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ARTICLE

Analytical insights on a behavioral macroeconomic model

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Abstract

In this paper, we provide a detailed analytical treatment of the behavioral macroeconomic model by De Grauwe and Ji (2020 Structural reforms, animal spirits, and monetary policies. *European Economic Review* 124, 103395). Although the model's dynamics is governed by a high-dimensional nonlinear law of motion, we are able to derive necessary and sufficient conditions for the local asymptotic stability of its fundamental steady state. Specifically, we find that under the authors' baseline parameter setting, the fundamental steady state is locally asymptotically stable, implying that the dynamics of booms and busts only arise when exogenous shocks hit the system. However, we also identify conditions under which boom-bust dynamics emerge temporarily endogenously from within the model. By doing so, we may contribute to a deeper understanding of how booms and busts can arise in such a framework – insights that central banks can use to design more effective monetary policies.

Keywords: behavioral macroeconomics; heterogeneous expectations; bounded rationality; stability analysis

JEL classifications: E12; E32; C62

1. Introduction

In this paper, we provide a detailed analytical treatment of the behavioral macroeconomic model proposed by De Grauwe and Ji (2020). Unlike traditional DSGE models, De Grauwe and Ji (2020) assume that individuals face cognitive limitations that prevent them from fully comprehending the complexity of the economy. As a result, they rely on simple heuristics to guide their behavior, offering a more realistic representation of economic agents compared to models that assume perfect rationality. However, this does not imply that agents are irrational. On the contrary, they continuously evaluate the performance of these heuristics and adopt those that perform better. Since agents can employ different heuristics, heterogeneity emerges among them, and their interactions gives rise to endogenous business cycles – a characteristic absent in standard DSGE models. In traditional DSGE models, by contrast, business cycles are driven by exogenous shocks combined with the slow adjustment of wages and prices.

More specifically, in De Grauwe and Ji (2020), agents can choose between two expectation rules to forecast output and inflation. They endogenously update their choices based on past forecasting errors, demonstrating a willingness to learn from their mistakes. Since this approach explains the observed dynamics in output and inflation, it enhances our understanding of how booms and busts in economic activity may arise. In particular, De Grauwe and Ji (2020) show that the dynamics of their behavioral model is driven by correlations in beliefs, which, in turn, generate waves of optimism and pessimism. Similar versions of this powerful framework have been studied, for instance, in De Grauwe (2011), De Grauwe (2012), De Grauwe and Ji (2019a, b) and De Grauwe and Ji (2022). Other works considering heterogeneous expectations in a New Keynesian framework include Branch and McGough (2009, 2010), Branch and Evans (2011), Massaro (2013), Anufriev et al. (2013), Hommes and Lustenhouwer (2019), and Assenza et al. (2021). For a survey see, for instance, Hommes (2021), Branch and McGough (2018), or Franke and Westerhoff (2017).

Due to the endogenous switching between heterogeneous heuristics, De Grauwe's framework becomes highly nonlinear, making it difficult to derive analytical insights. For this reason, numerical methods have primarily been used to analyze its dynamics. However, De Grauwe and Ji (2020) provide the first analytical characterization of this framework. Unfortunately, they focus only on a special case – one that does not correspond to the parameter setting used to illustrate the model's dynamics. One important goal of this contribution is to provide a detailed analytical treatment of the full model. We find that the dynamics of De Grauwe and Ji (2020)'s model is driven by an 11-dimensional nonlinear law of motion. Nonetheless, we are able to characterize the fundamental steady state and derive conditions under which it is locally asymptotically stable. Surprisingly, for the parameter setting they consider, we find that the fundamental steady state is locally asymptotically stable. This implies that, in the absence of exogenous shocks, the model's dynamics can be characterized by fixed-point dynamics. As will become clear, booms and busts emerge due to exogenous shocks and the way these shocks are endogenously amplified by the model's nonlinear features.

Fortunately, we also identify conditions under which boom-bust dynamics emerge endogenously, at least temporarily. By identifying conditions that trigger endogenous business cycles, we may contribute to a deeper understanding of how booms and busts can arise from within the model – insights that central banks can use to design more effective monetary policies. Our results suggest that active monetary policy is crucial when the steady state becomes unstable. If, in such a situation, the model generates boom-bust dynamics and the central bank does not respond with sufficient strength, these boom-bust dynamics will further amplify and may explode. However, when the deterministic system is locally stable, the central bank may not need to implement an active monetary policy to ensure convergence toward equilibrium.

Of course, we are not the first to analytically investigate a New Keynesian framework with heterogeneous expectations and boundedly rational agents. For instance, Hommes and Lustenhouwer (2019) consider a very similar setup. However, they also analyze only a special case. In their setup, agents switch between the same two heuristics based on past forecasting errors, resulting in identical expectation-formation behavior. As a result, the core structure of the model is the same, and with some adjustments, one model can be transformed into the other, and vice versa. Therefore, my analysis may also help generalize their analytical results. While the results depend on the specific model setup and cannot be directly transferred, my analysis may serve as a roadmap for analytically studying such behavioral New Keynesian models.

The remainder of this paper is organized as follows. We will briefly review the model by De Grauwe and Ji (2020) in Section 2. In Section 3, we will derive the dynamical system of their deterministic model and provide an analytical characterization of the fundamental steady state and its local asymptotic stability. In Section 4, we show the model's stochastic dynamics and use our analytical results to explain the functioning of their model. Section 5 concludes our paper.

2. The behavioral model

In this section, we recap the behavioral model by De Grauwe and Ji (2020), which describes the interplay between an aggregate demand equation, an aggregate supply equation, and a Taylor rule. To be more precise, y_t represents the output gap at time t, while π_t and r_t denote the rate of inflation and the nominal interest rate, respectively. Aggregate demand is specified as

$$y_t = a_1 \tilde{E}_t y_{t+1} + (1 - a_1) y_{t-1} + a_2 (r_t - \tilde{E}_t \pi_{t+1}) + \nu_t, \tag{1}$$

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where $0 < a_1 < 1$, $a_2 < 0$ and $\tilde{E}_t$ is a heterogeneous expectations operator. Aggregate supply is assumed to be of the New Keynesian Phillips curve type, defined by

$$\pi_t = b_1 \tilde{E}_t \pi_{t+1} + (1 - b_1)\pi_{t-1} + b_2 y_t + \eta_t, \tag{2}$$

where $0 < b_1 < 1$ and $b_2 > 0$. Note that both equations contain a lagged dependent variable, capturing the fact that there is some inertia when consumers and producers optimize their decisions. Moreover, demand and supply shocks are described by the two error terms ν_t and η_t, respectively, which are assumed to be normally distributed with mean zero and a constant standard deviation, i.e., $\nu_t \sim \mathcal{N}(0, \sigma_\nu)$ and $\eta_t \sim \mathcal{N}(0, \sigma_\eta)$.

The central bank adjusts the nominal interest rate according to the following Taylor rule

$$r_t = d_3[\pi^T + d_1(\pi_t - \pi^T) + d_2 y_t] + (1 - d_3) r_{t-1} + u_t, \tag{3}$$

where $d_1, d_2 \geq 0$, $0 < d_3 < 1$ and π^T is the inflation target. Accordingly, the central bank raises r_t when the inflation rate observed exceeds the inflation target or when the output gap increases; parameters d_1 and d_2 measure the intensity at which it does inflation targeting and output stabilization, respectively. Parameter d_3 measures how strongly the central bank smooths the interest rate. Finally, interest rate shocks are captured by $u_t \sim \mathcal{N}(0, \sigma_u)$.

Since cognitive limitations prevent agents from forming rational expectations, De Grauwe and Ji (2020) assume that agents rely on simple heuristics to forecast the future. In particular, agents can choose between two types of forecasting rules to predict the future output gap. When agents follow the fundamental expectation rule, they use the steady-state value of the output gap (which is normalized at 0) as forecast, while they rely on the previously observed output gap when they choose the naive predictor. De Grauwe and Ji (2020) formalize the two forecasting rules by

$$\tilde{E}^f_t y_{t+1} = y^* = 0 \tag{4}$$

and

$$\tilde{E}^e_t y_{t+1} = y_{t-1}, \tag{5}$$

whose weighted average yield the market forecast, i.e.,

$$\tilde{E}_t y_{t+1} = \alpha_{f,t} \tilde{E}^f_t y_{t+1} + \alpha_{e,t} \tilde{E}^e_t y_{t+1} = \alpha_{e,t} y_{t-1}, \tag{6}$$

where $\alpha_{f,t}$ and $\alpha_{e,t}$ represent the market shares of agents using the fundamental and the naive heuristic, respectively. It is the case that

$$\alpha_{f,t} + \alpha_{e,t} = 1. \tag{7}$$

Agents switch between the two expectation rules with respect to a certain criterion of success. This is assumed to be the forecast performance (utility), determined by the negative (infinite) sum of past squared forecasting errors, i.e.,

$$U_{j,t} = -\sum_{k=1}^{\infty} \omega_k (y_{t-k} - \tilde{E}^j_{t-k-1} y_{t-k})^2, \quad j = f, e,$$

where $U_{f,t}$ and $U_{e,t}$ define the utilities of using the fundamental and naive rule at time t, respectively, and $\omega_k = (1 - \rho)\rho^k$ represents geometrically declining weights. Since they add up to unity, a mathematical reformulation is given by

$$U_{j,t} = \rho U_{j,t-1} - \rho(1 - \rho)(y_{t-1} - \tilde{E}^j_{t-2} y_{t-1})^2, \quad j = f, e, \tag{8}$$

where $0 < \rho < 1$ can be interpreted as a memory parameter.[1] Accordingly, utility measures depend more strongly on past forecasting errors if ρ increases.

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Following Brock and Hommes (1997, 1998), these utilities are evaluated by applying the multinomial discrete choice model by Manski and McFadden (1981). Thus, the market shares of the fundamental and naive forecasting rule evolve according to

$$\alpha_{j,t} = \frac{\exp(\gamma U_{j,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})}, \quad j = f, e, \tag{9}$$

where $\gamma \geq 0$ is the intensity of choice. Accordingly, the greater the past forecast performance of a forecasting rule, the more agents will follow it. Note that γ can also be interpreted as agents' willingness to learn from past performance. For $\gamma = 0$, this willingness is zero, and both market shares will equal $1/2$. For $\gamma = \infty$, all agents are willing to switch to the forecasting rule with the greater utility, i.e., its market share is given by one.

De Grauwe and Ji (2020) assume that agents can also choose between two simple heuristics when they forecast future inflation. Similarly, they can either use a fundamental or a naive forecasting rule. When relying on the fundamental rule, agents believe in the inflation target announced by the central bank, and use it as their forecast. Agents who do not trust the inflation target rely on the naive predictor, and use the last observed value of inflation. This can be expressed formally by

$$\tilde{E}_t^f \pi_{t+1} = \pi^T \tag{10}$$

and

$$\tilde{E}_t^e \pi_{t+1} = \pi_{t-1}, \tag{11}$$

whose weighted average again yields the market forecast, i.e.,

$$\tilde{E}_t \pi_{t+1} = \beta_{f,t} \tilde{E}_t^f \pi_{t+1} + \beta_{e,t} \tilde{E}_t^e \pi_{t+1} = \beta_{f,t} \pi^T + \beta_{e,t} \pi_{t-1}, \tag{12}$$

where

$$\beta_{f,t} + \beta_{e,t} = 1. \tag{13}$$

The market shares of agents who trust the inflation target and those who do not are updated according to the same mechanism as in the case of output forecasting. We therefore have that

$$\beta_{j,t} = \frac{\exp(\gamma A_{j,t})}{\exp(\gamma A_{f,t}) + \exp(\gamma A_{e,t})}, \quad j = f, e, \tag{14}$$

where

$$A_{j,t} = \rho A_{j,t-1} - \rho(1-\rho)(\pi_{t-1} - \tilde{E}_{t-2}^j \pi_{t-1})^2, \quad j = f, e, \tag{15}$$

defining the forecast performances realized by the fundamental and naive rule, respectively. Thus, if relying on the announced inflation target yields the better forecast, the market share of agents using it will increase. However, if it turns out not to be very accurate, more and more agents will switch to the naive rule to forecast inflation.

3. Analytical results

In this section, we analyze the underlying deterministic framework of the model by De Grauwe and Ji (2020), and characterize the fundamental steady state. In doing so, we also derive analytical conditions for its local asymptotic stability. While De Grauwe and Ji (2020) characterize only a special case of their model analytically, we provide an analysis of the full model. As we will see, analytical insights are not precluded, although the model dynamics is due to a high-dimensional nonlinear law of motion. Moreover, our analysis will reveal a number of surprising results with respect to the functioning of the model by De Grauwe and Ji (2020).

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3.1 Dynamical system

To derive the dynamical system of the deterministic model, we first drop all random variables, i.e., we set $\sigma_\nu = \sigma_\eta = \sigma_u = 0$. Moreover, we follow De Grauwe and Ji (2019b) and normalize the model by expressing inflation rates and interest rates as deviations from the inflation target π^T. Thus, we introduce $\hat{\pi}_t = \pi_t - \pi^T$, $\hat{r}_t = r_t - \pi^T$ and $\tilde{E}_t\hat{\pi}_{t+1} = \tilde{E}_t\pi_{t+1} - \pi^T$. Equations (1)–(3) then become

$$y_t = a_1\tilde{E}_t y_{t+1} + (1-a_1)y_{t-1} + a_2(\hat{r}_t - \tilde{E}_t\hat{\pi}_{t+1}), \tag{16}$$

$$\hat{\pi}_t = b_1\tilde{E}_t\hat{\pi}_{t+1} + (1-b_1)\hat{\pi}_{t-1} + b_2 y_t, \tag{17}$$

and

$$\hat{r}_t = c_1\hat{\pi}_t + c_2 y_t + c_3\hat{r}_{t-1}, \tag{18}$$

where $c_1 = d_1d_3$, $c_2 = d_2d_3$, and $c_3 = 1 - d_3$. Furthermore, we get

$$\tilde{E}_t^f\hat{\pi}_{t+1} = 0 \tag{19}$$

and

$$\tilde{E}_t^e\hat{\pi}_{t+1} = \pi_{t-1} - \pi^T, \tag{20}$$

yielding

$$\tilde{E}_t\hat{\pi}_{t+1} = \beta_{e,t}\hat{\pi}_{t-1} \tag{21}$$

and

$$A_{j,t} = \rho A_{j,t-1} - \rho(1-\rho)(\hat{\pi}_{t-1} - \tilde{E}_{t-2}^j\hat{\pi}_{t-1}), \quad j = f, e. \tag{22}$$

Taking into account also equations (6), (8), (9), and (14), and by introducing the auxiliary variables $z_t = y_{t-1}$, $x_t = z_{t-1}$, $p_t = \hat{\pi}_{t-1}$, and $q_t = p_{t-1}$, we are able to summarize the model by De Grauwe and Ji (2020) by the following 11-D nonlinear discrete dynamical system

$$S: \begin{cases} y_t = \dfrac{a_1\alpha_{e,t}y_{t-1} + (1-a_1)y_{t-1} - a_2\beta_{e,t}\hat{\pi}_{t-1}(1-c_1b_1) + a_2c_1(1-b_1)\hat{\pi}_{t-1} + a_2c_3\hat{r}_{t-1}}{1 - a_2c_2 - a_2c_1b_2} \\ \hat{\pi}_t = b_1\beta_{e,t}\hat{\pi}_{t-1} + (1-b_1)\hat{\pi}_{t-1} + b_2y_t \\ \hat{r}_t = c_1\hat{\pi}_t + c_2y_t + c_3\hat{r}_{t-1} \\ U_{e,t} = \rho U_{e,t-1} - \rho(1-\rho)(y_{t-1} - x_{t-1})^2 \\ U_{f,t} = \rho U_{f,t-1} - \rho(1-\rho)(y_{t-1})^2 \\ A_{e,t} = \rho A_{e,t-1} - \rho(1-\rho)(\hat{\pi}_{t-1} - q_{t-1})^2 \\ A_{f,t} = \rho A_{f,t-1} - \rho(1-\rho)(\hat{\pi}_{t-1})^2 \\ z_t = y_{t-1} \\ x_t = z_{t-1} \\ p_t = \hat{\pi}_{t-1} \\ q_t = p_{t-1} \end{cases}, \tag{23}$$

where

$$\alpha_{e,t} = \frac{\exp(\gamma U_{e,t})}{\exp(\gamma U_{f,t}) + \exp(\gamma U_{e,t})}$$

and

$$\beta_{e,t} = \frac{\exp(\gamma A_{e,t})}{\exp(\gamma A_{f,t}) + \exp(\gamma A_{e,t})}.$$

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Accordingly, the model dynamics is driven by the iteration of map S, which describes the state of the system at time t, defined by y_t, $\hat{\pi}_t$, $\hat{r}_t$, $U_{e,t}$, $U_{f,t}$, $A_{e,t}$, $A_{f,t}$, z_t, x_t, p_t, and q_t, as a function of the state of the system at time $t-1$, i.e., y_{t-1}, $\hat{\pi}_{t-1}$, $\hat{r}_{t-1}$, $U_{e,t-1}$, $U_{f,t-1}$, $A_{e,t-1}$, $A_{f,t-1}$, z_{t-1}, x_{t-1}, p_{t-1}, and q_{t-1}.[2]

3.2 Steady state and local asymptotic stability

We are now ready to explore the existence of a steady state, which can be done by looking for a constant solution to the dynamical system S. Proposition 1 states that a steady state exists at which we have $\pi^* = \pi^T$. Since the central bank aims to keep inflation at its target rate, we call it the fundamental steady state.

Proposition 1. *The model's dynamical system (23) gives rise to the fundamental steady state* $s^* = (y^*, \hat{\pi}^*, \hat{r}^*, U_e^*, U_f^*, A_e^*, A_f^*, z^*, x^*, p^*, q^*) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) = 0$, *implying that* $\pi^* = \pi^T$.

Proof. When imposing the fact that expectations are realized at the steady state, i.e., $\tilde{E}y^* = y^*$ and $\tilde{E}\pi^* = \pi^*$, it follows from (16) and (17) that a steady state must satisfy

$$y^* = a_1 y^* + (1 - a_1) y^* + a_2(\hat{r}^* - \hat{\pi}^*)$$

and

$$\hat{\pi}^* = b_1 \hat{\pi}^* + (1 - b_1)\hat{\pi}^* + b_2 y^*,$$

yielding $\hat{r}^* = \hat{\pi}^*$ and $y^* = 0$, respectively. Then, the Taylor rule

$$\hat{r}^* = c_1 \hat{\pi}^* + c_2 y^* + c_3 \hat{r}^*$$

can be solved for $\hat{\pi}^* = \hat{r}^* = 0$, implying that $\pi^* = \pi^T$. It follows that $p^* = q^* = 0$, $z^* = x^* = 0$, $U_e^* = A_e^* = 0$, $U_f^* = -\rho y^{*2} = 0$, and $A_f^* = -\rho \hat{\pi}^{*2} = 0$. A steady-state solution is therefore given by $s^* = (y^*, \hat{\pi}^*, \hat{r}^*, U_e^*, U_f^*, A_e^*, A_f^*, z^*, x^*, p^*, q^*) = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0) = 0$.

Note that $\pi^* = \pi^T$ implies that the steady state coincides with expectations of the fundamental rule. However, the naive heuristic also generates perfect forecasts at the steady state. Thus, both predictors yield the same forecast, and the difference in forecasting performance is zero. Consequently, half of the agents rely on the fundamental heuristic and the other half on the naive heuristic to forecast inflation, i.e., $\beta_f^* = \beta_e^* = 0.5$. The same is true for the two output forecasting rules. Since $y^* = 0$, both the fundamental and the naive expectation rule yield a perfect forecast, resulting again in the same forecasting performance, and therefore in a uniform distribution among agents, i.e., $\alpha_f^* = \alpha_e^* = 0.5$. However, the dynamical system (23) may also give rise to further nonfundamental steady states at which $\tilde{E}y^* \neq y^*$ and $\tilde{E}\pi^* \neq \pi^*$.[3]

A key contribution of our paper is to provide a (complete) local stability analysis of the fundamental steady state for the model by De Grauwe and Ji (2020). The results are summarized by the following proposition.

Proposition 2. *The fundamental steady state* $s^* = 0$ *is locally asymptotically stable if and only if*

$$c_1 > \frac{a_1 b_1 (1 - c_3) + 2a_2(b_2(1 - c_3) - b_1 c_2)}{4a_2 b_2}$$

and

$$1 - \frac{(-2 + a_1)(-2 + b_1)(-4 + a_1 + b_1 + a_2 b_2 + 2a_2 c_2 - a_2 b_1 c_2 - 2c_3)c_3}{8(-1 + a_2(b_2 c_1 + c_2))^2}$$
$$> \frac{(-2 + a_1)^2(-2 + b_1)^2 c_3^2}{(4 - 4a_2(b_2 c_1 + c_2))^2} - \frac{(-2 + a_1)(-2 + b_1) - 2(-4 + a_1 + b_1 + a_2 b_2)c_3}{-4 + 4a_2(b_2 c_1 + c_2)}$$

are simultaneously satisfied.

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Proof. We take the dynamic variables in the same order as they appear in (23), and find that the Jacobian matrix, computed at the fundamental steady state, has the following block structure

$$J(s^*) = \begin{bmatrix} H_1 & 0_{(3,4)} & 0_{(3,4)} \\ 0_{(4,3)} & \rho I_4 & 0_{(4,4)} \\ H_2 & 0_{(4,4)} & H_3 \end{bmatrix}, \tag{24}$$

where matrix H_1 is given by

$$H_1 = \begin{bmatrix} \frac{-2+a_1}{-2+2a_2(b_2c_1+c_2)} & \frac{a_2+a_2(-2+b_1)c_1}{-2+2a_2(b_2c_1+c_2)} & -\frac{a_2c_3}{-1+a_2(b_2c_1+c_2)} \\ \frac{(-2+a_1)b_2}{-2+2a_2(b_2c_1+c_2)} & \frac{-2+b_1+a_2b_2-a_2(-2+b_1)c_2}{-2+2a_2(b_2c_1+c_2)} & -\frac{a_2b_2c_3}{-1+a_2(b_2c_1+c_2)} \\ \frac{(-2+a_1)(b_2c_1+c_2)}{-2+2a_2(b_2c_1+c_2)} & \frac{(-2+b_1+a_2b_2)c_1+a_2c_2}{-2+2a_2(b_2c_1+c_2)} & -\frac{c_3}{-1+a_2(b_2c_1+c_2)} \end{bmatrix},$$

$0_{(m,n)}$ denotes the null (m, n) matrix, pI_4 is the 4-D identity matrix multiplied by the memory parameter, i.e.,

$$\rho I_4 = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & \rho & 0 & 0 \\ 0 & 0 & \rho & 0 \\ 0 & 0 & 0 & \rho \end{bmatrix},$$

and

$$H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad H_3 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Since $J(s^*)$ is a lower triangular block matrix, the eigenvalues can be determined by computing the eigenvalues of blocks H_3, ρI_4, and H_1 separately. It immediately follows that four eigenvalues are equal to 0, and four are given by ρ. Due to $0 < \rho < 1$, they are smaller than one in absolute value. The remaining three eigenvalues, say λ_1, λ_2, and λ_3, are the ones of block H_1, from which we get the characteristic polynomial $P_{H_1}(\lambda) = \lambda^3 + \kappa_1\lambda^2 + \kappa_2\lambda + \kappa_3$, where $\kappa_1 = -\frac{-4+a_1+b_1+a_2b_2+2a_2c_2-a_2b_1c_2-2c_3}{-2+2a_2(b_2c_1+c_2)}$, $\kappa_2 = \frac{-((-2+a_1)(-2+b_1))+2(-4+a_1+b_1+a_2b_2)c_3}{-4+4a_2(b_2c_1+c_2)}$ and $\kappa_3 = \frac{(-2+a_1)(-2+b_1)c_3}{-4+4a_2(b_2c_1+c_2)}$. A set of necessary and sufficient conditions to have all roots of a cubic polynomial inside the unit circle is given by (see Gardini et al. (2021)):

(i) $1 + \kappa_1 + \kappa_2 + \kappa_3 > 0$

(ii) $1 - \kappa_1 + \kappa_2 - \kappa_3 > 0$

(iii) $1 - \kappa_2 + \kappa_1\kappa_3 - \kappa_3^2 > 0$

(iv) $|\kappa_3| < 1$

Since we have $0 < a_1, b_1 < 1$, $a_2 < 0$, $b_2, c_1(= d_1d_3), c_2(= d_2d_3) > 0$ and $0 < c_3(= 1 - d_3) < 1$, the denominator of κ_3 is in absolute terms always greater than 4, while its numerator is always smaller than 4. Condition (iv) therefore always holds, and we consider only the three remaining conditions. From condition (*ii*), however, we obtain $-16 - 16c_3 - a_1b_1 - a_1b_1c_3 + 4a_1 + 4b_1 +$

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$4a_1c_3 + 4b_1c_3 + 2a_2b_2(1 + c_3) + 2a_2c_2(4 - b_1) + 4a_2b_2c_1 < 0$, which, given the model's parameter restrictions, is always fulfilled. This can be easily seen as $a_2 < 0$, $-16 - 16c_3 - a_1b_1 - a_1b_1c_3 < -16$ and $4a_1 + 4b_1 + 4a_1c_3 + 4b_1c_3 < 16$. Thus, the fundamental steady state $s^* = 0$ is locally asymptotically stable if (*i*) and (*iii*) hold simultaneously, i.e., if and only if

$$c_1 > \frac{a_1b_1(1-c_3) + 2a_2(b_2(1-c_3) - b_1c_2)}{4a_2b_2} \tag{25}$$

and

$$\begin{aligned} &1 - \frac{(-2+a_1)(-2+b_1)(-4+a_1+b_1+a_2b_2+2a_2c_2-a_2b_1c_2-2c_3)c_3}{8(-1+a_2(b_2c_1+c_2))^2} \\ &\quad > \frac{(-2+a_1)^2(-2+b_1)^2c_3^2}{(4-4a_2(b_2c_1+c_2))^2} - \frac{(-2+a_1)(-2+b_1)-2(-4+a_1+b_1+a_2b_2)c_3}{-4+4a_2(b_2c_1+c_2)}. \end{aligned} \tag{26}$$

As shown above, the local stability analysis is performed by determining the eigenvalues of the Jacobian matrix, evaluated at the fundamental steady state. A sufficient condition for the local stability of a fixed point is that all eigenvalues are inside the unit circle in the complex plane, i.e., smaller than one in modulus. Due to the particular structure of (24), eight out of the 11 eigenvalues can be determined immediately. Among these eight eigenvalues, four are equal to zero, while the remaining four are given by $0 < \rho < 1$. If stability conditions (25) and (26) are simultaneously satisfied, the remaining three eigenvalues also lie inside the unit circle, and the fundamental steady state is locally asymptotically stable. However, as soon as one of the two conditions is violated, at least one eigenvalue crosses the unit circle, and the steady state becomes unstable. If inequality (26) is violated, while (25) is satisfied, we have one real eigenvalue inside the unit circle and a pair of complex eigenvalues that is larger than one in absolute value (see Gardini et al. (2021)). In such a situation, model dynamics may explode cyclically.[4] When (25) gets violated and (26) holds, we have one real eigenvalue that becomes larger than +1, while the other two (complex or real) lie inside the unit circle. In this case, model dynamics may explode monotonically.[5] Now suppose the central bank faces a situation in which at least one of these conditions gets violated. Then, it can try to re-establish the stability of the steady state by adjusting the parameters of its monetary policy rule. As demonstrated in the next section, achieving steady-state stability depends on dampening self-fulfilling waves of optimism and pessimism that drive the economy away from equilibrium.

4. Model dynamics

Before we illustrate our analytical results and the dynamical behavior of the deterministic model, we begin this section by visualizing the stochastic dynamics of the model. In doing so, we rely on the same parameter setting as in De Grauwe and Ji (2020). Note, however, that we have adjusted parameter ρ (see Footnote 1). The parameters are $a_1 = 0.5$, $a_2 = -0.2$, $b_1 = 0.5$, $b_2 = 0.05$, $d_1 = 1.5$, $d_2 = 0.5$, and $d_3 = 0.5$, implying that $c_1 = d_1d_3 = 0.75$, $c_2 = d_2d_3 = 0.25$, and $c_3 = 1 - d_3 = 0.5$, $\gamma = 2$, and $\rho = 0.65$.[6] Moreover, the standard deviations of the three shocks are equal to 0.5.

4.1 Stochastic dynamics

For reasons of illustration, we follow De Grauwe and Ji (2020) and first introduce an index of market sentiments, reflecting how optimistic or pessimistic agents' forecasts are. These animal spirits are defined by

$$S_t = \begin{cases} \alpha_{e,t} - \alpha_{f,t} = \alpha_{e,t} - (1-\alpha_{e,t}) = 2\alpha_{e,t} - 1 & \text{if } y_{t-1} > 0 \\ -\alpha_{e,t} + \alpha_{f,t} = -\alpha_{e,t} + (1-\alpha_{e,t}) = -2\alpha_{e,t} + 1 & \text{if } y_{t-1} < 0 \end{cases},$$

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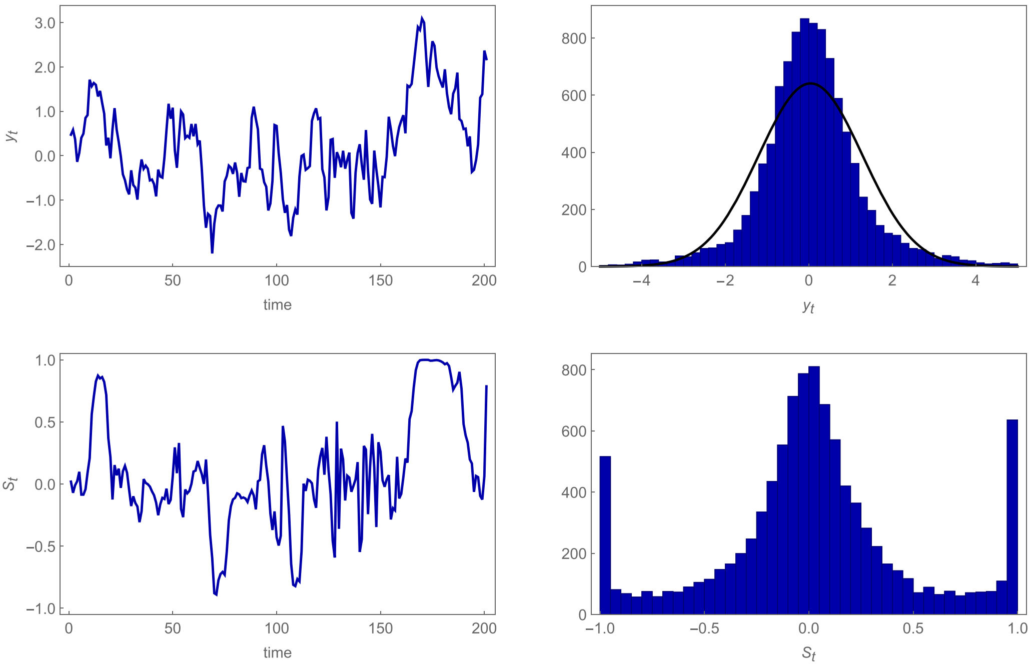

Figure 1. Stochastic dynamics. The panels on the left-hand side show movements of the output gap and animal spirits for a sample of 200 periods, while their frequency distributions are presented on the right-hand for the full 10, 000 periods. The parameters are $a_1 = 0.5$, $a_2 = -0.2$, $b_1 = 0.5$, $b_2 = 0.05$, $d_1 = 1.5$, $d_2 = 0.5$, and $d_3 = 0.5$, implying $c_1 = d_1d_3 = 0.75$, $c_2 = d_2d_3 = 0.25$ and $c_3 = 1 - d_3 = 0.5$, $\gamma = 2$, $\rho = 0.65$, and $\sigma_v = \sigma_\eta = \sigma_u = 0.5$.

where $S_t \in [-1, 1]$. Let us assume the last period's output gap is positive (negative). If this is the case, we have that agents who rely on the naive predictor forecast a positive (negative) output gap, while those using the fundamental heuristic make a pessimistic (optimistic) forecast. The market sentiment can be captured by subtracting the fraction of pessimists from the fraction of optimists, i.e., $\alpha_{e,t} - \alpha_{f,t}$ if $y_{t-1} > 0$ and $\alpha_{f,t} - \alpha_{e,t}$ if $y_{t-1} < 0$. Of course, index S_t becomes positive (negative) if the fraction of optimists (pessimists) exceeds the fraction of pessimists (optimists), and is equal to 0 if $\alpha_{e,t} = \alpha_{f,t} = 0.5$.

In Figure 1, we depict a representative simulation run based on 10, 000 observations. The panels on the left-hand side show movements of the output gap and animal spirits for a sample of 200 periods, respectively. Since the model by De Grauwe and Ji (2020) was calibrated such that time units can be considered to be quarters, 200 periods correspond to a time span of about 50 years. As can be seen, the output gap shows a strong cyclical movement in the time domain. These movements of output are triggered by the waves of optimism and pessimism depicted in the lower left-hand side panel. If optimists dominate, we observe a positive output gap, while the output is below its equilibrium when pessimists dominate. Between observations 160 and 180, we even observe a situation where all agents extrapolate a positive output gap, i.e., S_t becomes equal to +1. Obviously, the output gap shows extreme positive movements during this period. The high correlation between these two variables can be explained by the self-fulfilling nature of expectations. When a wave of optimism (pessimism) is set in motion, aggregate demand increases (decreases), and those who made a optimistic (pessimistic) forecast benefit as their forecasting rule performs better. As a result, more and more agents switch to being optimists (pessimists) by which, in turn, aggregate demand is stimulated (dampened) and a boom (bust) may be created.

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The right-hand panels show the output gap and animal spirits in the frequency domain for the full 10, 000 periods. In the upper panel, we compare the frequency distribution of the output gap to the one of a normal distribution with identical mean and standard deviation, represented by the black line. As can be seen, the output gap is not normally distributed. De Grauwe and Ji (2020) find a higher concentration around the mean, thinner shoulders, and more probability mass in the tails. This non-normality can be explained by the lower panel. While the distribution of the animal spirits shows a high concentration around 0, we also observe a concentration of observations at the extreme values of $+1$ and -1. As extreme optimism (pessimism) creates extreme positive (negative) movements, the distribution of the output gap reveals fat tails.

4.2 Deterministic dynamics

As demonstrated above, the model proposed by De Grauwe and Ji (2020) is able to replicate several key empirical statistical properties, and the simulated data closely resembles the dynamics observed in the real world. However, the model's dynamics depends strongly on exogenous shocks affecting the economy. Specifically, three different shocks, each with a standard deviation of 0.5, hit the system in every period. Given the amplitude of the dynamics, these shocks are relatively large.

Now, we set $\sigma_\nu = \sigma_\nu = \sigma_u = 0$ and illustrate the dynamics in the time domain in Figure 2. From top to bottom, the panels show the evolution of the output gap, the inflation rate, and the interest rate for 50 periods, respectively. At $t = 0$, the dynamics is hit by a single exogenous demand shock of 0.5. As can be seen, all three state variables quickly converge towards their equilibrium. The reason for this is provided by our analytical results. For the given parameter setting, the characteristic polynomial $P_{H_1}(\lambda)$ is given by $\lambda^3 - 1.93144\lambda^2 + 1.2435\lambda - 0.265957$, from which we obtain $\lambda_1 = 0.55$ and $\lambda_{2,3} = 0.69 \pm 0.08i$. Since $|\lambda_{1,2,3}| < 1$, the fundamental steady state is locally stable. Moreover, we have two eigenvalues that are complex, which is why the adjustment towards the steady state is cyclical. If no further shock hits the dynamics, the economy remains at its steady state.

Now, suppose a series of random shocks hits the dynamics, initiating a wave of optimism or pessimism. Once such a wave is set in motion, the nonlinear features of the model kick in and amplify these shocks. Since the shocks are i.i.d, it is the model's nonlinear forces due to which we find non-normality in the distribution of the output gap. In this way, the model transforms normally distributed shocks – without any boom-bust characteristics – into dynamics of booms and busts. However, without any of these exogenous shocks, the dynamics can be characterized by fixed-point dynamics.

Interestingly, the fundamental steady state would also be stable if the central bank were inactive, i.e., if $d_1 = 0$ and $d_2 = 0$, implying $c_1 = 0$ and $c_2 = 0$. In this case, the characteristic polynomial of matrix H_1 would be $P_{H_1}(\lambda) = \lambda^3 - 2.005\lambda^2 + 1.315\lambda - 0.28125$, yielding the eigenvalues $\lambda_1 = 0.5$, $\lambda_2 = 0.69$, and $\lambda_3 = 0.81$. Since all eigenvalues lie inside the unit circle, the system would still converge to the steady state, even if there were no inflation targeting and no output stabilization. This result differs from the findings of standard DSGE models. Woodford (2003) and Galí (2008), for instance, show that the inflation parameter d_1 must exceed one to ensure stability of the model. However, other New Keynesian models with heterogeneous expectations and boundedly rational agents – such as those proposed by Hommes and Lustenhouwer (2019) and Salle et al. (2013) – also suggest that convergence to the steady state can be ensured even if the Taylor Principle is not satisfied. Interestingly, Angeletos and Lian (2023) arrive at a similar result. They translate a dynamic game among consumers into the standard representative agents New Keynesian model and assume that social memory is imperfect. Under this perturbation, they demonstrate that the equilibrium is unique and given by the fundamental solution, regardless of monetary policy.

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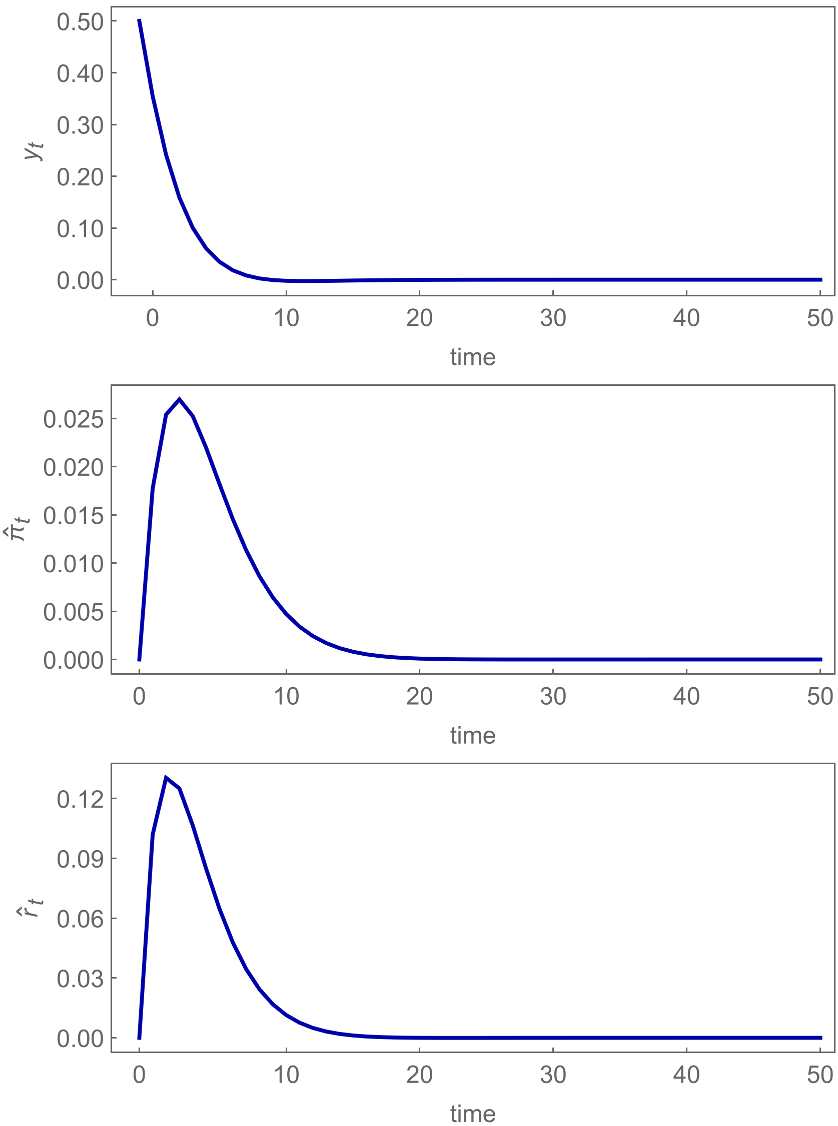

Figure 2. Deterministic dynamics for the base parameter setting. The panels show, from top to bottom, the evolution of the output gap, the inflation rate, and the interest rate for 50 periods, respectively. The parameters are $a_1 = 0.5$, $a_2 = -0.2$, $b_1 = 0.5$, $b_2 = 0.05$, $d_1 = 1.5$, $d_2 = 0.5$, and $d_3 = 0.5$, implying $c_1 = d_1 d_3 = 0.75$, $c_2 = d_2 d_3 = 0.25$ and $c_3 = 1 - d_3 = 0.5$, $\gamma = 2$, $\rho = 0.65$, and $\sigma_\nu = \sigma_\eta = \sigma_u = 0$.

This raises the question: why does this result occur in the model we consider? To answer this, let us consider the example described above where $c_1 = 0$ and $c_2 = 0$. The fundamental steady state is locally stable, implying that half of the agents rely on the fundamental predictor and the other half on the naive predictor, i.e., $\alpha_f^* = \alpha_e^* = 0.5$. Then, a positive demand shock occurs, causing the output gap to increase. As the output gap moves away from equilibrium, the naive heuristic, making a positive forecast, becomes more attractive. However, agents update their choice of heuristic with a time delay of three periods. Thus, at the time of the shock, the fraction of those relying on the fundamental predictor remains at 50 percent, and their mean-reverting power is strong enough to push the output gap back towards its equilibrium value. Once agents adjust their choices,

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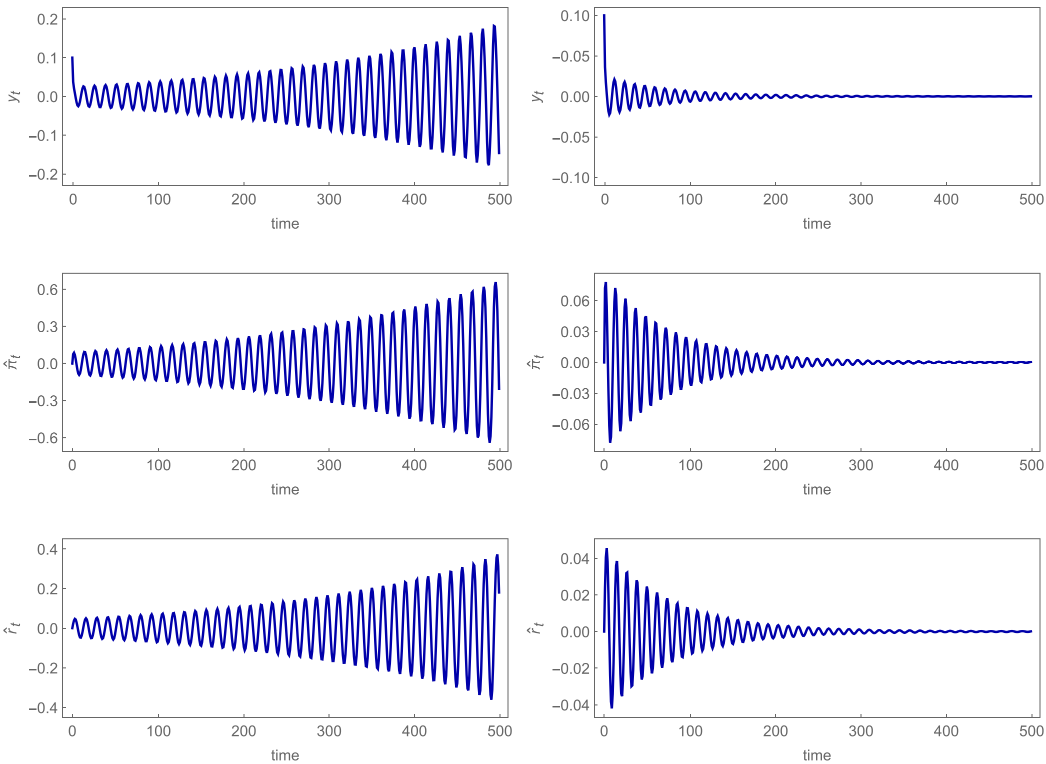

Figure 3. Deterministic dynamics for a different parameter setting. The panels show, from top to bottom, the evolution of the output gap, the inflation rate, and the interest rate for 500 periods, respectively. On the left-hand (right-hand) side, the parameters are $a_1 = 0.8$, $a_2 = -1$, $b_1 = 0.8$, $b_2 = 2$, $c_1 = 0.2$ ($c_1 = 0.23$), $c_2 = 0.2$, $c_3 = 0.8$, $\gamma = 2$, $\rho = 0.65$, and $\sigma_v = \sigma_\eta = \sigma_u = 0$.

the fraction of those making a positive forecast increases. However, by then, the output gap is already converging toward equilibrium, making the fundamental predictor attractive again. As a result, the fraction of agents using the naive predictor gradually decreases, while the fraction of those following the fundamental predictor increases. This process continues – without any central bank intervention – until equilibrium is reached, with both fractions equal to 0.5. Nonetheless, the central bank can accelerate this stabilization process by increasing its policy parameters. In doing so, it can reduce the amplitude of fluctuations – much like in Angeletos and Lian (2023) – and help the economy return to equilibrium more quickly. However, if the system is regularly hit by exogenous shocks, the volatility of the adjustment dynamics of the output gap and the inflation rate may become unacceptably high, forcing the central bank to intervene, even though the system is dynamically stable.

Next, we illustrate that there are parameter settings within the defined parameter ranges for which the fundamental steady state is unstable. In Figure 3, we assume the following parameter values: $a_1 = 0.8$, $a_2 = -1$, $b_1 = 0.8$, $b_2 = 2$, $c_1 = 0.2$, $c_2 = 0.2$, $c_3 = 0.8$, $\gamma = 2$, $\rho = 0.65$, and $\sigma_v = \sigma_\eta = \sigma_u = 0$, and display the dynamics in the left-hand panels.[7] At $t = 0$, we assume an exogenous demand shock of 0.1. The panels show again, from top to bottom, the evolution of the output gap, the inflation rate, and the interest rate in the time domain (for 500 periods), respectively. As evident from the figure, the system generates cyclical dynamics. However, these boom-bust dynamics eventually explode and diverges to infinity. Applying our analytical results to this parameter setting reveals that the first stability condition is satisfied, while the second one

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is violated. From (26), we obtain, for instance, that parameter c_1 must exceed 0.207482 to ensure stability of the fundamental steady state. Given that $c_1 = 0.2$, the dynamics diverge whenever an exogenous shock disrupts the system.

However, the central bank can achieve stability of the steady state by increasing the intensity at which it does inflation targeting. As soon as parameter c_1 exceeds its critical value of 0.204782, the system may converge to its equilibrium. In the right-hand panels of Figure 3, we show how the dynamics change when the central bank increases its inflation parameter up to $c_1 = 0.23$. The output gap, the inflation rate and the interest rate now appear to cyclically converge to their equilibrium values.

What economic mechanism leads to this stabilization? The key is to mitigate the intensity of self-fulfilling waves of optimism and pessimism that drive the economy away from equilibrium. When monetary policy responds strongly enough to deviations in inflation and output, it can counteract expectation-driven fluctuations, thereby reducing excessive belief-switching and, in turn, mitigating boom-bust cycles. For instance, suppose we observe a strong increase in aggregate demand. Those who made a positive output forecast will benefit, encouraging more agents to become optimistic, which further stimulates aggregate demand. If the central bank does not raise interest rates sufficiently, aggregate demand will continue to rise, potentially leading to a boom. However, if the central bank responds strongly enough, the increase in aggregate demand will be dampened, causing fewer agents to become optimists and thus reducing the boom.

5. Conclusions

In this paper, we provide a detailed analytical treatment of the behavioral macroeconomic model by De Grauwe and Ji (2020). In particular, we provide necessary and sufficient conditions for the local asymptotic stability of the model's fundamental steady state. Interestingly, our analytical results reveal that the parameter setting by De Grauwe and Ji (2020) yields a locally stable fundamental steady state, implying that the model's dynamics is largely driven by exogenous shocks. However, we also identify conditions under which boom-bust dynamics emerge temporarily endogenously from within the model. Overall, our findings contribute to a deeper understanding of how booms and busts emerge in such a framework and provide insights into how monetary policy can be utilized to stabilize these dynamics.

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Notes

1 See the Appendix for more details. Note that De Grauwe and Ji (2020) define the utility by $U_t = -\sum_{k=0}^{\infty} \omega_k (y_{t-k-1} - \tilde{E}_{t-k-2} y_{t-k-1})^2$, yielding $U_t = \rho U_{t-1} - (1-\rho)(y_{t-1} - \tilde{E}_{t-2} y_{t-1})^2$.

2 In their analyis, De Grauwe and Ji (2020) introduce the simplifying assumptions that $\pi^T = 0$, $\rho = 0$, $a_1 = 1$, and $b_1 = 1$, yielding a 6-D dynamical system.

3 These steady states cannot be expressed analytically. In footnote 5, we provide a parameter setting for which we find these nonfundamental steady states numerically.

4 If the modulus of the two complex conjugate eigenvalues is exactly 1, the dynamics remain bounded, and the system does not explode. Instead, it exhibits cyclical behavior, where the output gap, inflation, and interest rate oscillate around their fundamental steady-state values.

5 For parameter values outside the defined range of values, model dynamics may even converge to one of the nonfundamental steady states. For $a_1 = 0.5$, $a_2 = -0.2$, $b_1 = -0.5$, $b_2 = 0.05$, $c_1 = 1.5$, $c_2 = 0.5$, $c_3 = 0.5$, $\gamma = 2$, and $\rho = 0.5$, for instance, model dynamics either converges to $s_2^* = (-0.65, 1.81, 2.39, 0, -0.21, 0, -1.64, -0.65, -0.65, 1.81, 1.81)$ or $s_3^* = (0.65, -1.81, -2.39, 0, -0.21, 0, -1.64, 0.65, 0.65, -1.81, -1.81)$.

6 In De Grauwe and Ji (2020), the memory parameter is set to $\rho = 0.5$.

7 This parameter setting is chosen for illustrative purposes, but similar results can be obtained with more realistic values.

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Appendix

The utilities of using the fundamental and naive rule at time t are given by

$$U_{j,t} = -\sum_{k=1}^{\infty} (1-\rho)\rho^k (y_{t-k} - \tilde{E}^j_{t-k-1} y_{t-k})^2, \quad j = f, e,$$

which can easily be rewritten as

$$= -(1-\rho)\rho(y_{t-1} - \tilde{E}^j_{t-2} y_{t-1})^2 - (1-\rho)\rho^2(y_{t-2} - \tilde{E}^j_{t-3} y_{t-2})^2 \\ -(1-\rho)\rho^3(y_{t-3} - \tilde{E}^j_{t-4} y_{t-3})^2 - \cdots$$

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Since we can write that

$$\rho U_{j,t-1} = -\rho\Big((1-\rho)\rho(y_{t-2} - \tilde{E}^j_{t-3}y_{t-2})^2 + (1-\rho)\rho^2(y_{t-3} - \tilde{E}^j_{t-4}y_{t-3})^2 + \cdots\Big),$$

we obtain

$$\begin{aligned} U_{t,j} - \rho U_{j,t-1} = &-(1-\rho)\rho(y_{t-1} - \tilde{E}^j_{t-2}y_{t-1})^2 \\ &-(1-\rho)\rho^2\Big((y_{t-2} - \tilde{E}^j_{t-3}y_{t-2})^2 - (y_{t-2} - \tilde{E}^j_{t-3}y_{t-2})^2\Big) \\ &-(1-\rho)\rho^3\Big((y_{t-3} - \tilde{E}^j_{t-4}y_{t-3})^2 - (y_{t-3} - \tilde{E}^j_{t-4}y_{t-3})^2\Big) - \cdots \end{aligned}$$

Since the last terms cancel out, we get the expression defined in (8), i.e.,

$$U_{t,j} = \rho U_{j,t-1} - (1-\rho)\rho(y_{t-1} - \tilde{E}^j_{t-2}y_{t-1})^2, \quad j = f, e.$$

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