P 79. Let $G = A \cup B$ be a finite group written as the union of two disjoint non-empty subsets, and let φ be a permutation of the elements of G. Prove that

$$A\varphi(B) = B\varphi(A) .$$

(For example, $AB = BA$ and $AB^{-1} = BA^{-1}$. This problem arose in conversation with G. Baumslag.)

H. Schwerdtfeger, McGill University

P 80. Among all partitions $n = m_1 + \ldots + m_k$ of n, which maximizes $m_1 m_2 \ldots m_k$?

A. Evans, McGill University

P 81. Denote by $G(n;m)$ a graph of n vertices and m edges. Prove that every $G(n; [n^2/4] + 1)$ contains at least $2[n/2] + 1$ edges which occur in some triangle of our graph. The result is sharp. (Turán proved that every $G(n; [n^2/4] + 1)$ contains at least one triangle, and Rademacher proved that it contains at least $[n/2]$ triangles.)

P. Erdős

P 82. (Conjecture) The Stirling Numbers S_n^m of the first kind are defined by

$$x(x-1)\ldots(x-n+1) = S_n^1 x + S_n^2 x^2 + \ldots + S_n^n x^n .$$

Prove that for fixed n,

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$$f(m) = |S^m_{n-1} / S^m_n|$$

is monotone decreasing in m.

G. P. Patil, McGill University

P 83. A group G is generated by a and x subject to the relations $x^2 = axa$ and x is of finite order $n \not\equiv 0$ mod 3. Show that G is Abelian.

N. Mendelsohn, University of Manitoba

P 84. Nagata Masayoshi and Matsumura Hideyuki (Sûgaku 13 (1961/62), p. 161; Math. Rev., Sept. 1963, p. 457) proved that if $1 \leq a_1 < a_2 < \ldots < a_n \leq 2n-1$ are n integers, then every $m > a_n$ can be written in the form

$$(1) \qquad \sum_{i=1}^{n} c_i a_i = m, \quad c_i \geq 0 \text{ integers.}$$

If $1 \leq a_1 < \ldots < a_n = 2n$ and $(a_1, a_2, \ldots, a_n) = 1$ prove that (1) holds for $m \geq 2n + 2$. Determine the exact bounds if $a_n = 2n + 1$.

More generally: Let $1 \leq a_1 < a_2 < \ldots < a_n = 2n + k$ and assume that $(a_1, a_2, \ldots, a_n) = 1$. Define $f(n, k)$ as the smallest integer so that every $m \geq f(n, k)$ can be written in the form (1). I have not determined $f(n, k)$ for $k \geq 2$; no doubt for each fixed k it can be done, but I do not see a general solution.

P. Erdős

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P 85. Denote by $f_1(n)$ the number of Abelian groups of order n and by $f_2(n)$ the number of semi-simple rings of order n (see I. G. Connell, this Bulletin 7(1964), 23-34. Prove that

$$\overline{\text{Lim}} \; \log f_i(n)/(\log n/\log\log n) = k_i \,, \quad i = 1, 2,$$

and determine the constants k_i.

P. Erdős

SOLUTIONS

P 6. (Conjecture). If $a_1 < a_2 < \dots$ is a sequence of positive integers with $a_n/a_{n+1} \to 1$, and if for every d, every residue class (mod d) is representable as the sum of distinct a's, then at most a finite number of positive integers are not representable as the sum of distinct a's.

P. Erdős

In its present generality the conjecture is false; this is shown by an example due to J. W. S. Cassels, On the representation of integers as the sums of distinct summands taken from a fixed set, Acta Sci. Math. Szeged 21(1960), 111-124 (Math. Rev. 24(1962), A103). See also P. Erdős, On the representation of large integers as sums of distinct summands taken from a fixed set, Acta Arith. 7(1961/62), 345-354 (Math. Rev. 26(1963), no. 2387).

P 27. Prove that

$$\sum_{n=1}^{\infty} \sum_{\substack{d \mid F_n \\ d>1}} d^{-1/2} < \infty \,, \quad F_n = 2^{2^n} + 1 \,.$$

P. Erdős

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