primarily it seems to be aimed, then the opening chapter, on mathematical prerequisites, is rather frightening. This is probably due partly to its condensed nature and partly to an unfamiliar philosophical ring in the language and notation used—thus the authors speak of " the truth set of a statement " rather than " the set representing an event ". It would be unfortunate if this acted as a deterrent to the reader because the book continues with a very readable account of the theory of finite Markov chains.

The treatment is very systematic and problems are well isolated so that they can be discussed without extraneous difficulties. In this respect the device of making ergodic states absorbing is used with great effect. Moreover the mathematics introduced is kept at as elementary a level as possible. One would expect that use of a fairly wide range of matrix theory would be required in this subject, because a finite Markov chain is completely defined by an initial probability vector and a transition matrix. But the authors succeed in carrying the theory through on elementary matrix operations, without using any canonical reduction theory. And fairly simple matrix expressions are arrived at for the quantities of practical interest such as means and variances of passage times.

Early in the text a long list of examples of simple Markov chains is given and these are used throughout for illustrative purposes. There are exercises for the reader at the end of each chapter, and in the final chapter applications in a wide variety of subjects are considered.

This would be a very useful text-book for an undergraduate course on finite Markov chains. It is self-contained and as elementary as is consistent with mathematical precision. As a reference book it suffers from the disadvantage of having no index, and a non-mathematical reader would probably have considerable difficulty in following the book in its entirety. S. D. SILVEY

KAC, MARK, *Statistical Independence in Probability, Analysis and Number Theory* (*Carus Mathematical Monographs No.* 12) (Math. Association of America, and John Wiley & Sons, 1959), xiv+93 pp., \$3.

It used to be fashionable to say that " probability theory is just measure theory plus the concept of statistical independence ". In a sense this is true; we can equate probabilities with measures, random variables with measurable functions, and expectations with integrals, and after defining the new concept of statistical independence we are then in a position to state and prove the strong law of large numbers, which relates probabilities to frequency ratios and so puts measure theory at the service of the statistician. One of the aims of this book is to show that " statistical " independence is not after all a new notion, but one already active below the surface in many branches of pure mathematics; thus the book opens with the remark that Vieta's formula

$$\frac{\sin x}{x} = \prod_{k=1}^{\infty} \cos \frac{x}{2^k}$$

can lead one straight to the statistical independence of the Rademacher functions and thence to the now standard mathematical model for coin tossing. The author then discusses " normal " numbers (e.g. Champernowne's number 0.1234567891011...), the convergence of $\Sigma \pm c_k$, Weyl's equidistribution theorem, and results in number theory due to Davenport, Erdös, Halberstam, Kac, Renyi, Schoenberg and Turan. A final section links ergodic theory with continued fractions (Khintchine's theorem is proved by Ryll-Nardzewski's method, using the individual ergodic theorem of G. D. Birkhoff). This final section helps to dissipate any feeling the reader might otherwise have acquired, that probabilists are *only* interested in independence situations. In fact the greater part of modern work in probability theory is devoted to the study of stochastic processes (families of non-independent random variables); for a good account of these the reader may turn to another book by Professor Kac appearing almost simultaneously with the present one. For both we are much in his debt.

D. G. KENDALL

MASSEY, H. S. W., AND KESTELMAN, H., *Ancillary Mathematics* (Sir Isaac Pitman & Sons, London, 1959), 990 pp., 75s.

The contents of this book are based on the recently revised syllabus for mathematics as an ancillary subject in the Special Honours Degree courses in Physics and Chemistry in the University of London, and the book is offered as a text-book primarily for first-year students. Its size is due in part to the authors' desire to include as many as possible of the topics proposed for the new syllabus, and in part to their conviction that a more detailed and rigorous training in mathematics is an essential prerequisite to further study in these fields. Some of the more recondite parts have been put in smaller type, an acknowledgment of the fact that they may be difficult for the immature student and may be omitted at a first reading.

The real number system is discussed in the opening chapter, a real number being defined as an infinite decimal, and linked with this is the notion of the limit of a sequence and the general principle of convergence. There is also an account of some inequalities useful in the discussion of convergence. The student beginning a course of mathematics at university will find this chapter formidable, and many with experience of teaching at this level will not be able to share the authors' view that so early an initiation into rigorous mathematics is sound teaching practice however desirable from other viewpoints.

The second chapter is an account of some elementary functions of a single variable introducing the ideas of continuity and convergence. Differentiation and differentials are the topics of the third chapter, the concept of a differential being carefully explained. The chapter on infinite series is in the precise style which characterises the book and is followed by accounts of the exponential function, defined as an infinite series, the logarithmic and hyperbolic functions. Maxima and minima, Taylor's series, and approximate solution of equations are grouped, discussion of them being based on the first mean-value theorem.

The chapter on determinants with applications to electrical problems is orthodox but less so are those on plane analytical geometry, calculus methods being used extensively, and the concept of an invariant being introduced. In addition to the straight line and circle the properties of the conics are investigated from the cartesian, polar and tangent-polar equations, in preparation for a discussion on central orbits in a subsequent chapter, and the general equation of the second degree is discussed.

In the first of three chapters on partial differentiation and related topics (the second and third chapters are later in the book), the elementary notions including Taylor's formula are dealt with competently, and no less satisfying are the subsequent chapters which contain non-linear transformations, differentials, curvilinear co-ordinates, and maxima and minima of two variables. The chapter on curve-tracing has less appeal and may prove difficult reading for the student.

The chapter on complex numbers does not differ much from the usual treatment found in undergraduate text-books and touches on the theory of functions of a complex variable and transformations. There is a short discussion of the field properties of complex numbers at the end.

There are four chapters devoted to the integral calculus and related topics in the first of which integration as the inverse operation to differentiation is considered and methods of integration are discussed extensively. The properties of the definite integral, defined as an increment in the primitive of the integrand, are discussed in