

Some topics in non-singular rings

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A ring R is right non-singular if and only if Q_R , the maximal quotient ring of R , is von Neumann regular. A ring R is said to be a right *P.P.* ring if every principal right ideal of R is projective. A right *P.P.* ring is right non-singular.

Chapter 1 is expository. In the first three sections of Chapter 2, the condition that Q_R is left torsion-free in the sense of Hattori ([2]) is considered. When R is commutative and non-singular it is shown that this condition is equivalent to the left flat epimorphic hull of R being a regular ring. The fourth section of the chapter is an investigation of the left flat epimorphic hull of a not necessarily commutative non-singular ring and some sufficient conditions are found for it to be regular. Ideals of a commutative non-singular ring R which are contractions of ideals in the maximal quotient ring of R are considered in Section 5.

Chapter 3 is principally concerned with right *P.P.* rings and several characterisations of these rings are collected. It is shown that a ring R is right *P.P.* if and only if every principal right ideal of R is flat and there exists a ring monomorphism $\phi : R \rightarrow S$, where S is a regular ring which is left torsion-free, in the sense of Hattori, when considered as an R -module.

A right R -module A_R is said to be right projectively torsion-free (P.T.F.) if for every $a \in A$ there exist $s_1, s_2, \dots, s_n \in R$ and

$a_1, a_2, \dots, a_n \in A$ such that $a = \sum_{j=1}^n a_j s_j$ and if $ax = 0$ for some

Received 25 September 1973. Thesis submitted to Monash University, February 1973. Degree approved, August 1973. Supervisor: Professor P.D. Finch.

$x \in R$, then $e_j x = 0$ for all $1 \leq j \leq n$. It is shown that these form a hereditary torsion-free class of $\text{mod}R$ if and only if each right complement ideal of R is generated by an idempotent and R is right non-singular. Characterisations of right *P.P.* rings and Baer rings are also presented through these modules.

Chapter 4 is a discussion of right semi-hereditary rings. Chapter 5 is concerned with the epimorphic hull ([3]) of a commutative non-singular ring and a simple construction is given for it. It is also shown that the epimorphic hull of a commutative non-singular ring satisfies a certain universal property.

References

- [1] M.W. Evans, "On commutative *P.P.* rings", *Pacific J. Math.* 41 (1972), 687-697.
- [2] Akira Hattori, "A foundation of torsion theory for modules over general rings", *Nagoya Math. J.* 17 (1960), 147-158.
- [3] Hans Heiner Storrer, "Epimorphismen von kommutativen Ringen", *Comment. Math. Helv.* 43 (1968), 378-401.