Abstract

We describe an organizing framework for the study of infinitary combinatorics. This framework is Čech cohomology. It describes ZFC principles distinguishing among the ordinals of the form ω_n. More precisely, this framework correlates each ω_n with an $(n+1)$-dimensional generalization of Todorcevic's walks technique, and begins to account for that technique's "unreasonable effectiveness" on ω_1.

We show in contrast that on higher cardinals κ, the existence of these principles is frequently independent of the ZFC axioms. Finally, we detail implications of these phenomena for the computation of strong homology groups and higher derived limits, deriving independence results in algebraic topology and homological algebra, respectively, in the process.

Abstract prepared by Jeffrey Bergfalk.
E-mail: jeffrey.bergfalk@univie.ac.at

Luca Carai, ***New Directions in Duality Theory for Modal Logic***, New Mexico State University, USA, 2021. Supervised by Guram Bezhanishvili. MSC: Primary 03B45, 06F25; Secondary 03B44, 54C30. Keywords: compact Hausdorff space, continuous relation, Gödel translation, intuitionistic logic, modal logic, tense logic.

Abstract

In this work we present some new contributions towards two different directions in the study of modal logic. First we employ tense logics to provide a temporal interpretation of intuitionistic quantifiers as "always in the future" and "sometime in the past." This is achieved by modifying the Gödel translation and resolves an asymmetry between the standard interpretation of intuitionistic quantifiers.

Then we generalize the classic Gelfand–Naimark–Stone duality between compact Hausdorff spaces and uniformly complete bounded archimedean ℓ-algebras to a duality encompassing compact Hausdorff spaces with continuous relations. This leads to the notion of modal operators on bounded archimedean ℓ-algebras and in particular on rings of continuous real-valued functions on compact Hausdorff spaces. This new duality is also a generalization of the classic Jónsson-Tarski duality in modal logic.

Abstract taken directly from the thesis.
E-mail: lcarai@unisa.it
URL: https://www.proquest.com/openview/5d284dbfb954383da9364149fa312b6f/1?pq-origsite=gscholar&cbl=18750&diss=y

Colin Jahel, ***Some Progress on the Unique Ergodicity Problem***, Université Claude Bernard Lyon 1, Villeurbanne, France, 2021. Supervised by Lionel Nguyen Van Thé and Todor Tsankov. MSC: Primary 37B05; Secondary 22F50, 03C15, 43A07. Keywords: Amenability, unique ergodicity, dynamics of topological groups, Fraïssé limits.

Abstract

This thesis is at the intersection of dynamics, probability and model theory. It focuses on a specialization of the notion of amenability: unique ergodicity.

Let G be a Polish group, i.e., a topological group whose topology is separable and completely metrizable. We call a G-flow the action of G on a compact space. A G-flow is said to be minimal if every orbit is dense.

A famous theorem of Ellis states that any Polish group G admits a unique universal minimal flow that we denote $\mathrm{M}(G)$. This means that for any minimal G-flow X there is a surjective G-map from $\mathrm{M}(G)$ to X. G is said to be amenable if every G-flow admits an invariant probability measure, and uniquely ergodic if every minimal flow admits a unique invariant probability measure.

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The notion of unique ergodicity relating to a group was introduced by Angel, Kechris and Lyons. They also ask the following question which is the main focus of the thesis: Let G be an amenable Polish group with metrizable universal minimal flow, is G uniquely ergodic?

Note that unique ergodicity is an interesting notion only for relatively large groups, as it is proved in the last chapter of this thesis that locally compact non compact Polish groups are never uniquely ergodic. This result is joint work with Andy Zucker.

The thesis includes proofs of unique ergodicity of groups with interesting universal minimal flows, namely the automorphism group of the semigeneric directed graph and the automorphism group of the 2-graph.

It also includes a theorem stating that under some hypothesis on a ω-categorical structure M, the logic action of $\mathrm{Aut}(M)$ on $\mathrm{LO}(M)$, the compact space of linear orders on M, is uniquely ergodic. This implies unique ergodicity for the group if its universal minimal flow happens to be the space of linear orderings. It can also be used to prove non-amenability of some groups for which the action of $\mathrm{Aut}(M)$ on $\mathrm{LO}(M)$ is not minimal. This result is joint work with Todor Tsankov.

Finally, the thesis also presents a proof that under the assumption that the universal minimal flows involved are metrizable, unique ergodicity is stable under group extensions. This result is joint work with Andy Zucker.

Abstract prepared by Colin Jahel.
E-mail: cjahel@andrew.cmu.edu
URL: http://math.univ-lyon1.fr/~jahel/doc/These.pdf

LILING KO. ***Towards Finding a Lattice that Characterizes the $> \omega^2$-Fickle Recursively Enumerable Turing Degrees.*** University of Notre Dame, USA. 2021. Supervised by Peter Cholak. MSC: 03D25, 03D55. Keywords: Fickle, Turing degrees, recursively enumerable, lattices, computable approximation, embedding.

Abstract

Given a finite lattice L that can be embedded in the recursively enumerable (r.e.) Turing degrees $\langle \mathcal{R}_{\mathrm{T}}, \leq_{\mathrm{T}} \rangle$, we do not in general know how to characterize the degrees $\mathbf{d} \in \mathcal{R}_{\mathrm{T}}$ below which L can be bounded. The important characterizations known are of the L_7 and M_3 lattices, where the lattices are bounded below $\mathbf{d}$ if and only if $\mathbf{d}$ contains sets of "*fickleness*" $> \omega$ and $\geq \omega^{\omega}$ respectively. We work towards finding a lattice that characterizes the levels above ω^2, the first non-trivial level after ω. We introduced a lattice-theoretic property called " 3-*directness*" to describe lattices that are no "*wider*" or "*taller*" than L_7 and M_3. We exhaust the 3-direct lattices L, but they turn out to also characterize the $> \omega$ or $\geq \omega^{\omega}$ levels, if L is not already embeddable below all non-zero r.e. degrees. We also considered upper semilattices (USLs) by removing the bottom meet(s) of some 3-direct lattices, but the removals did not change the levels characterized. This leads us to conjecture that a USL characterizes the same r.e. degrees as the lattice on which the USL is based. We discovered three 3-direct lattices besides M_3 that also characterize the $\geq \omega^{\omega}$-levels. Our search for a $> \omega^2$-candidate therefore involves the lattice-theoretic problem of finding lattices that do not contain any of the four $\geq \omega^{\omega}$-lattices as sublattices.

Abstract prepared by Liling Ko.
E-mail: ko.390@osu.edu
URL: http://sites.nd.edu/liling-ko/