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Systems Design Using Solution-Compensation Spaces with Built-In Tolerance Applied to Powertrain Integration

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Abstract

Complexity in systems design can be reduced by computing permissible ranges for some crucial design variables that need to be defined in an early design phase. These ranges are calculated such that there is sufficient tolerance for the remaining design variables in later design phases, while still achieving the overall system design goals. A new algorithm for this approach is presented and applied to the design of a vehicle powertrain mount system. The results show large permissible ranges for mount positions while maintaining sufficient tolerance for mount stiffnesses.

Keywords: complex systems, computational design methods, early design phase, optimisation, solution space engineering

1. Introduction

In the automotive sector, the requirements for powertrain development are driven as never before by regulatory framework conditions and even more by climate policy. This policy follows the overwhelming consensus that individual mobility and road transport have had and still have a decisive influence on climate change and that the contribution to the generation of greenhouse gases must be minimized. This globally effective requirement leads to a disruptive technology shift from internal combustion to electrified and hydrogen powertrains. Electrification (hybrid and battery-electric) is also accompanied by a trend toward more software-controlled customer functions and services, such as dynamic range determination or autonomous driving and parking. The new drive technologies, electrification and software-based customer functions challenge the mechanical development. On the one hand, there are new functional, technical detail requirements and solution concepts that must be developed in the system context with electrics/electronics (EE) and therefore have the character of new developments instead of manageable adaptation and variant designs. On the other hand, there are process-related changes, which concern the further shortening of development processes, as well as the saving of hardware validation cycles. In addition to that, there are organizational adjustments with the shift of development capacities from mechanical development and design to EE and software development. In summary, the pressure on mechanical engineers is increasing enormously and with it the willingness to use new digital methods for computer-aided, model-based development in compliance with functional requirements. Solution Space Engineering (SSE) is a promising set-based top-down approach introduced by Zimmermann and von Hössle (2013) to integrate requirements of different disciplines on a system level and generate multi-dimensional solution boxes that can be represented by a permissible interval for each design variable on component level hence achieving a complete decoupling. This means that each design variables value can be modified independently of all other design variables within the permissible range. Component designers in a distributed development environment thereby obtain flexibility to generate solutions independent of the design of other components of the system already in the early design phase, immediately after the concept phase is completed. Flexibility in this context indicates that a system meets all requirements even if properties of subsystems or components may change within a particular range. Thus the greater the flexibility the more possibilities for a developer to design components or subsystem. The maturity of this method, the associated tools as well as the data provision and quality is on a level that it can be successfully applied in architectural design; see for example Königs and Zimmermann (2017), Poulain et al. (2018), Stumpf et al. (2020). Unfortunately, the complete decoupling of all design variables achieved by interval-based solution spaces leads to a great loss of good solutions thus the flexibility that remains for a single design variable may becomes insufficient small for high dimensional problems. Erschen et al. (2018) presented a strategy to accommodate this effect by the optimization of 2-dimensional linear constrained permissible areas, which leads to a pairwise coupling of design variables. Another method that leads to a pair-wise coupling of design variables is proposed by Harbrecht et al. (2019). It contains the extension of the SSE approach by the rotation of 2-dimensional solution boxes as an additional degree of freedom. Both approaches obtain larger solution spaces. For a further minimization of loss of solution space, Daub et al. (2020) presented the calculation of arbitrarily shaped solution spaces for linear problems. In contrast, Vogt et al. (2018) introduced the calculation of so-called solution-compensation spaces (SCS). This approach distinguishes between early- and late-decision variables. Early-decision variables relate to a high degree of uncertainty and therefore need more flexibility in the development process. Late-decision variables are adjustable in a later stage of the development process, and therefore no flexibility in form of permissible intervals is needed. The idea is to maximize the early-decision solution box under the conditions that a minimum of one single set of late-decision parameter values that ensure compliance with the systems requirements remains. Funk et al. (2019) extended this method by sequentially updating SCS multiple times in the development process. However, it has not been possible yet to provide flexibility for late-decision design parameters with the stated approaches. This severely limits the specification of late-decision design variables because they need to be completely free adjustable and no tolerances for, e.g., manufacturing can be considered. Hence, the SCS approach conflicts with many engineer's approaches in practice. One example is the design of powertrain mounting systems, in particular the powertrain integration of both, hybrid and pure electric drive units. This is because of a necessary tolerance range for mount stiffnesses due to scattering of material properties and ageing processes that cannot be prevented. In this paper we present the extension to so-called solution-compensation spaces with built-in tolerance ranges (SCSBT) that is based on SCS but provides flexibility also for late-decision variables. In contrast to other set-based design approaches (see, e.g., Shallcross et al. (2020) for an overview), the goal of the novel SCSBT method is the projection of the set of all good solutions that ensure enough flexibility for later design steps onto a subset of crucial design variables, namely early-decision variables. This leads to both, maximum flexibility in early design stages as well as sufficient tolerance ranges for later design decisions and manufacturing. The content of this paper is structured as follows: in section 2, we derive the problem statement of SCSBT. A novel algorithm to calculate SCSBT for problems that are linear with respect to the late-decision variables is presented in chapter 3. Section 4 explains the real-world application to powertrain integration and the advantages of the novel approach in practice, whereas section 5 shows the conclusion and an outlook.

2. General Problem Statement

In this chapter, we show the idea of SCSBT by comparing it with the SSE and the SCS approach within a 2-dimensional design problem before we deduce the mathematical formulations.

2.1. Solution-compensation Spaces with Built-in Tolerance (SCSBT)

Figure 1 shows a 2-dimensional design space with linear constraints, which limit the complete solution space coloured in bright green. Every design point $x = (x_a, x_b)$ that lies within the complete solution space is called a good design. The solution of the conventional SSE approach is depicted as a green box Ω that is maximum for a complete decoupling of the design variables. The permissible intervals

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are defined by the edges of the box, which leads to a small permissible interval Ω_a. In contrast, the SCS and SCSBT approaches split the design variables into early- and late-decision variables x_a (e.g., the position of a bearing) and x_b (e.g., the stiffness of a bearing), respectively to maximize only the early-decision solution intervals $\Omega_{a,\mathrm{SCS}}$ and $\Omega_{a,\mathrm{SCSBT}}$, respectively. The 1-dimensional early-decision solution boxes (intervals) of the SCS projection ($\Omega_{a,\mathrm{SCS}}$) and the SCSBT projection ($\Omega_{a,\mathrm{SCSBT}}$) is drawn in blue and violet, respectively. In both cases, the late-decision design x_b depends on the choice of the early-decision design x_a. Only after the value of x_a is fixed, a permissible value for x_b can be found. However, if x_a is chosen at the boundary of $\Omega_{a,\mathrm{SCS}}$, only one permissible design for x_b is left. In contrast, the calculation of $\Omega_{a,\mathrm{SCSBT}}$ considers a problem specific minimal tolerance range $I_{b,\min}$ to gain necessary flexibility for the late-decision design x_b.

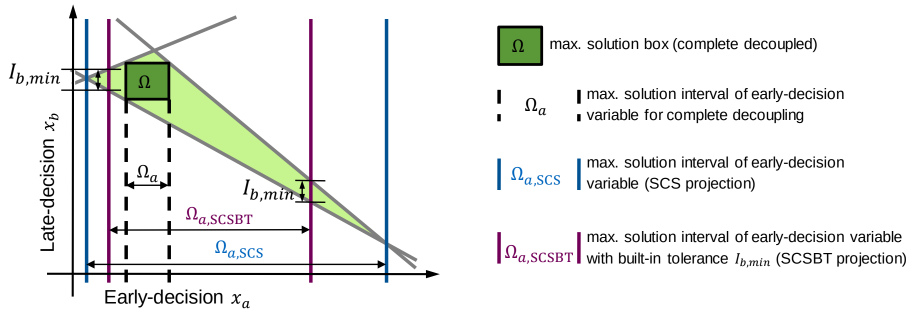

Figure 1. Complete decoupled solution box, early-decision solution space projection by both, SCS and SCSBT

This means that in most cases $\Omega_{a,\mathrm{SCSBT}}$ is smaller than $\Omega_{a,\mathrm{SCS}}$. However, there is the guarantee that the flexibility for the late-decision design is sufficient if the early-decision design lies within $\Omega_{a,\mathrm{SCSBT}}$.

2.2. Mathematical Description of SCSBT

In the following, we consider a design problem with $d \in \mathbb{N}$ design variables $x_i \in \mathbb{R}, i = 1 \dots d$, that are divided into $p \in \mathbb{N}$ early- and $q \in \mathbb{N}$ late-decision variables and it holds $d = p + q$. The number of the design problems quantities of interest is denoted by $m \in \mathbb{N}$. Let the closed set $\Omega_{\mathrm{ds}} \subset \mathbb{R}^d$ be the design space and consider a set of constraints

$$f(x) \leq f_{\mathrm{c}}, \tag{2.1}$$

with the performance function $f : \mathbb{R}^d \to \mathbb{R}^m$ and the critical values $f_{\mathrm{c}} \in \mathbb{R}^m$ that define the complete solution space Ω_{c} related to Zimmermann and von Hössle (2013) such that

$$\Omega_{\mathrm{c}} = \{\, x \in \Omega_{\mathrm{ds}} \mid f(x) \leq f_{\mathrm{c}} \,\}. \tag{2.2}$$

A design point $x \in \Omega_{\mathrm{ds}}$ is the vector of all design variables, whereby the first p and the last q entries represent the early- and the late-decision variables, respectively:

$$x = (x_1, x_2, \dots, x_d) = (x_a, x_b). \tag{2.3}$$

The design space Ω_{ds} can be represented by the Cartesian product $\Omega_{\mathrm{ds}} = \Omega_{\mathrm{ds},a} \times \Omega_{\mathrm{ds},b}$, with $\Omega_{\mathrm{ds},a} \subset \mathbb{R}^p$ and $\Omega_{\mathrm{ds},b} \subset \mathbb{R}^q$ as the early- and late-decision design space, respectively. For a given early-decision design point $x_a \in \Omega_{\mathrm{ds},a}$, the complete late-decision solution space $\Omega_{\mathrm{c},b}$ is defined as

$$\Omega_{\mathrm{c},b} = \{\, x_b \in \Omega_{\mathrm{ds},b} \mid (x_a, x_b) \in \Omega_{\mathrm{c}} \,\}. \tag{2.4}$$

To achieve decoupling of the design variables, we define independent intervals $I_i = [x_i^{\mathrm{l}}, x_i^{\mathrm{u}}]$, with x_i^{l} and $x_i^{\mathrm{u}}, i = 1 \dots d$ as the lower and upper boundaries of each design variable, respectively. The Cartesian product of these intervals represent a box-shaped set

$$\Omega = I_1 \times I_2 \times \dots \times I_d. \tag{2.5}$$

Similar to equation (2.5) we define an early-decision box-shaped set as

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$$\Omega_a = I_1 \times I_2 \times \ldots \times I_p \tag{2.6}$$

and a late-decision box-shaped set as

$$\Omega_b = I_{p+1} \times I_{p+2} \times \ldots \times I_{p+q}. \tag{2.7}$$

If a box-shaped set is a subset of the complete solution space, it is called solution box. For unknown constraints (2.1) the complete solution-space may be arbitrary, and the maximum number of different solution boxes that need to be rated for optimization purposes is infinity. Therefore, a size measurement μ is introduced that can be calculated, e.g., by the volume of a set Ω

$$\mu(\Omega) = \int_\Omega d\Omega. \tag{2.8}$$

The maximization of a solution box Ω that leads to a complete decoupling of all design variables, related to the SSE approach and introduced by Zimmermann and von Hössle (2013) reads

$$\begin{aligned} &\max \mu(\Omega) \\ &s.t.\ \Omega \subset \Omega_c, \end{aligned} \tag{2.9}$$

with the complete solution space Ω_c, defined in equation (2.2).Vogt et al. (2018) introduced the SCS method and the idea of maximizing the early-decision solution box Ω_a by the optimization problem

$$\begin{aligned} &\max_{\Omega_a \subset \Omega_{ds,a}} \mu(\Omega_a) \\ &s.t.\ \Omega_{c,b}(x_a) \neq \emptyset,\ \forall x_a \in \Omega_a, \end{aligned} \tag{2.10}$$

where $\Omega_{c,b}(x_a)$ is the projection of the late-decision complete solution space for a single early-decision design x_a, defined in equation (2.4). This approach assumes that there is no flexibility for the late-decision variables needed, because the constraints of optimization problem (2.10) only require a nonempty projection of the late-decision complete solution space. The novel approach, presented in this paper additionally requires a specific size of a late-decision solution box Ω_b, as subset of the projected late-decision complete solution space (equation (2.4)). This leads to the optimization problem

$$\begin{aligned} &\max_{\Omega_a \subset \Omega_{ds,a}} \mu(\Omega_a) \\ &s.t.\ \forall x_a \in \Omega_a,\ \exists \Omega_b \in \mathcal{S}_b : \Omega_b \subset \Omega_{c,b}(x_a), \end{aligned} \tag{2.11}$$

with the set $\mathcal{S}_b$ of all permissible late-decision solution boxes Ω_b that ensure sufficient flexibility, which is defined as

$$\mathcal{S}_b = \{\Omega_b = \prod_{i=p+1}^{p+q}[x_i^l, x_i^u] \subset \Omega_{ds,b} \mid x_i^u - x_i^l \geq 2t_i^r|x_i^c| + 2t_i^a, i = p+1 \ldots p+q\}, \tag{2.12}$$

where $t_i^a \in \mathbb{R}_+$ and $t_i^r \in \mathbb{R}_+$ are the required absolute and relative tolerance, respectively, and $x_i^c = \frac{1}{2}\left(x_i^l + x_i^u\right), i = p+1 \ldots p+q$ is the centre point of the late-decision solution box. The constraints definition (2.12) can be interpreted as a minimum interval width for each dimension of the late-decision solution box Ω_b. To cope with different types of tolerance requirements it is possible to specify an absolute tolerance t_i^a that is constant, and a relative tolerance t_i^r that depends on the centre of the associated dimension i of Ω_b. To compare the achievement of these flexibility requirements, we define the slack s based on definition (2.12) as a measure of the minimal additional flexibility of all dimensions of $\Omega_b = \prod_{i=p+1}^{p+q}[x_i^l, x_i^u]$

$$s(\Omega_b) = \min_{i=p+1 \ldots p+q} \alpha_i\left((x_i^u - x_i^l) - 2t_i^a - t_i^r\left|x_i^l + x_i^u\right|\right), \tag{2.13}$$

with the normalization constants α_i. If s is negative, there is too little flexibility for at least one late-decision variable. If $s(\Omega_b) \geq 0$, the late-decision solution box Ω_b is permissible. Considering the definition (2.13), problem (2.11) can be formulated as a nested optimization problem

$$\begin{aligned} &\max_{\Omega_a \subset \Omega_{ds,a}} \mu(\Omega_a) \\ &s.t.\ s^*(x_a) \geq 0, \forall x_a \in \Omega_a, \end{aligned} \tag{2.14}$$

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whereby $s^*(x_a)$ is the solution of the inner optimization problem

$$\max_{\Omega_b \subset \Omega_{\text{ds},b}} s(\Omega_b) \quad (2.15)$$

$$s.t.\ (x_a, x_b) \in \Omega_c, \forall x_b \in \Omega_b.$$

If the result $s^*(x_a)$ is negative, there is no permissible late-decision solution box Ω_b and the corresponding x_a is not a good early-decision design. In contrast, $x_a$is a good early-decision design if $s^*(x_a) \geq 0$, which means, a permissible late-decision solution box exists.

3. Hybrid x_b-linear SCSBT Algorithm

In chapter 2 the problem statement for box-shaped solution-compensation spaces with built-in tolerance ranges (SCSBT) was introduced, and a nested optimization problem was formulated (equation (2.14) and (2.15)). To solve this problem for black box solution space constraints, large numerical costs can be expected. This is because if no information about the constraints are given, only stochastic sample-based algorithms, such as presented by Zimmermann and von Hössle (2013), can be used. This may lead to a sample-based box maximization algorithm for optimizing the early-decision solution box, whereby the evaluation of each sample point necessitates the execution of a sample-based box maximization algorithm for optimizing a late-decision solution box. Hence, the computational effort increases unreasonable for high dimensional problems in an industrial environment. In this chapter we present a hybrid SCSBT algorithm that can be applied for so-called x_b-linear solution space constraints, which are linear regarding the late-decision variables x_b. The complete solution space Ω_c of a x_b-linear system can be represented by

$$\Omega_\text{c} = \{\, (x_a, x_b) \in \Omega_\text{ds} \mid G(x_a)x_b \leq \text{g}_\text{c}(x_a) \,\}, \quad (3.1)$$

with the functions $G: \mathbb{R}^p \to \mathbb{R}^{m \times q}$ and $g_c: \mathbb{R}^p \to \mathbb{R}^m$. The dependencies of the early-decision variables are included in $G(\cdot)$ and $g_\text{c}(\cdot)$ and can still be non-linear. Considering definition (3.1) the maximization of a late-decision solution box leads to a linear program comparable to the box optimization for linear problems presented by Erschen et al. (2015). Therefore, we define the vector $\xi_b = (x_b^\text{l}, x_b^\text{u}) \in \Omega_{\text{bs},b} = \Omega_{\text{ds},b} \times \Omega_{\text{ds},b}$ as a representation of the lower and upper late-decision boundaries x_b^l and x_b^u, respectively. The slack $s^*(x_a)$ for x_b-linear solution space constraints is therefore the solution of the inner optimization problem

$$\max_{\substack{\xi_b \subset \Omega_{\text{bs},b} \\ s \in \mathbb{R}}} s \quad (3.2)$$

$$s.t.\ \ \alpha_i\left((x_i^\text{u} - x_i^\text{l}) - 2t_i^\text{a} - t_i^\text{r}\left|x_i^\text{l} + x_i^\text{u}\right|\right) \geq s,\ \ i = p+1 \ldots p+q$$

$$\tilde{G}\,\xi_b \leq \tilde{g}_\text{c},$$

whereby the projected late-decision solution space constraints for the corresponding early-decision design x_a are represented by $\tilde{g}_\text{c} = g_\text{c}(x_a)$ and $\tilde{G} = (\tilde{G}_1, \tilde{G}_2) \in \mathbb{R}^{m \times 2q}$, with

$$\tilde{G}_{1,ij} = \begin{cases} 0 & \text{for } G_{ij}(x_a) \geq 0 \\ G_{ij}(x_a) & \text{otherwise} \end{cases}, \quad \tilde{G}_{2,ij} = \begin{cases} G_{ij}(x_a) & \text{for } G_{ij}(x_a) \geq 0 \\ 0 & \text{otherwise} \end{cases}, \quad {\scriptstyle i=1\ldots m \atop j=1\ldots q.} \quad (3.3)$$

Note that the absolute value in the constraints of problem (3.2) can be easily reformulated by two linear constraints and a case distinction. The linear program (3.2) can now be solved by common optimization methods, e.g., the simplex algorithm, and due to the linearity, the computational costs are low enough to solve real-world applications with the nested optimization problem (2.14). Figure 2 visualizes the idea of the hybrid x_b-linear approach on a 4-dimensional problem. The left part depicts the early-decision design space whereas the right shows the projection of the late-decision design space for one single early design x_a. To find an early-decision solution box a Monte Carlo based algorithm, presented by Zimmermann and von Hössle (2013) is used. For each sample evaluation the solution of the linear program, presented in this chapter, needs to be solved. This is what happens on the right side of figure 2: the projected solution space constraints are linear and the dashed box depicts the minimum tolerance for the late decision-variables in its size. If and only if there exists a solution

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box that is equal or larger than the dashed box in all dimensions, then the slack s is positive and the associated early-decision design x_a is good.

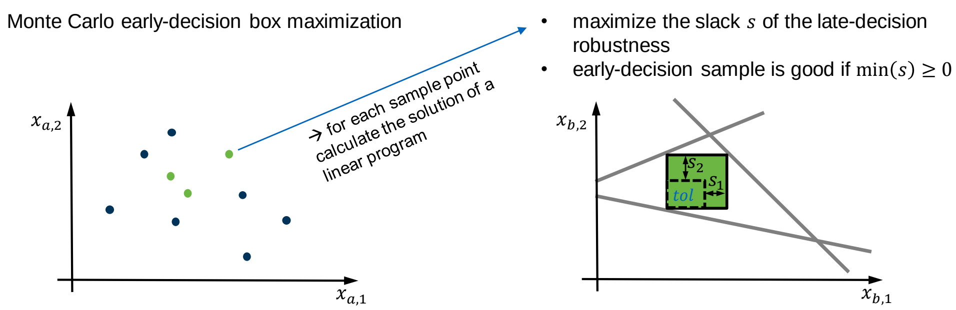

Figure 2. Schematic representation of the hybrid SCSBT algorithm; left: Monte Carlo early-decision solution box maximization; right: linear program for maximizing the slack s of the late-decision flexibility

4. Application to the Design of a Powertrain Mount System

Main tasks of a powertrain mount system (PMS) are to support the static and dynamic loads introduced by the engine gearbox assembly (EGA) as well as the isolation and reduction of undesirable noise and vibrations (e.g., idle state, road induced) described, e.g., by Angrosch et al. (2015). In the literature, different approaches to design PMS are referred. For the architecture design of PMS, Kang et al. (2016) proposed a real options approach for hybrid electric vehicles to determine optimal design considering uncertainty and different time of decision regarding design and pricing of PMS. Analytical methods by decoupling the torque-roll axis (TRA) introduced by Jeong and Singh (2000) and the proposed mode decoupling approach by Angrosch et al. (2015), which focus on the kinetic energy distribution of eigenmodes, are based on deterministic design variables. In contrast, the presented reliability-based approach from Lü et al. (2021) considers uncertainty caused by imprecise information during development to ensure robust design optimization of PMS. For the PMS design of conventional combustion engines, Königs and Zimmermann (2017) present the application of the SSE methodology that decouples all design variables with interval-type solution spaces on component level by taking requirements from many disciplines into account. However, in modern hybrid powertrain concepts, meeting all the noise, vibration, harshness (NVH) requirements leads to a major challenge. The use of small two- or three-cylinder combustion engines results in unfavourable dynamic excitations, while at the same time the electric drive component provides extraordinarily high maximum torques (Shangguan, 2009). Despite this, achieving conflicting objectives means that solution spaces become very small, and developers have insufficient flexibility in the early phase of product development to cope with the uncertainties due to the high level of system complexity. In the following we apply the novel SCSBT approach to the design of a hybrid powertrain mounting system. The objective hereby is not to design the system completely but to maximize permissible intervals for the early-decision variables, namely the positions of the engine mounts that ensure sufficient flexibility for the design of the mount stiffnesses, which represent the late-decision variables. This allows for maximum flexibility for positioning of the engine mounts in the early design phase. Once the positions of the mounts are determined, the permissible ranges for the stiffnesses that ensure sufficient flexibility for further development and production tolerances are easy to calculate.

4.1. Modelling of the Powertrain Mount System Regarding NVH Criteria

Figure 3 shows the CAD model of the considered PMS that contains the left and right main mount at the upper side of the EGA as well as the torque rod at the lower. The PMS consist of a 3-cylinder combustion engine, an electric drive unit and the gearbox. The modelling of the PMS is implemented as six degree of freedom rigid body and the mounts are represented by three-dimensional orthogonal springs. For reasons of simplification, damping properties are neglected. The inertia properties of the

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PMS are the mass $m = 230\text{kg}$ with the center of mass at $x = 78\text{mm}, y = 45\text{mm}, z = 270\text{mm}$ with respect to the vehicle coordinate system and the inertia tensor $I = \begin{pmatrix} 9.92 & 0.05 & 0.65 \\ 0.05 & 7.77 & -0.91 \\ 0.65 & -0.91 & 8.73 \end{pmatrix} \text{kg m}^2$.

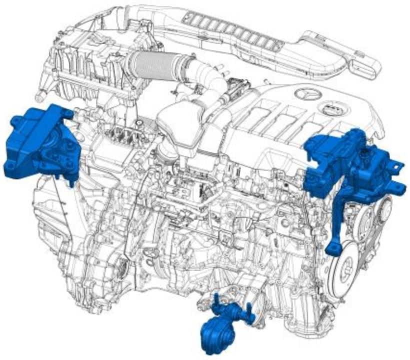

Figure 3. Powertrain system including combustion engine, electric drive unit and gearbox as well as highlighted left mount, right mount and torque rod

The considered quantities of interest (QOI) and their dependencies are depicted in figure 4 and will be explained in the following. The dynamic behaviour of the EGA is mainly characterised by the eigenmodes and their respective frequencies. By modification of the mounting system topology, modal behaviour can be adjusted with respect to the required quantity of interest. The eigenvalue problem is defined as

$$(K_R - \omega_i^2 M)v_i = 0, \tag{4.1}$$

where $M \in \mathbb{R}^{6\times6}$ is the mass matrix of the EGA, $K_R \in \mathbb{R}^{6\times6}$ is the assembled stiffness matrix determined by positions $P_1, P_2, P_3 \in \mathbb{R}^3$ and stiffnesses $K_1, K_2, K_3 \in \mathbb{R}^{3\times3}$ of the engine mounts and $\omega_i^2 \in \mathbb{R}$ and $v_i \in \mathbb{R}^6$ are the i-th eigenvalue and the i-th eigenvector, respectively. QOI 1 and 2, shown in figure 4, are related to the PMS eigenfrequencies $f_i = \omega_i/(2\pi), i = 1 \ldots 6$, which are restricted by the primary ride frequencies up to 5 Hz as a lower bound as well as the vehicle chassis frequencies about 15-20 Hz as an upper bound (Angrosch et al., 2015; TrellborgVibracustic, 2014).

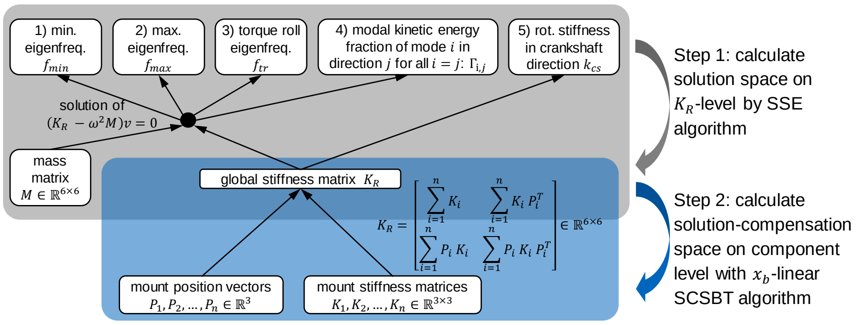

Figure 4. Quantities of interest of a powertrain mount system (1-5) and the two-step top-down approach to calculate SCSBT on component level with the mount positions and stiffnesses as early- and late-decision variables, respectively

The QOI 3, which is the corresponding frequency f_{tr} to the mode in TRA direction, needs to be at least 50% below the idle excitation frequency. For the PMS we are looking at, the idle excitation caused by the engine is characterized by dominant frequency located at approximately 19 Hz, which depends on the idle speed and the motorization (e.g., number of cylinders). Besides the value of the eigenfrequencies, fully decoupled eigenmodes are desirable in order to decouple external excitations from dominant vibration directions of the EGA (Xu et al., 2018). To evaluate the degree of decoupling of the eigenmodes (QOI 4 in figure 4), the modal kinetic energy distribution (Angrosch et al., 2015)

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$$\Gamma_{i,j} = \frac{v_{i,j} M_{j,j} v_{i,j}}{v_i^T M v_i}, \tag{4.2}$$

with $M_{j,j}$ as the j-th diagonal element of the mass matrix M and $v_{i,j}$ as the j-th element of the i-th eigenvector v_i is introduced. If the kinetic energy of the i-th mode is concentrated in one direction j, i.e. $\Gamma_{i,j} \approx 1$, the eigenmode is called fully or nearly decoupled. QOI 5 represents another important aspect, which is the global rotational stiffness $k_{cs} \in \mathbb{R}^+$ that represents the overall composed behaviour due to drive torque of the entire mounting system in crankshaft direction. It is bounded by a lower critical value to limit the rotational displacement of the EGA in crankshaft direction. Table 1 lists all the critical values for the QOI we derived in this chapter.

Table 1. Requirements on the considered powertrain mount system

Nr.	Quantity of interest	Symbol	Lower critical value	Upper critical value	Unit
1)	Minimum eigenfrequency	$f_{\min}$	4		Hz
2)	Maximum eigenfrequency	$f_{\max}$		15	Hz
3)	Eigenfrequency of torque roll mode	f_{tr}		8.8	Hz
4)	Modal kinetic energy fractions	$\Gamma_{i,i}, i = 1 \dots 6$	0.95		1
5)	Rotational stiffness in crank shaft direction	k_{cs}	18000		Nm/rad

The novel top-down approach for the design of PMS needs to be carried out in two steps, as shown in figure 4. This is because of the eigenvalue problem, which leads to a non-linear mapping between both, the early- and late-decision variables and the quantities of interest. Since the rigid body stiffness matrix K_R completely represents the design variables on a higher level of system modelling, it is possible to calculate permissible intervals for the entries of the stiffness matrix that ensure compliance with the requirements on system level in the first step. One possibility for a direct top-down calculation from a desired system response to the stiffness matrix is the solution of an inverse eigenvalue problem. Even though there are many approaches to solve inverse eigenvalue problems that can be found in literature, e.g., Chu and Golub (2002), they are not directly applicable in an interval-type set-based manner, but for optimal design approaches. However, the goal of this approach is not to find an optimal design but maximize permissible intervals and since the solution of an eigenvalue problem (bottom-up mapping) is well explored and the computational effort is small, the first step, shown in figure 4, is realized by a stochastic SSE algorithm presented by Zimmermann and von Hössle (2013). Once the permissible intervals for the entries of the stiffness matrix K_R are calculated, the novel SCSBT algorithm can be applied since the mapping between K_R and the design variables is x_b-linear.

4.2. Results

The early-decision design space of the presented application is 8-dimensional, including the positions in x, y, z direction of the left and the right mount as well as the position in y, z direction of the torque rod. In contrast, the late-decision design spaces includes the stiffnesses in x, y, z direction of the left and the right mount as well as the stiffness in x direction of the torque rod hence is 7-dimensional. The left part of figure 5 shows the result of the SCSBT maximization within the early-decision design space, which contains all information about the positions of the engine mounts. The red lines describe the bounds of the solution box in all dimensions. Within these bounds, designers can place the mounts arbitrarily and have the guarantee that there exist sufficiently large ranges of stiffness values for the mounts and consequently sufficient design flexibility, such that all requirements on the system level are fulfilled. To demonstrate this, the right part of figure 5 depicts the projections of the solution space onto the late-decision variables for two different mount positions (represented by the red and violet early design points on the left). It can be seen that the late-decision solution box related to the red early design differs completely from the late-decision solution box related to the violet early design. Thus, the late-decision solution space projection depends strongly on the early-decision design. One

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can say that any early design choice within the early-decision solution box can be compensated by any late design within a specific lade-decision solution box in a sense that all system requirements are met. This leads to a huge advantage in an early design phase due to the fact, that the positions of the mounts are in conflict with a large number of other components that need to be placed in the front part of the vehicle, e.g., the steering column, suspension elements or the exhaust system. Hence, small changes of the package concept may require the modification of the mount positions. In a conventional development process this would cause an iteration loop to check for requirement compliance. The SCSBT, in contrast, allows the modification of mount positions within the permissible ranges without the necessity of an iteration loop and thus provides maximum flexibility and efficiency for the early design phase.

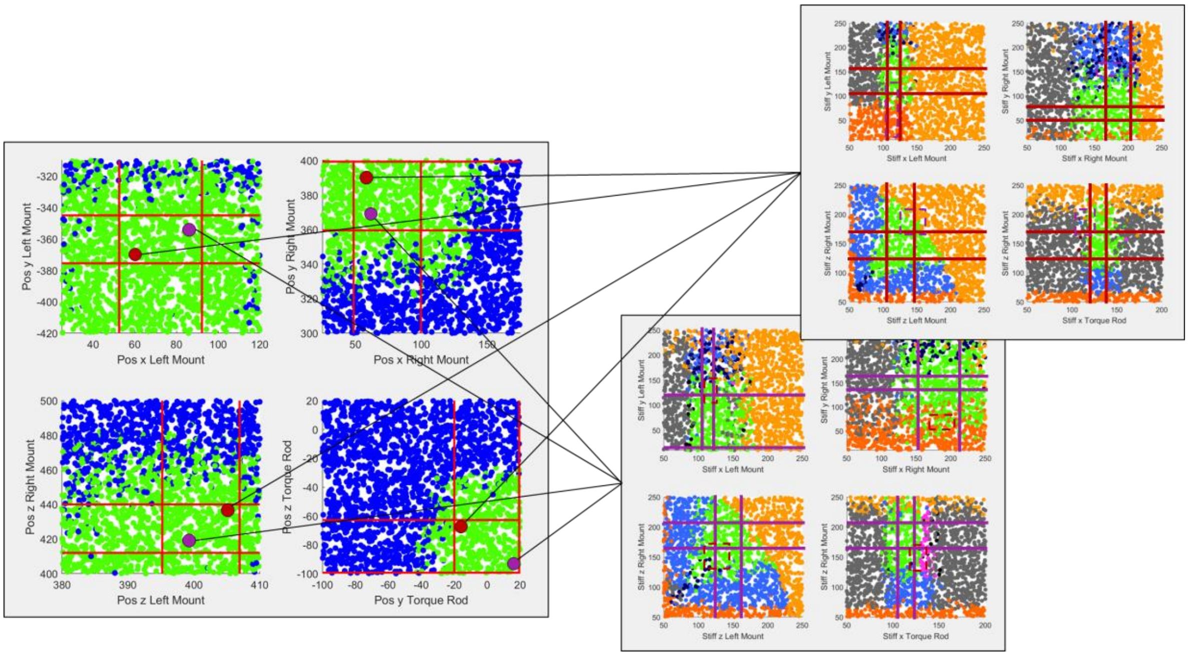

Figure 5. Solution-compensation space projection with built-in tolerance: maximized solution box of mount positions (left) and emerging solution boxes for mount stiffnesses (right); shown are two possible mount positions (red and violet points left), which lead to different late-decision solution boxes (right); green dots represent good design points, other colors indicate violation of at least one requirement

5. Conclusion and Outlook

One strategy to achieve flexibility for crucial design variables in complex systems development is the differentiation of early- and late-decision variables to calculate solution-compensation spaces (SCS) presented by Vogt et al. 2018. However, this approach is not applicable to problems where an assignment of a minimum tolerance range for all design variables in development process is necessary. This paper proposes the idea and the problem formulation that extend the SCS approach to provide flexibility also for late-decision variables by solution-compensation spaces with built-in tolerance (SCSBT). Furthermore, we introduced a hybrid algorithm to calculate SCSBT for so-called x_b-linear systems, which are linear with respect to the late-decision variables. Although this is a significant limitation, it could be shown that this algorithm can be applied even if the problem is not x_b-linear, as for the design of engine mount systems. This was achieved by a two-step calculation that first leads to permissible intervals for the global stiffness matrix of a powertrain mount system (PMS). In the second step the novel algorithm for x_b-linear systems project these permissible intervals onto component level and solution intervals for mount positions have been maximized. In contrast to some top-down approaches, the goal of this method is not to completely design a system at once, but to maximize the flexibility for the early-decision design and still providing sufficient flexibility for the late-decision design, once the early-decision design is fixed. Future work should contain the research

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into additional algorithms to calculate SCSBT to extend the field of applications. In case of the PMS design there are potentials regarding the two-step calculation procedure, since this requires an approximation of the complete solution space on the system describing stiffness matrix level by box-shaped solution spaces. This is why there is still a loss of good designs that could be reduced. Future work should continue on exploring a better approximation of permissible stiffness matrices.

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