Numerical Methods of Curve Fitting, by P. G. Guest. Cambridge University Press, 1961. xiv + 422 pages. $13.50.

This book covers portions of both statistics and numerical analysis and is primarily intended for students and graduates in physics.

It is divided into three parts. Part I summarizes the treatment of observations of a single variable. Chapter 1 starts off with the elementary notions of probability (the simple frequency definitions), expectation, variance; and the estimation postulates of least-squares, minimum variance, and maximal likelihood. After establishment of the central limit theorem, the author discusses the properties of the normal frequency distribution in some detail. Chapter 3 is a short summary of some statistical tests, i.e. Student's (W. S. Gosset's) ratio test, and the F-test for homogeneity. Chapter 4 deals with discrete distributions and here the binomial and Poisson distributions are the inevitable subjects of discussion. Most of this material will, as is evident, be familiar to statisticians. However, the author intended Part I only to be a rapid summary of those portions of statistics used in the reduction of routine physical measurements. Many related subjects, e.g. analysis of variance, are deliberately omitted. Numerous references accompany the text for the benefit of those interested.

Part II (chapters 5 and 6) treats the fitting of straight lines. The author felt justified to discuss the linear case separately, "since most 'curves' fitted are straight lines". In chapter 5, after introduction of the notion of regression, the estimation of the experimental regression curve is discussed. Here the usual procedure is the application of the least-square principle. The maximal likelihood postulate can also be used as basis for estimation, and will in most cases give an estimation very close to that obtained by the least-square method. (The minimum variance postulate always leads to estimates which are identical with the least-square estimates - Markoff's theorem.) This part, dealing with the case where the regression curve is a straight line, is perhaps unnecessarily elaborate in its detail. Many examples are also worked out, complete with calculating scheme.

Part III (chapters 7-12) deals with the fitting of polynomial curves and of special types of curves. For the solution of the normal equations which appear in the least-square method of fitting a polynomial curve to a series of observations, the well-known Gauss-Doolittle method is developed at great length. The problem is also treated from the point of view of orthogonal polynomials (the fitted curve is written in terms of orthogonal polynomials). This is essentially similar to what K. Nielsen did in his book on "Methods in Numerical Analysis". Here, however, the relations between the orthogonal polynomials and the quantities occurring in the Doolittle scheme are emphasized. The two

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treatments are then combined in the familiar matrix form of Gauss' method. This lengthy discussion will probably be appreciated by the non-mathematician not too accustomed to matrix manipulation.

The least-square polynomial fitting problem is furthermore discussed in the case of a large number of data, where these observations are divided into a number of groups. The mean value in each group is then treated as a single observation. Attention is firstly given to equally-spaced observations of equal weight, and secondly to the general case when the spacing is non-uniform. Here the author investigates the questions of efficiency and bias in an interesting manner.

A short chapter is devoted to the non-polynomial fitting problem which so often materializes in practice. This is the case where we have a tabulated function of several variables which is to be approximated (in terms of least-squares) by a linear function of these variables. The non-linear case is linearized (due to W. Deming) by a change of variable or by using approximation methods based on a Taylor's series expansion.

The book concludes with five examples illustrating the commonest types of curve fitting problem; i.e. the straight line, the polynomial, the polynomial with equally-spaced observations, the linear function, and the non-linear function. The author also provides a useful guide to the more commonly used calculating schemes so as to assist the computor in selecting a method best suited to a given problem.

This book should prove interesting and of great help to the physics student or other research worker whose experiments lead to a curve fitting problem. It is written in a very clear style, and only knowledge of the calculus is presumed. Sporadic use is made of just the very simplest matrix operations. Since the author wanted to "obtain a better balance between theory and practice", the work literally teems with numerical examples (mostly taken from problems in physics) which are worked out, complete with a full calculating scheme. These are designed for a desk calculating machine, but are also applicable to most automatic computers.

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Théorie Analytique des Problèmes Stochastiques Relatifs à un Groupe de Lignes Téléphoniques avec Dispositif d'Attente, by F. Pollaczek. Mémorial des Sciences Mathématiques, Fascicule 150, Gauthier-Villars, Paris 1961.

This monograph discusses the stochastic problems associated with