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On the existence of solutions to discrete and continuous boundary value problems

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This thesis investigates boundary value problems for second-order, ordinary, differential equations and also the difference equations which discretely approximate them. More specifically, the investigation involves the *existence* of solutions to these continuous and discrete problems.

The study begins with difference equations in Banach spaces. Some discrete Nagumo conditions are presented for a priori bounds on first difference quotients of solutions to the discrete Dirichlet problem in Banach spaces. These bounds are applied, in conjunction with discrete maximum principles, to formulate existence theorems for solutions to the discrete Dirichlet problem in Hilbert spaces. The solutions are shown to converge to solutions of the (continuous) Dirichlet problem in an aggregate sense.

Next, finite-dimensional systems of discrete boundary value problems are examined and a unifying theory for many different kinds of discrete boundary conditions is formulated. This theory involves topological degree theory. Existence and convergence theorems follow, in conjunction with the above-mentioned discrete maximum principles and inequalities, for solutions to the: discrete Neumann; discrete periodic and discrete Sturm-Louiville problems, while also applying to nonlinear discrete boundary conditions. In the one-dimensional case, some remarks and corollaries are presented for solutions to the discrete Dirichlet problem, in the presence of lower and upper solutions.

The aforementioned discrete maximum principles are subsequently replaced with conditions involving the magnitude of the discrete Dirichlet boundary conditions. Some a priori bound theorems are obtained on *all* solutions to systems of equations for the discrete Dirichlet problem. The bounds are independent of the step-size and thus no "spurious" solutions to the discrete problem can exist. Existence and convergence theorems follow for solutions of the discrete Dirichlet problem.

The main advantage of all the aforenamed results is that they apply to *systems* of equations and appear to be new even for the discrete Dirichlet problem in the two-dimensional case.

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Next, the investigation turns to special types of systems of second-order, ordinary, differential equations which readily lend themselves to the lower and upper solution technique. Existence theorems for solutions to two-point discrete boundary value problems follow, once the necessary assumptions regarding the (possibly nonlinear) boundary conditions are introduced.

Continuous and discrete three- (or more) point discrete boundary value problems for second-order, scalar equations are analyzed, subject to nonlinear three- (or more) point boundary conditions. Existence theorems follow, once the necessary assumptions regarding the multipoint boundary conditions are introduced.

Finally, conditions are formulated under which at least three disctinct solutions to two-point discrete boundary value problems for scalar, second-order, ordinary, differential equations exist, via the use of two pairs of lower and upper solutions and simple properties from degree theory.

The main advantage of all the aforementioned results is that they unify and apply to a very wide range of discrete boundary value problems in the literature and beyond.

Throughout the work, the new results are applied to real-world examples from such areas as: plankton population growth, chemical reactor theory, internal heat generation and deflections of an elastic string.

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