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Cataclysmic variables: structure and evolution

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Abstract

I discuss the structure and evolution of cataclysmic variables, with a particular emphasis on the influence of the physics used in calculating the internal structure of the secondary. The available observational data is very rich, and can, in principle, be used to constrain the stellar physics. It is found that, in order to explain the lack of systems with periods in the range 2 – 3 hr, it is required that main sequence star become convective for masses below 0.3 $M_\odot$. This has little consequences on the equation of state, but constrains the opacities and the treatment of subphotospheric layers.

On discute la structure et l'évolution des variables cataclysmiques, en s'attachant plus particulièrement à l'influence de la physique de l'étoile secondaire. Les données observationelles, très abondantes, peuvent en principe être utilisées pour contraindre la physique stellaire. On trouve que, pour expliquer l'absence de systèmes entre 2 et 3 heures, il faut que les étoiles de la séquence principale deviennent convectives lorsque leur masse atteint 0.3 $M_\odot$. Ceci a peu de conséquences sur l'équation d'état, mais contraint les opacités et le traitement des couches sub-photosphériques.

19.1 Introduction

Cataclysmic variables (CV's) are binary systems containing a white dwarf and a normal star, which fills its Roche and transfers mass onto the compact

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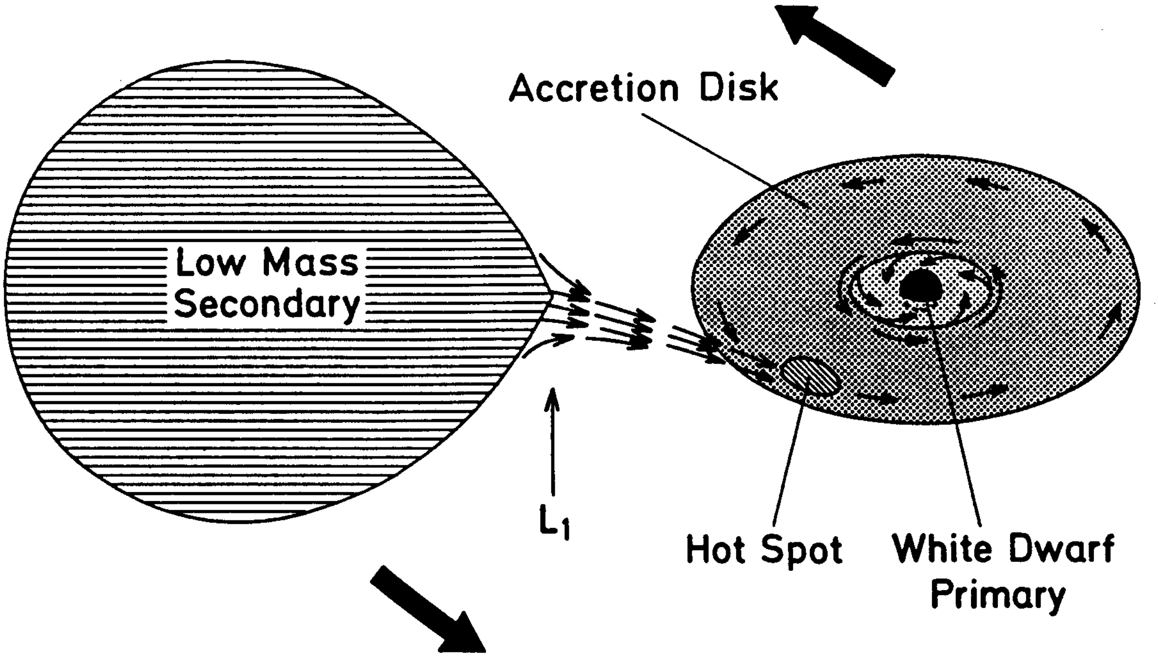

Fig. 19.1 Schematic view of a cataclysmic variable (from Ritter, 1985)

object. The orbital period is in the range 1 to 10 hr, although we know a few cases in which it is longer than one day or shorter than 1 hr. These systems are quite similar to low-mass X-ray binaries, with the difference that the primary star is a neutron star or a black hole. Figure 1 shows a schematic view of a cataclysmic variable. Because the orbital angular momentum of the accreting matter is high, matter cannot flow directly onto the white dwarf, and an accretion disc forms, except in the particular case of strongly magnetic white dwarfs. In the disc, mass flows inwards, whereas angular momentum is transported outwards to the outer edge of the disc, where it is transferred to the orbit via tidal effects. The disc therefore extends up to a significant fraction of the Roche radius of the primary (typically 0.8 or 0.9 times the Roche radius).

The mass transfer rate in these systems is of the order of 10^{-10} – 10^{-9} $M_{\odot}$ yr^{-1}, and is strongly variable on time scales ranging from days to years or more, giving rise to bolometric luminosities in the range 10^{32} – 10^{34} erg s^{-1}. Most of the optical light is emitted by the accretion disc itself, while infrared is emitted by the secondary (usually a K or M dwarf); the white dwarf and the boundary layer between the disc and the surface of the white dwarf are responsible for XUV emission.

We know several hundreds of these systems; their typical distance is of the order of 100 pc to a few hundred pc, so that the total number of cataclysmic variables in the galaxy is estimated to be about 200,000. We therefore have a wealth of available observational data that could be used to constraint models of low mass stars, which are normally quite difficult to observe. An interesting characteristic of these system is that, because the secondary fills its Roche lobe, there is a relation between the secondary radius and its mass, as we shall see later. This is particularly valuable for low mass stars, since the mass and radius determination is quite a difficult task in isolated low mass stars, and is model dependant. Another advantage of cataclysmic variables as compared to low mass X-ray binaries is, apart from the fact that they are much more numerous and therefore closer, that the effect of illumination of the secondary by radiation emitted by the primary is almost negligible, although some effect may still be present (Sarna, 1990). This is not the case for LMXB's, which may have an evolution rather different from that of CV's, due precisely to the illumination effect (see e.g. Ruderman et al., 1989; Podsiadlowski, 1991; Hameury et al., 1993)

19.2 Structure of cataclysmic variables

19.2.1 Orbital period distribution

Figure 2 shows the orbital period distribution of cataclysmic variables. It is seen that most systems have orbital periods below a few hours, with a peak at 1.5 – 4 hr. There are only 4 systems having a period shorter than 80 min, which is the so-called "minimum period", whereas there is a lack of systems between 2 and 3 hr; this interval is called the "period gap", although there are indeed a few systems in it.

As we shall see later, the orbital period distribution is an essential tool for the understanding the evolution of cataclysmic variables.

19.2.2 Primary and secondary masses

The masses of both stars are difficult to determine, since, in order to get a reliable information on the orbital velocity of both components, one requires double spectroscopic systems. In order to determine the masses, one moreover has to know the inclination i of the orbital plane of the system. This is for example the case in eclipsing systems, for which i has to be larger than 70 – 80°. Finally, one must also know where the observed lines are emitted from: this may be either from the whole surface of the secondary (this is usually the case of absorption lines as in normal stars), but lines

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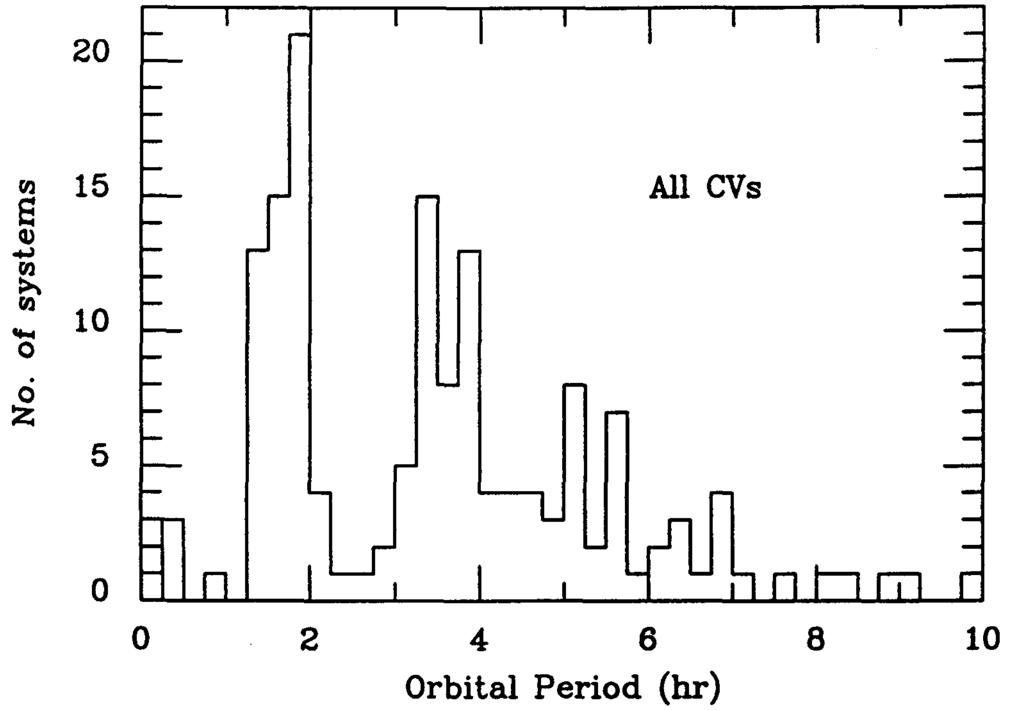

Fig. 19.2 Orbital period distribution of cataclysmic variables. Data from Ritter (1985)

may also originate from the vicinity of the L_1 region (in the case of emission lines due to heating of the secondary by hard radiation from the white dwarf). There are also emission lines from the accretion disc; in order to obtain the velocity of the white dwarf, one must assume circular symmetry for the accretion disc, which is certainly not true for its outer part, because of distortion due to tidal forces. With all these caveats, it is found that the average mass of the white dwarf in CVs is about 1 $M_\odot$, significantly larger than that of isolated white dwarfs, which is about 0.6 $M_\odot$. This difference, as has been shown by Ritter and Burkert (1986) is not due to differences in the formation mechanism of isolated and non-isolated white dwarfs, nor to the steady mass increase of the primary as a result of accretion (in fact as much mass is removed during novae explosions as is accreted in between), nor to systematic errors. This difference can be simply explained by selection effects, that tend to favour the observation of systems with a massive white dwarf, that has smaller radius, so that the luminosity for a given mass transfer rate is higher. After correction of the selection effects, the intrinsic average white dwarf mass turns out to be of the order of about 0.6 $M_\odot$, identical to that of isolated stars.

The secondary mass is always found to be in the range 0.1 – 1.0 $M_\odot$

19.2.3 Mass-radius relation

Kepler's third law relates the orbital separation a and period $P_{\rm hr}$ measured in hours via:

$$\frac{a}{R_\odot} = 0.50(M_1 + M_2)^{1/3} P_{\rm hr}^{2/3} \tag{1}$$

where M_1 and M_2 are respectively the primary and the secondary masses measured in solar masses. Because the secondary fills its Roche lobe, the secondary radius R_2 is determined by the Roche geometry, so that:

$$R_2 = R_{\rm L} = 0.46 \left(\frac{M_2}{M_1 + M_2}\right)^{1/3} a \tag{2}$$

(Paczyński, 1971) for $M_2 < M_1$ (see also Eggleton, 1983 for a more accurate and more general fit of $R_{\rm L}$. Combining Eqs. (1) and (2) gives the mass-radius relation:

$$\frac{R_2}{R_\odot} = 0.23 M_2^{1/3} P_{\rm hr}^{2/3} \tag{3}$$

which is independent of the primary mass. In addition, the average density of the secondary $< \rho >$ defined as $M_2/(4/3\pi R_2^3)$ is 110 $P_{\rm hr}^{-2}$ g cm^{-3}, and depends only on the orbital period of the system. For orbital periods of a few hours, $< \rho >$ is of the order of a few g cm^{-3}, typical of main sequence stars. If one makes the further assumption that the secondary is indeed on the main sequence, then $R_2 = M_2 R_\odot$ and the secondary mass is uniquely determined by the orbital period of the system via:

$$M_2 = 0.11 P_{\rm hr} \tag{4}$$

As we will see later, this assumption is not quite justified, but, at least for orbital periods larger than 3 hr, is not too extreme. A first important consequence can be deduced from this approximate relation: because mass is lost from the secondary, the orbital period of the system must decrease (again only for systems which have main sequence secondaries).

Ritter (1985) has shown that the secondary in many CVs is not too far from the main sequence. Fig. 2 shows the mass and radius of several cataclysmic variables, as compared with theoretical predictions. One can see first that, because of the mass-radius relation resulting from the geometry of the system, the error box in the plane M_2, R_2 is reduced to a line segment, and, more important, that the secondary in those systems is quite close to the main sequence, the 10 – 20 % difference in radii can be either intrinsic, or due to a systematic error in theoretical models.

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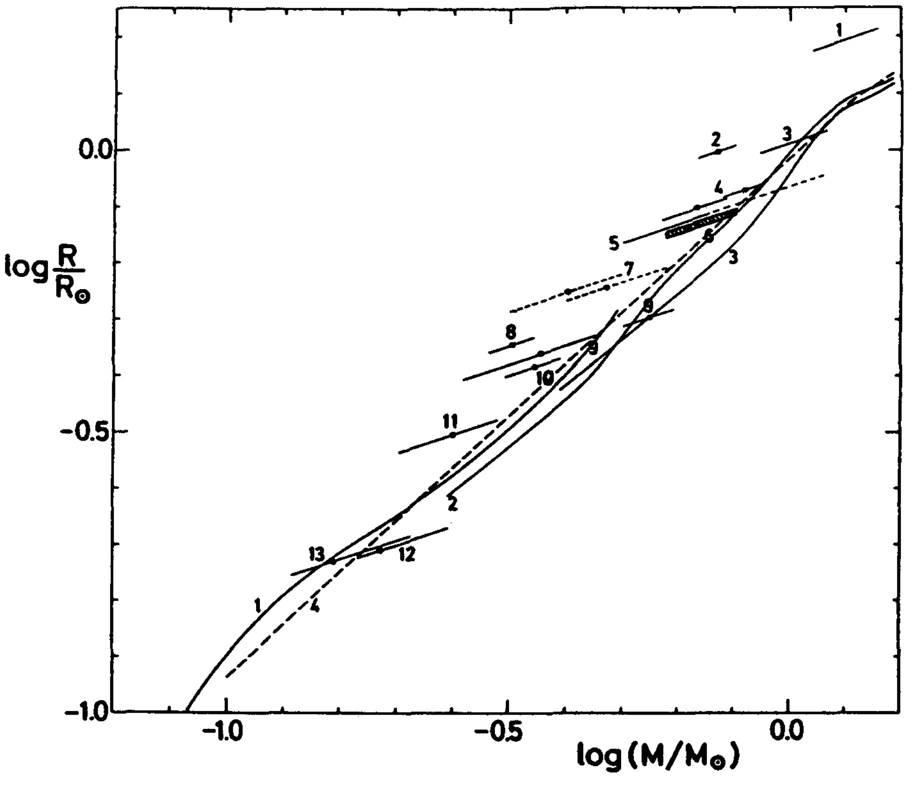

Fig. 19.3 Comparison of determined masses and radii of the secondary in 13 systems with the prediction of theoretical models of main sequence stars.

19.3 Evolution of cataclysmic variables

19.3.1 A preliminary approach

As we have seen, the typical mass transfer rate in cataclysmic variables is about 10^{-9} $M_\odot$ yr^{-1}, so that for a 1 $M_\odot$ secondary, the mass transfer time scale $t_{\dot{M}} = M_2/\dot{M}$ is about 10^9 yr, much shorter than the Hubble time. On the other hand, the nuclear time $t_{\rm nuc}$ is longer or much longer than the Hubble time for stars less massive than 1 $M_\odot$. This means that nuclear evolution is negligible, and that systems evolve under the effect of mass transfer. The Kelvin-Helmholtz time $t_{\rm K-H} \sim 10^7 M_2^{-3}$ becomes larger than $t_{\dot{M}}$ when the mass of the secondary reaches 0.3 – 0.4 $M_\odot$. This means that for larger masses, corresponding to periods larger than 3 – 4 hr, the secondary has time to adjust thermally to its new structure, modified by mass transfer, and remains on the main sequence. At shorter periods, it must deviate from main sequence, and it turns out that the secondary becomes degenerate. In these stars, the mass-radius relation is quite different from that of main sequence stars, and is $R_2 \propto M_2^{-1/3}$, so that applying the same argument as in section 1.2.3, one finds that $M_2 \propto P_{\rm hr}^{-1}$, so that the orbital period of short period systems must increase. The transition main-sequence – degenerate therefore corresponds to a minimum in the orbital period, as has been shown by Paczyński and Sienkiewicz (1981) and Rappaport et al. (1982).

19.3.2 Mass transfer; stability

The evolution of a binary system is governed by the variation of three important quantities: the total mass $M_1 + M_2$, the secondary radius R_2 equal to the Roche radius R_L, and the total angular momentum of the system J given by:

$$J = M_1 M_2 \left(\frac{Ga}{M_1 + M_2} \right)^{1/2} \tag{5}$$

Since the secondary must fill its Roche lobe at any time, $\dot{R}_2 = \dot{R}_L$, where the dot denotes time derivative. If one furthermore assumes for simplicity that the total mass of the system remains constant, then differentiation of Eq. (5) yields:

$$\frac{\dot{a}}{a} = 2\frac{\dot{J}}{J} - 2\frac{\dot{M}_2}{M_2}\left(1 - \frac{M_2}{M_1}\right) \tag{6}$$

$$\frac{\dot{R}_L}{R_L} = 2\frac{\dot{J}}{J} - 2\frac{\dot{M}_2}{M_2}\left(\frac{5}{6} - \frac{M_2}{M_1}\right) \tag{7}$$

In order to get the mass transfer rate, one needs to know the relation between $\dot{M}_2$ and $\dot{R}_2$; for a main sequence star, $\dot{M}_2/M_2 = \dot{R}_2/R_2$. Departure from thermal equilibrium must however be taken into account, which requires a mode detailed description analysis of the secondary.

The surface of a star is however not a well defined concept; matter extends above the photosphere, defined as the point where the optical depth is 2/3. Thus the prescription $R_2 = R_{\rm L}$ can only be a first order approximation, that is accurate to a few photospheric pressure scale height $H_{\rm p}$. It turns out that in the lower main sequence, $H_{\rm p}$ is a very small fraction (typically 10^{-4}) of the radius, so that the prescription $R_2 = R_{\rm L}$ is a very good approximation. This is not the case for systems in which the secondary is an evolved star; one would then require a more refined description of mass transfer. This is also the case if one is interested in short episodes of the secular evolution, during which the secondary radius does not vary by much more than a few scaleheight, as for example the turn on of mass transfer. The hydrodynamics of mass transfer have been studied in quite some details by Lubow and Shu (1975) who showed that the flow is approximately isothermal and reaches the sound velocity at the lagrangian point L_1. This enabled Ritter (1988) to determine the mass transfer rate as:

$$-\dot{M}_2 = \dot{M}_0 e^{-(R_2-R_L)/H_P} \tag{8}$$

where $\dot{M}_0$ is a quantity that depends of the photospheric temperature and density, as well as on the binary parameters.

19.3.2.1 Dynamical stability

A star subject to mass loss reacts on a short time scale according to:

$$\frac{\dot{R}_2}{R_2} = \xi_{\mathrm{ad}} \frac{\dot{M}_2}{M_2} \tag{9}$$

where ξ_{ad} is the adiabatic exponent of the star, defined solely by its structure ($\xi_{\mathrm{ad}} = -1/3$ for a fully convective star). The mass transfer rate is thus given by:

$$\frac{\dot{M}_2}{M_2} = \frac{2\dot{J}/J}{\xi_{\mathrm{ad}} + \frac{5}{3} - 2\frac{M_2}{M_1}} \tag{10}$$

It can be easily shown that mass transfer is stable only if the denominator in Eq. (10) is positive; otherwise, since $\dot{J}$ is negative (a binary system cannot gain angular momentum), the secondary mass would have to increase. If this is not the case, then mass transfer raises on a dynamical time scale, equal to the orbital period, leading to enormous mass transfer rates, much higher than what is observed in CVs. It is believed that, when mass transfer happens to be dynamically unstable, mass cannot be accreted onto the primary, and a common envelope forms.

The secondary mass must therefore be smaller than or approximately equal to the primary mass. As M_1 cannot exceed the Chandrasekhar mass, M_2 must be smaller than $1 - 1.4\ M_\odot$, and applying the mass-period relation derived in section 1.2.3 the case of main sequence stars, yields a maximum period of $10 - 15$ hr.

19.3.2.2 Thermal stability

In a similar way, it can be shown that the secondary is thermally stable against mass loss only if its mass in less than about the primary mass. If this were not the case, the mass transfer timescale would be of order of the Kelvin-Helmholtz time. This type of instability therefore leads to much weaker mass transfer rates; however, because the criteria for dynamical and thermal stability appear to be quite similar, it is not much of a surprise that most systems appear to be thermally stable (Ritter, 1985).

19.3.3 Angular momentum losses

The discussion in the previous section shows that angular momentum losses are required for mass transfer to occur at all, since Eq. (10) predicts $\dot{M}_2 = 0$ if $\dot{J} = 0$. Two mechanisms are invoked to account for the required $\dot{J}$.

19.3.3.1 Gravitational radiation

It was realized by Kraft et al. (1962) that gravitational radiation must play an important role in cataclysmic variables. Two massive bodies orbiting around each other emit gravitational radiation, just as charged particles would emit electromagnetic waves. Gravitational radiation carries aways angular momentum at a rate:

$$\frac{\dot{J}}{J} = -3.75 \times 10^{-9} M_1 M_2 (M_1 + M_2)^{1/3} P_{\rm hr}^{-8/3} \ \ {\rm yr}^{-1} \tag{11}$$

For orbital periods less than 2 hr, $\dot{J}$ is large enough to account for the observed mass transfer rates; however, for longer periods, this is not the case, and another mechanism must be efficient above the period gap.

19.3.3.2 Magnetic braking

Magnetic braking is the mechanism responsible for the slowing down of the rotation of isolated stars (Schatzman, 1962, 1965), and has been applied to cataclysmic variables by Verbunt and Zwaan (1981) who used the observed slowing down rate of isolated young G stars fitted by Skumanich (1972). Main sequence stars are magnetized, and loose mass via winds. These winds are coupled to the magnetic field, and co-rotate with the star, up to some radius $R_{\rm l}$ such as the energy density of the magnetic field does no longer exceed the kinetic energy of the wind. $\dot{J}$ is thus equal to $\dot{M}_{\rm w} R_{\rm l}^2 \Omega$, where $\dot{M}_{\rm w}$ is the wind mass loss rate, and Ω is the angular velocity of the star. In a binary system, there must also be some wind from the secondary, and the magnetic field must be stronger than in most isolated stars, since the rotation rate is much faster and the field is proportional to Ω. This mechanism brakes the rotation of the secondary, which co-rotates with the system because of tidal forces. The coupling of the secondary wind and magnetic field therefore leads to angular momentum losses from the binary system at a rate (Verbunt, 1984):

$$\frac{\dot{J}}{J} = -4 \times 10^{-9} \left(\frac{B}{300\text{G}}\right)^{4/3} \left(\frac{R_2}{R_\odot}\right)^{8/3} \left(\frac{\dot{M}_\text{w}}{10^{-10}\text{M}_\odot\text{yr}^{-1}}\right)^{1/3} \frac{(M_1 + M_2)^{1/3}}{M_1 M_2}\ \text{yr}^{-1} \qquad (12)$$

$\dot{J}$ may account for the observed mass transfer rates at periods larger than 3 hr if both the wind and the magnetic field are strong enough. Surface magnetic fields of the order of 300 G are expected in rapidly rotating stars (scaling the solar field to a rotation period of 3 hr precisely gives that value); strong winds are also expected in late type stars, which show intense coronal activity.

Other determinations of angular momentum losses via magnetic braking exist (see e.g. Mestel and Spruit, 1987 and Tout and Pringle, 1992); these are more realistic, but still lead to comparable results.

The angular momentum losses given by Eq. (12) is however too high for systems with periods less than 2 hr; this means that magnetic braking must either disappear or become inefficient below 3 hr. As can be noted from Eq. (4), the corresponding mass is of the order of 0.3 $M_\odot$, equal to the mass at which a main sequence star becomes fully convective. This coincidence lead to conjecture (Rappaport et al., 1983; Spruit and Ritter, 1983) that magnetic braking ceases when the secondary becomes fully convective, as in a fully convective star, magnetic field lines are not rooted in the radiative core, and the dynamo must have a different effect as compared to more massive stars. It must be emphasized at this point that this hypothesis is not based on very firm grounds, and that although most people agree that the angular momentum losses are strongly reduced when the orbital period reaches 3 hr, the disappearance of the secondary magnetic field for a fully convective star is questionable. It has for example been proposed that the secondary wind could be severely reduced at 3 hr (see e.g. Hameury et al., 1987), or that the magnetic field topology changes from a dipolar configuration to a higher multipole one when the secondary becomes fully convective (Taam and Spruit, 1989).

The main reason for the success of the disrupted magnetic braking hypothesis is that it explains in a natural way the period gap. As the characteristic time scale of mass transfer above 3 hr is only slightly larger than the Kelvin-Helmholtz time of the secondary, the secondary is bigger than what a main sequence would be. When $\dot{J}$ is strongly reduced, the secondary has time to adjust to the mass transfer, and joins the main sequence, therefore contracting. It detaches from the Roche lobe, and mass transfer stops. The systems now shrinks under the influence of gravitational radiation, until the Roche lobe catches the secondary surface, at which point mass transfer resumes.

Other explanations of the presence of a period gap have been put forward, such as modification of the nuclear burning processes due to ^{3}He mixing when the secondary becomes fully convective (D'Antona and Mazzitelli, 1982; Joss and Rappaport, 1983), but these are now much less favoured. It had also been proposed that a gap arises naturally because of the bimodal distribution of white dwarfs (He WD or CO WD), and the upper edge of the gap would then correspond to the minimum period of systems containing a CO white dwarf (Webbink, 1979; Paczyński and Sienkiewicz, 1983). Whereas the white dwarf composition is of importance, as will be discussed below, the required mass transfer rates required to get a minimum period at 3 hr seem to be excluded by observations.

19.3.4 Evolution of the primary: novae

Matter that accumulates at the surface of the white dwarf is unstable versus nuclear burning for a range of mass transfer rates corresponding to those observed in cataclysmic variables. After the thermonuclear explosion, a large amount of matter is ejected from the system. Novae explosion have two main effects on the evolution of the system. First, the total mass of the system does not remain constant; instead, novae observations as well as theoretical calculations show that as much mass is ejected as is accreted, so that the primary mass remains approximately constant (Kovetz and Prialnik, 1985; Truran and Livio, 1986). Another piece of evidence that M_1 remains constant comes from the period distribution of a subclass of CVs, the AM Her systems which have a strongly magnetic white dwarf rotating synchronously with the system. Hameury et al. (1989) have shown that this requires also that M_1 remains constant throughout the evolution of the system.

Ejection of matter leads to a small increase of the orbital separation of the system, and hence to some temporary reduction in the mass transfer rate. This effect, called the "hibernation scenario" (Shara et al., 1986) has been put forward to explain the fact that historical novae appear to have very low mass transfer rates. It does not however significantly affect the secular evolution of CVs, since the duration of the low mass transfer phase is much shorter than the interval between two novae explosions.

Another important effect of mass ejection during novae explosions is the interaction of the ejected matter and the secondary. This leads to angular momentum losses, which are quite difficult to calculate, especially since 2D computations are needed (Livio et al., 1990). A rough estimate leads to the conclusion that the effect could be comparable to angular momentum losses due to magnetic braking or gravitational radiation (McDonald, 1986; Livio et al., 1991). This effect has however not been taken into account in secular evolution calculations, simply because its magnitude is quite uncertain. This could be an important drawback in all "classical" models for CV evolution.

19.4 Numerical results of secular evolution

As we have seen, a simple analysis of mass transfer and angular momentum losses enables one to explain the basic characteristics of the orbital period distribution. The minimum period is due to the transition main sequence – degenerate sequence; the period gap results from the cessation of magnetic braking when the secondary becomes fully convective, i.e. when its mass is reduced to about 0.3 $M_\odot$, corresponding to an orbital period of 3 hr; the maximum period corresponds to the maximum secondary mass for dynamically stable mass transfer onto a white dwarf whose mass cannot exceed the Chandrasekhar limit. In order to proceed further, one requires a more detailed description of the internal structure of the secondary that takes into account departure from thermal equilibrium, so that one can get the secondary radius $R_2(M_2, t)$ as a function of both mass and time.

Two approaches have been used to estimate the thermal response of the secondary. Polytropic models (Rappaport et al., 1982, 1983; Hameury et al., 1987; Kolb and Ritter, 1992) in which the secondary is modelled as the superposition of two polytropes (one for the radiative core and the other for the convective envelope) have been largely used. The independent variables in these models are the mass and specific entropy of each polytropic component; one can therefore determine the radius variation as a function of the mass transfer rate and of the entropy variation, proportional to the luminosity excess. The mass transfer rate is then simply given by an equation similar to (7). These models must be calibrated, as they contain several free parameters, and are therefore not very accurate. The computing time is however severely reduces as compared to more detailed models, which enables the calculation of a large number of evolutionary sequences with different initial parameters. Detailed stellar models have also been used (Paczyński and Sienkiewicz, 1983; McDermott and Taam, 1989; D'Antona et al., 1989; Hameury, 1990). These are much more accurate, but quite slower; they are required for the calibration of polytropic codes. Contrary to polytropic models, these models require a refined prescription for the mass transfer rate, which is usually taken from Eq. (8). This prescription with a small value of H_p/R_2 makes the evolution codes quite sensible to numerical noise, and for this reason, larger values of H_p are sometimes taken (see e.g. Hameury 1990). This approximation merely affects the mass transfer turn-on and turn-off phases, as the main effect of this prescription for mass transfer is to keep R_L and R_2 close to within a few scaleheight.

19.4.1 Standard model

Figure 1.4 shows the evolution of a CV in the framework of what will be referred to as the standard model. The ingredients of this models are the following: the primary mass is 0.7 $M_\odot$, and is assumed to remain constant throughout the evolution; mass ejected during novae explosions is assumed to have the same specific angular momentum as that of the white dwarf. The initial secondary mass is 0.6 $M_\odot$. The magnetic braking law is taken from Mestel and Spruit (1987). Concerning the secondary structure, the ingredients are basically the same as in Dorman et al. (1989). The distortion of the secondary by tidal forces is neglected; this introduces a small systematic error: the minimum period for example is shifted by about 10 % (Nelson et al., 1985). The opacities are taken from Alexander (1975) for temperatures less than 10^4 K, and from Cox and Tabor (1976) at higher temperatures. The equation of state has been interpolated from the tables of Fontaine et al. (1977), and the nuclear reaction rates are taken from Harris et al. (1983) and Fowler et al. (1975), with screening corrections from Graboske et al. (1973). It must be noted that in short period systems, the ^{3}He(^{3}He,2p)^{4}He must be considered separately, since the central temperature is too low for nuclear equilibrium to be reached during the mass transfer timescale. Convection is treated by the mixing length theory, with $l/H_p = 1.5$. Finally, the scaled solar $T(\tau)$ relation of Krishna-Swamy (1966) is used in the photosphere, which accounts for the departure of the photosphere from a gray photosphere.

It is seen in Fig. 1.4 that the secondary is out of thermal equilibrium during the mass transfer phases, with a luminosity excess of about o factor 10. Similarly, ^{3}He is far from nuclear equilibrium; however, this has little consequences on the evolution, precisely because nuclear reactions are unimportant in determining the luminosity of the secondary when mass transfer is effective. Note that the difference between the actual and equilibrium helium abundances for the initial main sequence phase is due to the pres-

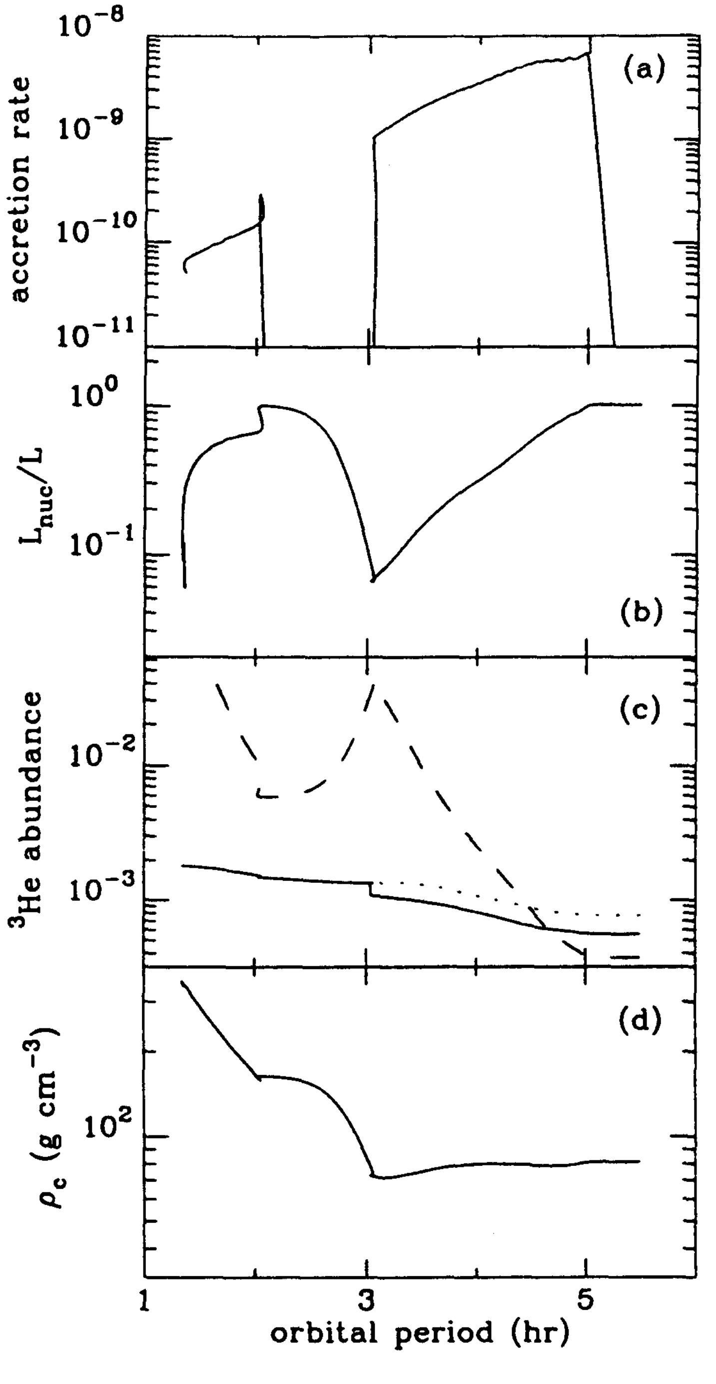

Fig. 19.4 Evolution of the standard model. Panel (a) shows the mass transfer rate, (b) the ratio of the nuclear versus total luminosity of the secondary, (c) the actual (solid line) and equilibrium (dashed line) ^{3}He abundance at the centre of the secondary, together with the average ^{3}He abundance (dotted line, and (d) the central density versus orbital period.

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Table 19.1. *Effects of the stellar parameters on binary evolution. The columns list the opacities, equation of state and boundary conditions used, the mixing length, the initial period P_i for which mass transfer starts, the upper and lower edge of the gap, P_u and P_l, and the minimum period P_{min}*

κ	EOS	BC	l/H_p	P_i	P_u	P_l	P_{min}
A	FGVH	NG	1.5	5.02	3.04	2.03	1.34
A	FGVH	NG	1.0	5.14	3.07	2.01	1.35
A	FGVH	NG	2.0	4.91	3.02	2.04	1.34
A	FGVH	G	1.5	5.23	3.51	1.92	1.44
A	P	NG	1.5	4.90	3.11	1.90	—
A	SC	NG	1.5	4.99	3.04	2.03	1.28
CS	FGVH	NG	1.5	4.77	2.83	2.07	< 1

ence of a small convective nucleus, so that He mixing leads to a different abundance from the local equilibrium value.

As can be seen, the predicted secular evolution accounts rather well for the observed period distribution. Hameury (1991) has investigated the influence of the assumed parameters on the position of both the minimum period and period gap, in order to determine whether one could constrain the modelisation of the secondary.

19.4.2 Effects of the stellar physics

Table 1.1 summarizes the main results concerning the position and width of the period gap, together with the minimum period, for various assumptions on the input stellar physics.

19.4.2.1 Equation of state

In order to evaluate the uncertainty on the period gap and minimum period that results from errors on the equation of state, evolution has been calculated using the EOS proposed by Paczyński (1969). The ionization fraction of each element is calculated using the Saha equation, and the dissociation of molecular hydrogen is calculated using van't Hoff's equation, as described in Vardya (1960), so that pressure ionization and Coulomb corrections are ignored. These assumptions are quite valid in the outer layers of the secondary, but totally fail in the interior. This is quite an extreme EOS, and still the difference with the standard model is not huge, except below the period gap. It turns out that the secondary never becomes degenerate, but

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becomes fully neutral when the central temperature becomes low. The use of a decent equation of state, such as that of Saumon and Chabrier (1991) inclluding the phase transition does not yield significant differences from the standard models.

19.4.2.2 Opacities

Table 1.1 shows that the opacities are the main source of error on the evolution. Using Cox and Stewart (1969) opacities instead those of Alexander (1975) results in a much shorter period gap, as well as a quite different minimum period, incompatible with observations. Cox and Stewart neglect the contributions of molecules such as H_2O, and their opacities are therefore largely underestimated for the temperatures and densities appropriate to the surface of red dwarfs. A major source of uncertainty is the formation and destruction of grains in convective atmospheres, as had been noted by Paczyński and Sienkiewicz (1983); this severely affect the position of the minimum period. The strong dependence of the secondary structure on opacities is not really a surprise, since the specific entropy of the convective envelope is determined by surface boundary conditions.

19.4.2.3 Treatment of the photosphere

As for the opacities, the treatment of the photosphere strongly affects the evolution of the system. If instead of using the scaled solar $T(\tau)$ of Krishna-Swamy (1966) one uses the simple gray atmosphere condition, $\kappa P = 2/3g$, in which κ is the opacity at the photosphere, defined at the position at which the optical depth is 2/3, P the pressure and g the gravity, quite significant changes are obtained for the position of the period gap. This implies that model atmosphere have to be constructed for low luminosity, low mass stars in order to get reliable evolutionary tracks. It should however be noted that the $T(\tau)$ relation used here is a fair approximation for low mass main sequence stars, as shown by VandenBerg et al. (1983), and that more accurate atmosphere models should not lead to important changes in the calculations presented here.

19.4.2.4 Convection

Changing the ratio of the mixing length and the pressure scale height only affects the superadiabatic layers, which, for the stars considered here, are not very important. Convection parameters have some importance only for relatively massive stars (above $0.5\,M_\odot$), in which the subphotospheric layers are not very dense and the luminosity is high, so that the departure from adiabaticity is more pronounced. The value of the minimum period is, as expected, independent of the mixing length value.

19.4.3 Influence of binary parameters

The evolution of a particular system depends indeed on the initial masses of the secondary and white dwarf, as well as on the magnetic braking law used. Because the exact determination of the mass transfer rate is impossible, the width of the period gap is a free parameter that is used to adjust the strength of magnetic braking. The position of the period gap is however not dependant on the braking low used, as shown by Hameury (1991), but is more directly related to the physics of the secondary. The initial secondary mass has little influence on the evolution, as long as it is sufficient for the secondary thermal disequilibrium to be large enough before crossing the period gap. This occurs for $M_2 > 0.4$ $M_\odot$. On the other hand, evolution strongly depends on the white dwarf mass, since $\dot{J}/J$ depends on M_1 both in the case of gravitational radiation and of magnetic braking. This means that an individual evolutionary track is not necessarily representative of the observed systems, but a weighted average must be performed, especially if one is interested in reproducing the orbital period distribution, and not only three characteristic parameters. This point will be discussed in more details in section 1.4.5.

19.4.4 Particular cases

As can be seen from Fig. 1.2, all cataclysmic variables do not follow the evolutionary scheme described here. Although not very numerous, there are systems below the minimum period and inside the period gap; there are also some long period systems. These must have followed some special evolution, that will be briefly discussed in the following.

19.4.4.1 Long period ($P_{\rm orb} > 10$ hr) systems: evolved secondaries

There are three systems with an orbital period longer than 15 hr: GK Per (48 hr), U Sco (30 hr), and V394 Cra (18.2 hr). These cannot obviously contain main sequence stars, since they would have a mass by far exceeding that of the white dwarf, and mass transfer would be unstable. The secondaries in long period systems are subgiant whose nuclear evolution drive the evolution of the system. Their evolution has been discussed by Webbink et al. (1983). The mass-period relation for these systems is given by (King,

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1988):

$$P = 16.5 \left(\frac{M_c}{0.25 M_\odot} \right)^{7.65} M_2^{-0.5} \text{ days} \tag{13}$$

where M_c is the mass of the helium core. It is seen that the period depends very sensitively upon M_c, and that the orbital period of these systems increase with time, as the helium core grows; long periods (up to a few days) can be obtained. This is confirmed by numerical models of the evolution of binaries containing an evolved secondary (Pylyser and Savonije, 1988a,b).

19.4.4.2 Ultrashort systems ($P_{orb} < 1$ hr) systems: He secondaries

Two systems have confirmed periods below 80 min: GP Com (46 min) and AM Cvn (18 min). These systems cannot have evolved through the sequence described above; they are too compact to contain H-rich main sequence or degenerate secondaries, but instead contain He stars that may or may not be degenerate. These systems have been studied in quite some details (see e.g. Rappaport and Joss, 1984; Tutukov et al., 1985; Nelson et al., 1986; Fedorova and Ergma, 1989). He rich secondaries have a much smaller radius than H-rich secondaries, and hence have shorter orbital periods. The formation of such systems is not well understood; they are clearly very rare.

19.4.4.3 Systems in the period gap

¿From the period histogram shown in Fig. 1.3, it can be seen that there are some systems inside the period gap. This is not really unexpected, since depending on the initial secondary mass, systems may form at periods between 2 and 3 hr. Except in very special cases, they are fully convective, and evolve down to the minimum period. It seems however that the period gap il less pronounced for a subclass of cataclysmic variables, the AM Her systems, that contain a strongly magnetic white dwarf rotating synchronously with the orbit. This will be further discussed in the next section.

19.4.4.4 The case of magnetic systems

Cataclysmic variables containing a magnetized white dwarf are divided into two subclasses: systems in which the interaction between the dipoles of the primary and secondary is strong enough to force co-rotation of the white dwarf with the orbit, the so-called AM Her systems or polars, and systems in which the white dwarf spin period is shorter than the orbital period (the DQ Her systems, or intermediate polars). Magnetic systems have received much attention, still many questions are left unanswered (see King, 1993 for a recent review), The fact that the average orbital period of AM Her systems is shorter than that of DQ Hers lead Chanmugam and Ray (1984) and King et al. (1985) to propose that intermediate polars must synchronize when the orbital separation has become small enough and evolve into polars. The issue is still controversial (see e.g. Lamb and Melia, 1987; Hameury et al., 1987), but it now appears that, although most AM Her systems must have been born as intermediate polars, most observed intermediate polars will never become synchronous.

The period distribution of AM Her systems have two interesting features: (1) there is an accumulation of systems at a period of 114 min, and (2) the period gap is much less pronounced for these systems. An explanation of (1) has been suggested by Hameury et al. (1988): as systems emerge from the gap, their mass transfer is high and they spend a long time at about that period (see Fig. 1.4). This imposes tight constraints on the white dwarf mass distribution in these systems, as well as on angular momentum losses (Hameury et al., 1988; Ritter and Kolb, 1992). It is however not clear whether the spike is real, or will disappear as the number of detected systems increases. Point (2) may indicate that there is mass transfer while AM Her systems evolve through the gap, and that magnetic braking is either not interrupted or never effective. Wu and Wickramasinghe (1993) and Wickramasinghe (1993) suggested that the strong white dwarf field would prevent open field lines from the secondary, and consequently magnetic braking would be impossible. On the other hand, Schmidt et al. (1986) and Frank et al. (1993) argued that the interaction of the secondary wind with the white dwarf magnetic field would lead to enhanced magnetic braking that would persist even after the secondary has become fully convective.

19.4.5 Predicted orbital distribution

As mentioned earlier, the evolutionary tracks obtained for different binary parameters must be convolved by the distribution of initial masses and periods to obtain the intrinsic orbital period distribution of cataclysmic variables. This has been done by Hameury et al. (1990) for AM Her systems only, and more recently by Kolb (1993) and Shafter (1992) for all systems. Both Kolb (1993) and Shafter (1992) used Politano's (1988, 1990) and/or de Kool (1992) differential formation rates of cataclysmic variables, but Shafter assumption that the secondary lies on the main sequence is extremely crude, whereas Kolb used a bi-polytrope code. Kolb (1993) main results are: (1) only 1% of CVs are located below the period gap; (2) the intrinsic period distribution depends weakly on the details of magnetic braking; realistic

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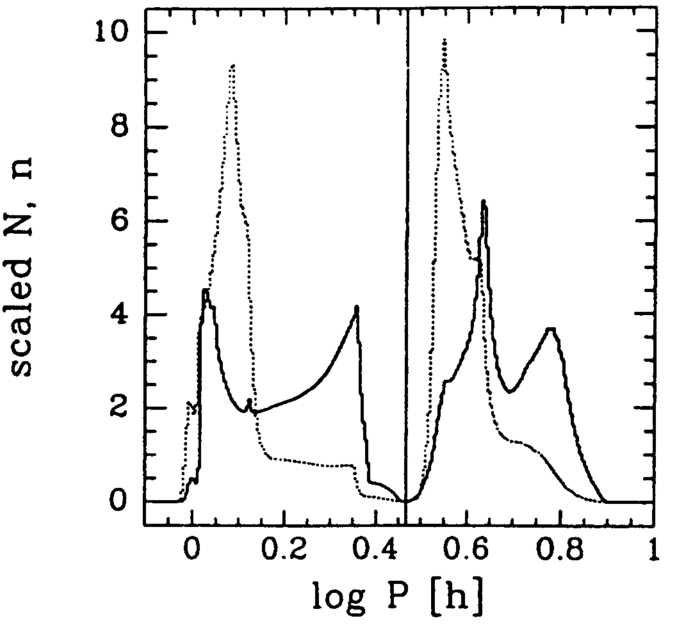

Fig. 19.5 Predicted intrinsic (dotted line) and observed (solid line) period distribution of cataclysmic variables (from Kolb, 1993).

braking laws from Mestel and Spruit (1987) or Verbunt and Zwaan (1981) lead to essentially the same results and (3) there is a much more significant dependence on the details of the stellar structure.

Once the intrinsic population of CVs is determined, it must be corrected for selection effects. These are quite difficult to estimate (see e.g. a discussion by Ritter and Burkert, 1986); in particular, they are different for each sub-class of cataclysmic variables, depending on how they are detected. Figure 1.5 shows one of Kolb's (1993) result in the case of a bolometric luminosity limited sample (i.e. the detection probability is proportional to the total luminosity to the power 3/2), which is not a very good approximation (most of the luminosity emitted by a CV is in the UV range, and hence undetectable). It is seen that this is in disagreement with observations, as far too many systems are found above the period gap, and that the predicted excess at the minimum period does not show up in the observed distribution. Assuming a visual magnitude limited sample would solve the first problem, whereas it is quite difficult to explain why one does not observe an accumulation of systems at the minimum period. It is possible that this is due to some unforeseen selection effect; it is nevertheless worth noting that the secondary mass is of the order of 0.05 $M_\odot$ at the minimum period, and that the stellar structure for those low masses is not well understood.

19.5 Conclusion

Models for the evolution of cataclysmic variables are relatively sensitive to the assumed stellar physics, the strongest dependance being that due to the opacities and the treatment of the photospheric layers. If one were to know with reasonable accuracy the angular momentum losses from these systems, one would be able to test stellar models for masses in the range 0.05 – 0.5 $M_\odot$. The presence of a more massive compact companion certainly helps in determining the characteristics of the secondary. In order to test the stellar models, one would also need to know the white dwarf mass in a larger number of systems; observational selection effects should also be better understood. The subclass of magnetic systems appear very promising from that point of view, even though the presence of strong magnetic field from the white dwarf introduces a further complication. The disagreement of the observed and predicted orbital period distribution for systems close to the minimum period is also quite interesting in that it is unlikely to be solely due to selection effects, and might therefore tell us something on the structure of very low mass stars.

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