Martin Eugene Muldoon, Ph.D., Singular integrals whose kernels involve certain Sturm-Liouville functions, University of Alberta, Edmonton, Alberta, 1966. (Supervisor: L. Lorch)

The principal object of study here is the behaviour, as $\nu \to \infty$, of the integral

$$(1) \qquad \int_a^\infty f(t)\, w(\nu, t-x)\, dt ,$$

where $-\infty < a < x < \infty$, and for each $\nu > 0$, $w(\nu, t)$ is a suitably normalized solution, vanishing at ∞, of the differential equation

$$(2) \qquad d^2 w/dt^2 = [\nu^2 t + q(t)]\, w .$$

We assume that $q(t)$ is continuous for $a - x \le t < \infty$, and that $\int_{a-x}^\infty |t^{-1/2} q(t)|\, dt$ exists.

The following is a simplified form of one of the main results. Let $f(t)$ be defined on $a \le t < \infty$, where $-\infty < a < 0$, and suppose that (i) $f \in L(a, \infty)$, (ii) $f(0+)$ exists, and (iii) $f \in BV[a, 0]$; then

$$(3) \qquad \lim_{\nu \to \infty} \int_a^\infty f(t)\, w(\nu, t)\, dt = \frac{2}{3} f(0-) + \frac{1}{3} f(0+) .$$

We consider applications of (3) to integrals involving some well-known special functions which satisfy, on making appropriate changes of variable, differential equations of type (2). The special case $q(t) \equiv 0$ corresponds to $w(\nu, t) = \nu^{2/3} \mathrm{Ai}(\nu^{2/3} t)$, with the usual notation for the Airy function.

We show that if $w(\nu, t)$ is given by this expression involving the Airy Function, hypothesis (iii) in the above theorem cannot be weakened to: (iii') $f(0-)$ exists. This therefore raises the question of the summability analogue of (3). It turns out that the result corresponding to (3) holds under hypotheses (i), (ii) and (iii'), provided $w(\nu, t)$ in (3) is replaced by its Cesàro (C, k) mean, $k > 1/2$, and provided the integrals involved in the (C,k) process exist. This result is not valid for $k = 1/2$, at least in the case $w(\nu, t) = \nu^{2/3} \mathrm{Ai}(\nu^{2/3} t)$.

We show that when f belongs to a suitable class of functions and $\rho > 0$, the Gibbs phenomenon is exhibited at the point $x = 0$ by

$$\int_a^\infty f(t)\, \nu^\rho \; \mathrm{Ai}[\nu^\rho (t-x)]\, dt .$$

We consider some aspects of the Gibbs phenomenon as exhibited by this expression, in the case of (C, k) summability, $k > 0$.

Morton Abramson, Ph.D., Enumeration Methods in Combinatorial Analysis, McGill University, May 1967. (Supervisor: W.O.J. Moser)

Many elementary problems exist in Combinatorial Analysis which either have not been solved or are solved by using complicated difference equations and algebra. Elementary combinatorial methods are given in the thesis which solve a large number of such problems. One such class is the chessboard problems. For example, a simple combinatorial solution is given for the Rook-king problem: In how many ways can n kings be placed on an $n \times n$ board, one in each row and column so that no two attack each other? A more general problem, the enumeration of permutations of degree n containing exactly s of the $2(n-p+1)$, $p > 1$, sequences $(1, 2, 3, \ldots, p), (2, 3, 4, \ldots, p+1), \ldots, (n-p+1, n-p+2, n-p+3, \ldots, n)$ and/or $(p, \ldots, 3, 2, 1), (p+1, \ldots, 4, 3, 2), \ldots, (n, \ldots, n-p+3, n-p+2, n-p+1)$ is given.

Also formulae are given for the solution of a class of permutation problems called "restricted sequences". That is, the number of ways of choosing k integers from the n integers $1, 2, \ldots, n$ subject to certain restrictions. Further, each restricted choice corresponds to a restricted sequence of Bernoulli trials. Generalizations of some of Riordan's and Kaplansky's formulae and recurrence relations are given.