ (c_j\restriction_{V_0})$ for $j = 1, \ldots, n$.

These x_i, y_j satisfy (i) and (ii) by our choice of a_i', b_j'. For $v \in V_0$ we have $vx_i = 0$ for $1 \le i \le m$ and $vy_j = v\sigma$. So

$$\begin{aligned} vf(\vec{x}; y_j) &= v(x_1 f_1(\vec{x}; y_j) + \cdots + x_m f_m(\vec{x}; y_j) + y_j g(\vec{x}; y_j)) \\ &= vy_j g(\vec{x}; y_j) = (v\sigma)(\sigma^{-1} \circ (c_j\restriction_{V_0})) = vc_j. \end{aligned}$$

So (iii) also follows as claimed.

Case 4: $g(\vec{X}; Y)$ is not a constant and does not involve Y. So $g(\vec{X}; Y)$ involves nontrivially some X_i, say X_m. So write $g(\vec{X}; Y) = g(X_1, \ldots, X_m) = g(\vec{X})$. By (2.1), we choose n finite-dimensional subspaces $V_0^{(j)}$, $j = 1, \ldots, n$, satisfying the following:

- $\dim_D V_0^{(i)} = \dim_D V_0$ for $i = 1, \ldots, n$.
- The sum $\sum_{j=1}^n V_0^{(j)}$ is direct.
- $(\sum_{j=1}^n V_0^{(j)}) \cap (V_0 + \sum_{i=1}^m V_i + \sum_{i=1}^n U_i) = 0$.

Pick isomorphisms $\sigma_j\colon V_0 \to V_0^{(j)}$ for $j = 1, \ldots, n$. Define $c \in \operatorname{End}(_D V)$ satisfying

$$c\restriction_{V_0^{(j)}} = \sigma_j^{-1} \circ (c_j\restriction_{V_0}) \text{ for } 1 \le j \le n.$$

https://doi.org/10.4153/CMB-2010-041-1 Published online by Cambridge University Press

Pick $a'_i \in \operatorname{End}({}_D V)$, $1 \leq i \leq m$, such that

$$a'_i\restriction_{V_i} = a_i\restriction_{V_i} \quad \text{and} \quad a'_i\restriction_{V_0} = 0.$$

Clearly, g has smaller degree than f. Note that

$$(V_0^{(1)} \oplus \cdots \oplus V_0^{(n)}) \cap (V_0 + V_1 + \cdots + V_m) = 0.$$

We apply the induction hypothesis to $g(X_1, \ldots, X_m)$ with X_m playing the role of Y. So there exist $x_1, \ldots, x_{m-1}, x_m \in R$ such that

- $x_i\restriction_{V_0 \oplus V_i} = a'_i\restriction_{V_0 \oplus V_i}$ for $1 \leq i \leq m$.
- $g(x_1, \ldots, x_m)\restriction_{V_0^{(1)} \oplus \cdots \oplus V_0^{(n)}} = c\restriction_{V_0^{(1)} \oplus \cdots \oplus V_0^{(n)}}$.

Moreover, by the density of R in $\operatorname{End}({}_D V)$ there exist $y_j \in R$, $1 \leq j \leq n$, such that

$$y_j\restriction_{U_j} = b_j\restriction_{U_j} \quad \text{and} \quad y_j\restriction_{V_0} = \sigma_j.$$

Clearly, these x_i, y_j satisfy (i) and (ii). For $v \in V_0$, $vx_i = 0$ for $1 \leq i \leq m$, and $v\sigma_j \in V_0^{(j)}$ for $1 \leq j \leq n$. So we have for $1 \leq j \leq n$,

$$\begin{aligned} vf(\vec{x}; y_j) &= v(x_1 f_1(\vec{x}; y_j) + \cdots + x_m f_m(\vec{x}; y_j) + y_j g(\vec{x})) \\ &= v y_j g(\vec{x}) = (v\sigma_j)\sigma_j^{-1} \circ (c_j\restriction_{V_0}) = vc_j. \end{aligned}$$

Hence, (iii) follows as claimed. ■

We now turn to the proof of Theorem 1.3 Let R be a prime ring. Then the extended centroid C of R is a field; we refer the reader to [1] for details. Let $C\{X_1, X_2, \ldots\}$ denote the free algebra over C in noncommuting indeterminates $X_1, X_2, \ldots$. Let $RC\{X_1, X_2, \ldots\}$ denote the free product of the C-algebras RC and $C\{X_1, X_2, \ldots\}$. Elements of $RC\{X_1, X_2, \ldots\}$ (resp. of $C\{X_1, X_2, \ldots\}$) are called *generalized polynomial* (resp. *polynomial*). We call $f(X_1, X_2, \ldots, X_t)$ in $RC\{X_1, X_2, \ldots\}$ (resp. in $C\{X_1, X_2, \ldots\}$) a *generalized polynomial identity*, abbreviated as GPI (resp. *polynomial identity*, abbreviated as PI) if $f(x_1, \ldots, x_t) = 0$ for all $x_i \in R$. A prime ring R is called a GPI-ring (resp. a PI-ring) if it satisfies a nonzero GPI (resp. a nonzero PI). To prove Theorem 1.3 we need the following two lemmas (see [4, Lemmas 1 and 2]).

Lemma 2.1 *Let $S = M_n(D)$, where D is a division ring. If $ab^\ell = 0$ for some integer $\ell \geq 1$ where $a, b \in R$, then $ab^n = 0$.*

Lemma 2.2 *Let S be a simple Artinian ring and let T be a subset of S such that $uTu^{-1} \subseteq T$ for all invertible elements $u \in S$. Then either $\ell_S(T) = 0$ or $T = 0$, where $\ell_S(T)$ is the left annihilator of T in S.*

Proof of Theorem 1.3 Let $\rho \overset{\text{def.}}{=} aR$, a nonzero right ideal of R. Let $x_1, \ldots, x_m \in R$. By assuption, there exists an integer $n(x_i a) \geq 1$, depending on $x_1 a, \ldots, x_m a$, such that $af(x_1 a, \ldots, x_m a)^{n(x_i a)} = 0$ and so

$$f(ax_1, \ldots, ax_m)^{n(x_i a)} a = af(x_1 a, \ldots, x_m a)^{n(x_i a)} = 0. \tag{2.2}$$

https://doi.org/10.4153/CMB-2010-041-1 Published online by Cambridge University Press

Set $\overline{\rho} = \rho/\rho \cap \ell_R(\rho)$. Since R is a prime ring without nonzero nil one-sided ideals, so is the ring $\overline{\rho}$. In view of [8, Lemma 3], the extended centroid $\overline{C}$ of the prime ring $\overline{\rho}$ is canonically isomorphic to C. This induces a canonical isomorphism of free algebras $C\{X_1, X_2, \dots\}$ and $\overline{C}\{X_1, X_2, \dots\}$. Let $\overline{f}(X_1, \dots, X_m)$ denote the canonical image of $f(X_1, \dots, X_m)$. By (2.2), $\overline{f}(x_1, \dots, x_m)$ is nilpotent for all $x_1, \dots, x_m \in \overline{\rho}$. It follows from [11] that either $\overline{f}(X_1, \dots, X_m)$ is a polynomial identity for $\overline{\rho}$ or $\overline{\rho}$ is a finite matrix ring over a finite field. In either case, ρ itself is a PI-ring. Since R contains a nonzero PI right ideal, it is a GPI-ring. By Martindale's theorem [9, Theorem 3], RC has a minimal idempotent g such that $gRCg$ is a finite-dimensional central division C-algebra. Let H denote the socle of RC. Since $Ha \subseteq H$, for our purpose it suffices to assume $a \in H$ from the start.

We claim that $af(x_1, \dots, x_m)^{n(x_i)} = 0$ for all $x_1, \dots, x_m \in H$, where $n(x_i)$ is a positive integer depending on $x_1, \dots, x_m$. Suppose on the contrary that there exist $z_1, \dots, z_m \in H$ such that

$$af(z_1, \dots, z_m)^k \neq 0 \text{ for all } k = 1, 2, \dots.$$

Notice that H is a simple ring with nonzero socle. By Litoff's theorem [5], there exists an idempotent $e \in H$ such that $a, z_1, \dots, z_m \in eHe$. Moreover, $eHe = eRCe \cong \mathrm{M}_p(D)$ for some division ring $D \cong gRCg$ and for some integer $p \geq 1$. By Lemma 2.1 we see that

$$af(x_1, \dots, x_m)^p = 0 \text{ for all } x_1, \dots, x_m \in R \cap eRCe. \tag{2.3}$$

Case 1. Assume that C is a finite field. Pick an ideal $I \neq 0$ of R such that $IC \subseteq R$. Then $eRCe = eICe \subseteq R$ by the simplicity of $eRCe$. So (2.3) holds for all $x_1, \dots, x_m \in eRCe$. In particular, $af(z_1, \dots, z_m)^p = 0$, a contradiction.

Case 2. Assume that C is an infinite field. Pick an ideal $I \neq 0$ of R with $eIe \subseteq R$. Then (2.3) holds for all $x_i, \dots, x_m \in eIe$. Note that C is infinite. If we further choose I with $\alpha I \subseteq R$ for sufficiently, but finitely many, $\alpha \in C$, then by a Vandermonde argument (2.3) holds for all $x_i, \dots, x_m \in eICe$. Then $eICe = eRCe$ follows by the simplicity of $eRCe$. So $af(z_1, \dots, z_m)^p = 0$, a contradiction again.

This proves our claim. Set $V \overset{\text{def.}}{=} gRC$ and $D \overset{\text{def.}}{=} gRCg$. Then, by the density theorem, H acts densely on ${}_DV$. Suppose first that $\dim_D V = \infty$. Choose a vector $v \in V$ such that $va \neq 0$. By Theorem 1.1, there exist $x_1, \dots, x_m \in H$ such that $vaf(x_1, \dots, x_m) = va$ and so $vaf(x_1, \dots, x_m)^k = va \neq 0$ for all $k \geq 1$, a contradiction. Thus $\dim_D V < \infty$, implying that $R = RC = H \cong \mathrm{M}_p(D)$ for some integer $p \geq 1$. By Lemma 2.1, $af(x_1, \dots, x_m)^p = 0$ for all $x_1, \dots, x_m \in R$. The subset T of R consisting of all elements $f(x_1, \dots, x_m)^p$ for $x_1, \dots, x_m \in R$ clearly satisfies $uTu^{-1} \subseteq T$ for all invertible elements $u \in R$. Since $a \neq 0$, Lemma 2.2 asserts that $f(x_1, \dots, x_m)^p = 0$ for all $x_1, \dots, x_m \in R$. Applying [11], we see that either $f(x_1, \dots, x_m) = 0$ for all $x_1, \dots, x_m \in R$ or R is a finite matrix ring over a finite field. ■

https://doi.org/10.4153/CMB-2010-041-1 Published online by Cambridge University Press

References

[1] K. I. Beidar, W. S. Martindale, III, and A. V. Mikhalev, *Rings with Generalized Identities.* Monographs and Textbooks in Pure and Applied Mathematics 196. Marcel Dekker, New York, 1996.

[2] C.-L. Chuang, *On nilpotent derivations of prime rings*. Proc. Amer. Math. Soc. **107**(1989), no. 1, 67–71. doi:10.2307/2048036

[3] ———, *On ranges of polynomials in finite matrix rings*. Proc. Amer. Math. Soc. **110**(1990), no. 2, 293–302. doi:10.2307/2048069

[4] C.-L. Chuang and T.-K. Lee, *Rings with annihilator conditions on multilinear polynomials*. Chinese J. Math. **24**(1996), no. 2, 177–185.

[5] C. Faith and Y. Utumi, *On a new proof of Litoff's theorem*. Acta Math. Acad. Sci. Hungar **14**(1963), 369–371. doi:10.1007/BF01895723

[6] B. Felzenszwalb, *On a result of Levitzki*. Canad. Math. Bull. **21**(2) (1978), 241–242.

[7] B. Felzenszwalb and A. Giambruno, *Periodic and nil polynomials in rings*. Canad. Math. Bull. **23**(1980), no. 4, 473–476.

[8] T.-K. Lee, *Derivations and centralizing mappings in prime rings*. Taiwanese J. Math. **1**(1997), no. 3, 333–342.

[9] W. S. Martindale, III, *Prime rings satisfying a generalized polynomial identity*. J. Algebra **12**(1969), 576–584. doi:10.1016/0021-8693(69)90029-5

[10] T.-L. Wong, *Derivations with power central values on multilinear polynomials*. Algebra Colloq. **3**(1996), no. 4, 369–378.

[11] C.-T. Yeh and C.-L. Chuang, *Nil polynomials of prime rings.* J. Algebra **186**(1996), no. 3, 781–792. doi:10.1006/jabr.1996.0394

Department of Mathematics, National Taiwan University, Taipei 106, Taiwan
e-mail: chuang@math.ntu.edu.tw
tklee@math.ntu.edu.tw