

# Order and chaos in hydrodynamic BL Her models

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**Abstract.** Many dynamical systems of different complexity, e.g. 1D logistic map, the Lorentz equations, or real phenomena, like turbulent convection, show chaotic behaviour. Despite huge differences, the dynamical scenarios for these systems are strikingly similar: chaotic bands are born through the series of period doubling bifurcations and merge through interior crises. Within chaotic bands periodic windows are born through the tangent bifurcations, preceded by the intermittent behaviour. We demonstrate such behaviour in models of pulsating stars.

**Keywords.** convection, hydrodynamics, methods: numerical, stars: oscillations, Cepheids

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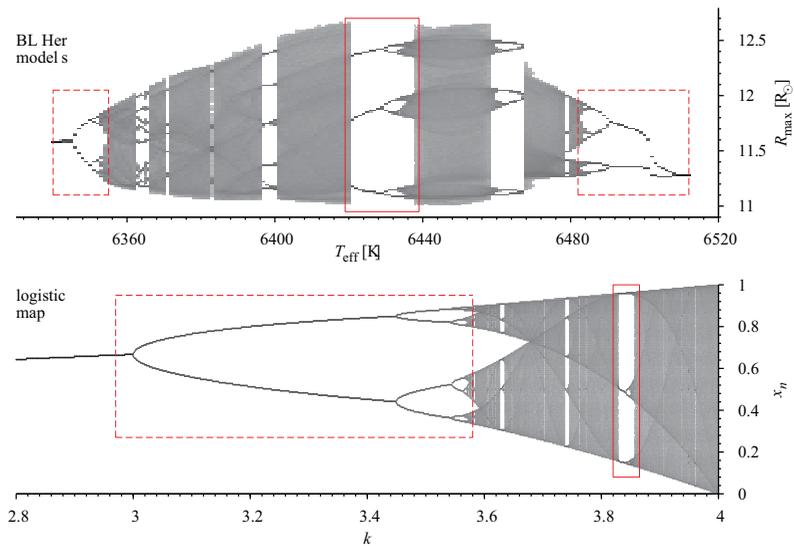
During our study of hydrodynamic BL Her-type models showing periodic and quasi-periodic modulation of pulsation akin to the Blazhko Effect (Smolec & Moskalik 2012) we also found the domains of chaotic behaviour. Here we report the initial analysis of these models, focusing on the universal behaviour they display.

All of our models were computed with the Warsaw nonlinear convective pulsation codes (Smolec & Moskalik 2008). The models have the same mass ( $0.55 M_{\odot}$ ) and chemical composition ( $X/Z = 0.76/0.0001$ ) and were computed along a horizontal strip ( $136 L_{\odot}$ ) in the H-R diagram with a maximum step in effective temperature of 1 K. Convective parameters are the same as in Smolec & Moskalik (2012). We note that the eddy-viscous dissipation was strongly decreased, resulting in too-large pulsation amplitudes as compared with observations. Yet, the models show a wealth of dynamical behaviours characteristic of deterministic chaos. Except for the period-doubling effect (Smolec *et al.* 2012), such behaviour was not detected in any BL Her star so far, but these models may help to understand the pulsation of more luminous, longer-period, irregular variables.

In Fig. 1 (top) we show the bifurcation diagram for our models: the histogram of possible values of maximum radii (gray shaded) plotted as a function of the control parameter for which we choose the effective temperature,  $T_{\text{eff}}$ . This diagram is compared with the bifurcation diagram for the simplest chaotic system, namely the iteration of the logistic map,  $x_{n+1} = kx_n(1 - x_n)$  (Fig. 1, bottom). Despite the huge differences between the two systems, striking similarities are apparent.

The chaotic bands are born through the period-doubling cascades (within dashed frames in Fig. 1). In case of our models the single-periodic (period-one) cycle (one point on the bifurcation diagram), found at both the cool and the hot edge of the computational domain, undergoes a series of period-doubling bifurcations en route to chaos. Period-two, period-four and period-eight cycles are all detected. In case of the logistic map the same scenario is computed as  $k$  is increased beyond  $k = 3.0$ .

Within the chaotic bands, periodic windows of order emerge. There are several such windows in case of our models, e.g. period-5, period-6, period-7 or period-9 windows. We stress that, contrary to recent claims (Plachy *et al.* 2013), stable periodic cycles are not necessarily caused by the resonances among pulsation modes, but may be an intrinsic



**Figure 1.** Bifurcation diagrams for hydrodynamic models (top) and the logistic map (bottom).

property of the chaotic systems. We focus our attention on parameter ranges indicated with solid frames in Fig. 1. Similarity between our models and the logistic map is apparent again. The well understood properties of simple logistic mapping allow us to explain the dynamics of the much more complex hydrodynamic models. As  $k(T_{\text{eff}})$  is increased the stable period-3 cycle is born through the tangent bifurcation. The bifurcation is preceded with *intermittent* behaviour (Pomeau & Manneville 1980) – evolution of the system is characterized by long phases of almost periodic behaviour interrupted with shorter bursts of chaos. As  $k(T_{\text{eff}})$  is increased further, the period-3 cycle undergoes a series of period-doubling bifurcations to form three chaotic bands. These bands finally merge through *interior crises* (Grebogi *et al.* 1982) – a bifurcation in which the volume occupied by chaotic attractor suddenly changes. The crises occurs when three chaotic bands collide with the unstable period-3 cycle born along with stable period-3 cycle in the tangent bifurcation. Detailed analysis of the presented models is in preparation.

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