Correspondence

DEAR EDITOR,

2 + 2 = 5

Note 106.20 quotes these words of George Orwell from an essay of 1939:-

> ‘ It is quite possible that we are descending into an age in which two plus two will make five when the Leader says so.’

Here is a proof of this fact, taken from my book *Comic Sections Plus*.

Addition can be performed with any objects. For example, you can add a number of spoons to a number of spoons and wind up with a number of spoons.

What happens if you add plus signs together? What, for example, is two plus signs added to two plus signs? Perhaps it should be

$$(+\ +) + (+\ +) = +\ +\ +\ +\ +,$$

or, in other words, $2 + 2 = 5$?

I would be interested to see how readers would refute this demonstration if a pupil were to offer it.

10.1017/mag.2023.31
Published by Cambridge University Press on behalf of The Mathematical Association

DES MACHALE
School of Mathematics, Applied Mathematics and Statistics
University College Cork, Cork, Ireland
e-mail: *d.machale@ucc.ie*

Feedback

On 106.12: Martin Lukarevski writes: Tran Quang Hung gives a proof of the Pythagorean theorem in n-dimensions. The three-dimensional case for a tetrahedron is of course the most interesting and is known as de Gua's theorem. It was published by J. P. de Gua de Malves in 1783, in [1] but it was already known to Descartes 1619-1621.

Recently de Gua's theorem was used for a derivation of Heron's formula [2], so a direct proof is desirable and we give one here. Let $OABC$ be a tetrahedron with right corner at vertex O and let $l = OA$, $m = OB$ and $n = OC$. Let the perpendicular from A to B meet BC at L. Then OL is also perpendicular to BC. From the right triangle OBC it follows that $CL = \dfrac{OC^2}{CB}$.

Hence

$$\begin{aligned} \text{Area}\,(ABC)^2 &= \tfrac{1}{4}BC^2AL^2 = \tfrac{1}{4}\left(OB^2 + OC^2\right)\left(AC^2 - CL^2\right) \\ &= \tfrac{1}{4}\left(m^2 + n^2\right)\left(l^2 + n^2 - \frac{n^4}{m^2 + n^2}\right) \\ &= \tfrac{1}{4}\left(l^2m^2 + m^2n^2 + n^2l^2\right) \\ &= \text{Area}\,(ABO)^2 + \text{Area}\,(BCO)^2 + \text{Area}\,(CAO)^2. \end{aligned}$$

https://doi.org/10.1017/mag.2023.31 Published online by Cambridge University Press