DEAR EDITOR,

Re: Paul Scott, Some recent discoveries in elementary geometry, *Math. Gaz.* **81** (Nov 1997), pp. 391-397 and I. Ward, The tritet rule, *Math. Gaz.* **79** (July 1995), pp. 380-382.

Readers may like to know of some earlier references which discuss the generalisation of Pythagoras' Theorem to 3-space. The first, originally published in 1962 is George Pólya, *Mathematical discovery*, Wiley (1981), p. 34. The others were collected as Note 62.23 in the *Gazette*: (1) Lewis Hull, (2) Hazel Perfect, (3) I. Heading, Pythagoras in higher dimensions: three approaches, *Math. Gaz.* **62** (October 1978) pp. 206-211.

Yours sincerely,

A. K. JOBBINGS

Bradford Grammar School, Keighley Road, Bradford BD9 4JP

DEAR EDITOR,

In Note 82.53 a proof is given for a test of divisibilty by 19. I offer a shorter proof.

Let the number to be tested be $N = 10a + b$ where b is the units digit. The reduced test number is given by $P = a + 2b$, so that $2N - P = 19a$. Therefore, $19|N$ if and only if $19|P$.

Yours sincerely,

E. J. PEET

3 Queensway, Newby, Scarborough YO12 6SJ

DEAR EDITOR,

In [1] Murray Humphreys and Nicholas Macharia show that the $(n + 1)$-digit number

$$k = \overline{a_n a_{n-1} \ldots a_0} = 10^n a_n + 10^{n-1} a_{n-1} + \ldots + a_0 \qquad (1)$$

is divisible by 19 if and only if

$$m = 10a_n + a_{n-1} + 2a_{n-2} + 4a_{n-3} + \ldots + 2^{n-2} a_1 + a_0 \qquad (2)$$

is divisible by 19. This is essentially a special case of the method of James Voss in [2] for determining divisibility by any integer s relatively prime to 10. The method hinges on using the multiplicative inverse of 10 (mod s). When $s = 19$, the multiplicative inverse is 2 because

$$2 \times 10 = 20 \equiv 1 \pmod{19}. \qquad (3)$$

If we multiply (1) by 2^{n-1} we get

$$2^{n-1}k = 2^{n-1}\left(10^n a_n + 10^{n-1} a_{n-1} + 10^{n-2} a_{n-2} + 10^{n-3} a_{n-3} + \ldots + 10a_1 + a_0\right)$$

$$= 2^{n-1}10^{n-1}10a_n + 2^{n-1}10^{n-1}a_{n-1} + 2^{n-2}10^{n-2}2a_{n-2} + 2^{n-3}10^{n-3}4a_{n-3}$$

$$+ \ldots + 2 \times 10 \times 2^{n-2} a_1 + 2^{n-1} a_0$$

$$= 20^{n-1}10a_n + 20^{n-1}a_{n-1} + 20^{n-2}2a_{n-2} + 20^{n-3}4a_{n-3} + \ldots + 20 \times 2^{n-2}a_1 + 2^{n-1}a_0.$$

Using (3),

$$2^{n-1}k \equiv 10a_n + a_{n-1} + 2a_{n-2} + 4a_{n-3} + \ldots + 2^{n-2}a_1 + 2^{n-1}a_0 \pmod{19}.$$

By (2),

$$2^{n-1}k \equiv m \pmod{19}$$

from which it is readily seen that 19 divides k if and only if 19 divides m.

References

1. Murray Humphreys and Nicholas Macharia, Tests for divisibility by 19, *Math. Gaz.* **82** (November 1998) pp. 475-477.
2. James E. Voss, Divisibility tests in ℕ, *The Fibonacci Quarterly* **36.1** (February 1998) pp. 43-44.

Yours sincerely,

ROGER F. HOUSE

Sonoma State University, Mathematics Department, 1801 E. Cotati Ave.

Rohnert Park, CA 94928 USA

e-mail: *rhouse@sonic.net*

More percentage problems

For the record, 1,101,887 members voted for conversion and 1,135,597 against—a difference of 33,710. A resounding victory? I don't think so. There is not even a percentage point in it.

From *The Sunday Times* 26 July 98 and sent in by Hamish Sloan.

Too much!

But the British mother, according to Unicef figures just released, either doesn't bother trying (34 per cent) or gives up within four months (73 per cent).

From a reference to breast-feeding in *The Times* 18 May 98 and sent in in by Hamish Sloan who observes 'Well, most mothers give over 100% don't they?!!'

Needed – a new agent

[The Duke of Buccleuch] owns 2,700 acres, equivalent, his agent calculates, to a mile-wide, 400-mile long corridor running from Scotland to London.

From *The Daily Telegraph TV & Radio Supplement* 27 February 99 and spotted by Harrold Farnsworth.