PROCEDURE FOR VLBI ESTIMATES OF EARTH ROTATION PARAMETERS REFERRED TO THE NONROTATING ORIGIN

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ABSTRACT. This paper investigates the practical use of the nonrotating origin (NRO) (Guinot 1979) for estimating the Earth Rotation Parameters from VLBI data, which is based on the rotational transformation between the geocentric celestial and terrestrial frames as previously derived by Capitaine (1990). Numerical checks of consistency show that the transformation referred to the NRO is equivalent to the classical one referred to the equinox and considering the complete "equation of the equinoxes" (Aoki & Kinoshita 1983). The paper contains the expressions for the partial derivatives of the VLBI geometric delay to be used for the adjustment of the pole coordinates, UT1 and deficiencies in the two celestial coordinates of the Celestial Ephemeris Pole (CEP) in the multiparameters fits to VLBI data. The use of the NRO is shown to simplify the estimates of these parameters and to free the estimated UT1 parameter from the model for precession and nutation.

1. INTRODUCTION

The classical procedure for estimating the Earth Rotation Parameters (ERP) from VLBI observables refers, due to historical reasons, to the equinox of date. This leads to a coordinate transformation from the Terrestrial Reference System (TRS) to the Celestial Reference System (CRS) in which the concepts of precession, nutation and the celestial Earth's angle of rotation are mixed.

As VLBI observations are nearly not sensitive to the position of the ecliptic (and therefore of the equinox), but only to the position of the equator, the use of a coordinate transformation from the TRS to the CRS based both on the nonrotating origin (NRO) (Guinot 1979) and on the two celestial coordinates (Capitaine 1990) of the Celestial Ephemeris Pole (CEP) should be more convenient for deriving the ERP from VLBI observations.

The purpose of this paper is to investigate the practical use of this proposed transformation in the computation of the geometric delay for VLBI estimates of the ERP; it contains numerical checks of consistency between this transformation and the classical one and gives the expressions of the partial derivatives of the geometric delay with respect to the fundamental parameters.

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2. DEFINITION, USE AND POSITIONING OF THE NRO

2.1. Definition

Let (Oxyz) be the instantaneous system based on the instantaneous equator, its corresponding pole P and the NRO; the NRO has been defined by Guinot (1979) by the kinematical condition that when P moves in the CRS, the system (Oxyz) has no component of instantaneous rotation along the equator with respect to the CRS (Figure 1).

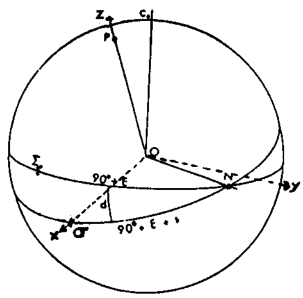

Figure 1: Kinematical definition of the NRO

The kinematical condition defining the NRO corresponds to a necessary concept to describe any motion of rotation along the moving equator. It allows to define a NRO in the CRS, denoted by σ, and also a NRO denoted by ϖ in the TRS, as an exact definition of the "instantaneous origin of the longitudes" (i.e. instantaneous prime meridian).

2.2. The use of the NRO for the representation of the Earth Rotation

The "stellar angle", $\theta = \widehat{\varpi O \sigma}$, gives the "specific Earth angle of rotation", such that $\dot{\theta}=\omega_Z$,and UT1 should therefore be conceptually defined as an angle proportional to θ (Guinot 1979).

2.3. Positioning of the NRO

The positioning of the NRO can be easily derived from the origin Σ_o on the fixed equator of the CRS by the use of the quantity s. A similar quantity s' is necessary for positioning ϖ in the TRS (Figure 2).

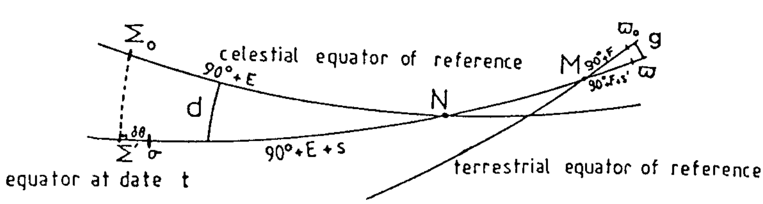

Figure 2: The positioning of the NRO

The quantity s can be written as (Guinot 1979): $s = \int^{t} (\cos d-1)\, \dot{E}\, dt$, which provides the position of σ on the moving equator as soon as the celestial motion of the CEP is known between the epoch t_0 and the date t. Its expression as a function of time has been shown to be nearly not sensitive to the model of the pole trajectory (Capitaine *et al.*1986).

3. THE COORDINATE TRANSFORMATION FROM THE TRS TO THE CRS TO BE USED IN THE VLBI EARTH ROTATION PARAMETERS ESTIMATES

Any procedure for parameter estimation from VLBI observables requires the calculation of the "geometric delay" : $\tau = -\vec{B}.\vec{K}/c$ (where $\vec{B}$ is the baseline vector, $\vec{K}$ is the unit vector pointing in the direction of the observed source and c is the velocity of light).

One step in such a calculation is to apply to the baseline vector in the TRS, the rotational transformation Q of coordinate frames from the TRS to the CRS:

$$[CRS] = Q\,[TRS],$$

Q being composed of several separate rotations.

3.1. The classical transformation

In the classical procedure Q is written as: $Q=Q_1.Q_2.Q_3$, such that, if $R_i(\eta)$ represents, as usual, the rotation matrix of angle η around the i-axis :

1) $Q_1 = R_3(\zeta_A).R_2(-\theta_A).R_3(z_A).R_1(-\varepsilon_A).R_3(\Delta\psi).R_1(\varepsilon_A+\Delta\varepsilon)$,
ε_A being the mean obliquity of the ecliptic at date t, and z_A, ζ_A, θ_A, $\Delta\varepsilon$, $\Delta\psi$ the usual precession and nutation quantities in right ascension, obliquity and ecliptic longitude respectively, referred to the mean ecliptic of epoch (or of date),

2) $Q_2 = R_3(-GST)$, GST being Greenwich True Sidereal Time at date t, including both the effect of Earth rotation and the precession and nutation in right ascension,

3) $Q_3 = R_1(y_p).R_2(x_p)$,
x_p and y_p being the "pole coordinates" of the CEP in the TRS,

3.2. The use of the NRO in the transformation from the TRS to the CRS

The use of the NRO allows us to separate Q into three independant rotation matrices, such that: $Q=Q_1.Q_2.Q_3$, each of the matrix Q_i corresponding to one component of the Rotation of the Earth around its center of mass:

1) $Q_1 = R_3(-E).\ R_2(-d).\ R_3(E).\ R_3(s)$,
for those rotations arising from the celestial motion of the CEP (see Fig 2), including the rotation s which takes into account the displacement of σ on the instantaneous equator due to the celestial motion of the CEP,

2) $Q_2 = R_3(-\theta)$ for the rotation of the Earth around the axis of the CEP,

3) $Q_3 = R_3(-s').R_3(-F).\ R_2(g).\ R_3(F) = R_3(-s'+x_py_p/2).R_1(y_p).R_2(x_p)$,
for those rotations arising from the terrestrial motion of the CEP (see Fig 2), including the rotation s' (Capitaine 1990) which takes into account the displacement of ϖ on the instantaneous equator due to polar motion.

4. NUMERICAL CHECKS OF CONSISTENCY OF THE COORDINATE TRANSFORMATIONS

The coordinate transformation from the TRS to the CRS has been applied to vectors in the terrestrial frame such that: ($r = 1$, $\varphi = 0°$, λ = 0h, 6h, 12h; $\varphi = 45°$, λ = 0h, 6h, 12h),
from t = 1900.0 to t = 2100.0, every 0.1 century.

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4.1. Numerical expressions to be used for the parameters referred to the NRO

In order to check the consistency of the coordinate transformation from the TRS to the CRS referred to the NRO with the classical one referred to the equinox, it is necessary to have consistent numerical expressions for the parameters used in the two transformations.

(1) Numerical relationship between θ and UT1
We have used the following relationship, which has been given by Capitaine *et al.* (1986) in order to be consistent with the conventional relationship between GMST and UT1 (Aoki *et al.* 1982): $\theta = 2\pi\,\{0.779\,057\,273\,264 + 1.002\,737\,811\,911\,354\,T_u\}$, (1)
T_u being the number of days elapsed since 2000 January 1, 12h UT1.

(2) Numerical expression for the celestial pole coordinates of the CEP
We have used the developments as functions of time of the coordinates X=sindcosE and Y=sindsinE of the CEP in the CRS as given by Capitaine (1990) with a consistency of 5×10^{-5}" after a century with the conventional developments for precession and nutation.

Each development, including both effects of precession and nutation, is the sum of a polynomial form of t, of periodic terms corresponding to the nutations and of pseudo-periodic terms arising from the cross terms between general precession and luni-solar nutations. It can be expressed as:

$$X(t) = X(t_0)+2004.310\,9''t-0.426\,65''t^2-0.198\,656''t^3+0.000\,014\,0''t^4+\Sigma_i(a_{io}+a_{i1}t)\sin(\omega_i t-\phi_i)$$
$$-0.000\,35''\sin2\Omega+\Sigma_j a'_{j1}t\cos(\omega_j t-\phi_j)+0.002\,04''t^2\sin\Omega+0.000\,16''t^2\sin2\odot+0.000\,06''t^2\cos\Omega \qquad (2)$$

$$Y(t) = Y(t_0)-22.409\,92''t^2+0.001\,836''t^3+0.001\,113\,0''t^4+\Sigma_i(b_{io}+b_{i1}t)\cos(\omega_i t-\phi_i)$$
$$+0.000\,13''\cos2\Omega+\Sigma_j b'_{j1}t\sin(\omega_j t-\phi_j)-0.002\,31''t^2\cos\Omega-0.000\,14''t^2\cos2\odot,$$

where $(a_{io}, b_{io})_{i=1,106}$ are the coefficients in longitude×sinε_o and obliquity of arguments $(\omega_i t-\phi_i)_{i=1,106}$ of the 1980 IAU nutation, and $a_{i1}, b_{i1}, a'_{j1}, b'_j$ are quantities lower than 5×10^{-5}", except for a few terms (5 for index i and 18 for index j).

(3) Numerical expression for the quantity s
We have used the numerical expression of s as derived, by the relation: $s=-\delta\theta-XY/2$, from the numerical values of X and Y and from the following numerical development of δθ, with an accuracy of 5×10^{-5}" after a century (Capitaine 1990):

$$\delta\theta = -0.003\,85''t+0.072\,59''t^3+0.002\,65''\sin\Omega+0.000\,06''\sin2\Omega$$
$$-0.000\,74''t^2\sin\Omega-0.000\,06''t^2\sin2\odot, \qquad (3)$$

which is equivalent to the expression of s previously given (Capitaine *et al.* 1986).

4.2. Internal checks for the transformation referred to the NRO

Three different forms of Q_1 have been tested, using the previous numerical developments (1), (2) (3) and a polar motion equal to zero (i.e. $x_p=y_p=s'=0$) :

(i): Q_1 as defined as a product of rotation matrixes, with: $E=\arctan(Y/X)$, $d=\arcsin\sqrt{X^2+Y^2}$

$$Q_1=R_3(-E).\,R_2(-d).\,R_3(E).\,R_3(s), \qquad (4)$$

(ii): Q_1 as given directly as a function of X and Y and s:

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$$Q_1 = \begin{pmatrix} 1-aX^2 & -aXY & X \\ -aXY & 1-aY^2 & Y \\ -X & -Y & 1-a(X^2+Y^2) \end{pmatrix} . R_3(s) \quad (5)$$

with $a = 1/(1+\cos d) = 1/2 + 1/8\,(X^2+Y^2) + \ldots$,

(iii): Q_1 as given with an accuracy better than 10^{-7}" after a century (Capitaine 1990), as a function of X, Y and $\delta\theta$ (symmetrical form of the matrix transformation Q_3):

$$Q_1 = \begin{pmatrix} 1-aX^2 & -2aXY+a^2X^3Y & X \\ 0 & 1-aY^2 & Y \\ -X(1+aY^2) & -Y(1-aX^2) & 1-a(X^2+Y^2) \end{pmatrix} . R_3(-\delta\theta) \quad (6)$$

The numerical checks of the transformation applied to the terrestrial vectors show the identity of the transformation (i) and (ii) and show moreover that (ii) and (iii) are in all cases equivalent with an accuracy better than 10^{-8}".

4.3. Numerical checks of consistency between the transformation referred to the NRO and the classical transformation

The proposed coordinate transformation using the most simple form (4) for Q_1 has been compared with the classical transformation for the terrestrial vectors as considered in the previous section, for the same period of time, assuming as previously, a polar motion equal to zero (i.e. $x_p=y_p=s'=0$). The numerical developments as given by (1), (2), (3) have been used for the transformation referred to the NRO, whereas for the classical transformation, $\zeta_A, z_A, \theta_A, \varepsilon_A$ are the precession parameters as given by Lieske *et al.* (1977), and $\Delta\psi$, $\Delta\varepsilon$ are the parameters as given by the IAU 1980 theory of nutation.

Greenwich Sidereal Time, GST, has been derived from the expression of Greenwich Mean Sidereal Time, $GMST_{0hUT1}$, as given by Aoki *et al*.(1982):

$$GMST_{0hUT1} = 24\ 110.548\ 41s + 8\ 640\ 184^s.812\ 866\ T'_u + 0.^s0931\ 04\ T'^2_u - 6.^s2\times10^{-6}\ T'^3_u, \quad (7)$$

with $T'_u = d_u/365\ 25$, d_u being the number of days elapsed since 2000 January 1, 12h UT1, taking on values ±0.5, ±1.5, ..., and from the periodic terms of the so-called "equation of the equinoxes", in two different forms:

(i) as: $GST = GMST + \Delta\psi\cos\varepsilon_A + 0.002\ 65" \sin\Omega + 0.000\ 06"\sin 2\Omega$, (8)
corresponding to the periodic terms as given by Woolard (1953), or by Aoki and Kinoshita (1983) for the "equation of the equinoxes" in a "wider sense",

(ii) as: $GST = GMST + \Delta\psi\cos\varepsilon_A$, (9)
corresponding to the "equation of the equinoxes" limited, as it is the general case, to the nutation in right ascension.

It should be noted that the expression (8) *of the equation of the equinoxes can only be obtained by using implicitely the concept of the NRO in order to express the accumulated precession and nutation on the moving equator between the epoch and the date.*

The numerical tests of consistency of the coordinate transformation referred to the NRO with the classical transformation referred to the equinox gives the following results *for the differences in the equatorial coordinates α and δ of the terrestrial vector in the CRS:*

(i) with the complete expression (8) for the equation of the equinoxes:

from 1900.0 to 2000.0: $0.4 \times 10^{-6}{}'' \leq \delta\alpha \leq 1 \times 10^{-4}{}''$

$0.4 \times 10^{-6}{}'' \leq \delta\delta \leq 1 \times 10^{-4}{}''$,

except for a very few cases for which the difference reachs $1.4 \times 10^{-4}{}''$

from 2000.0 to 2100.0: $0.1 \times 10^{-6}{}'' \leq \delta\alpha \leq 0.9 \times 10^{-4}{}''$

$0.5 \times 10^{-6}{}'' \leq \delta\delta \leq 0.9 \times 10^{-4}{}''$.

In most cases, from 1900.0 to 2100.0, the differences $\delta\alpha$, $\delta\delta$ are lower than $5 \times 10^{-5}{}''$, which is the order of the consistency of the used developments (1), (2), (3) with the conventional developments for precession, nutation, and Greenwich Sidereal Time (7) and (8).

(ii) with the incomplete expression (9) for the equation of the equinoxes, nearly identical results are obtained for $\delta\delta$, which is not sensitive to a rotation around the axis of the CEP, but periodic variations appear in the differences in right ascension, with the period of Ω and an amplitude of the order of 2 mas (see Table 1).

1900.0: $-2.2 \times 10^{-3}{}''$	1920.0: $-2.0 \times 10^{-3}{}''$	1940.0: $-1.1 \times 10^{-3}{}''$
1960.0: $-0.5 \times 10^{-5}{}''$	1980.0: $+1.2 \times 10^{-3}{}''$	2000.0: $+2.2 \times 10^{-3}{}''$

Table 1: examples of the differences in right ascension of the Gx terrestrial axis in the CRS resulting from the use of the incomplete expression (9) for the "equartion of the equinoxes"

Complementary calculations show, for example, that the difference is minimum in 1988 and 1997 and maximum in 1992 and 2002.

Such numerical checks show the consistency of the coordinate transformation referred to the NRO and using the numerical developments (1) for the stellar angle, (2) for the coordinates of the CEP in the CRS and (3) for the positioning of the NRO with the classical transformation referred to the equinox and using the conventional models for precession and nutation, Greenwich Mean Sidereal Time, as well as the complete expression (8) for the "equation of the equinoxes". Such numerical checks show moreover that the use of the incomplete equation of the equinoxes in the expression of Greenwich Sidereal Time at the date of the observation, as it is presently the case in the reduction of VLBI data, leads to a spurious periodic rotation around the axis of the CEP with an amplitude of a few mas. Such a rotation is probably mainly absorbed in the estimated UT1, which includes therefore a periodic error of this amplitude.

This results show the advantage of using the NRO as an explicit origin fo reckoning the Earth's rotation in order to derive an accurate UT1 parameter from the observations.

5. PROCEDURE FOR ERP ESTIMATES FROM THE VLBI OBSERVABLES

5.1. General case of the partial derivatives of the geometric delay with respect to the ERP

The ERP affect the expression of the VLBI geometric delay only through the orientation of the Earth as a whole. Therefore, if τ is the geometric delay in the geocentric frame, we have, for each of the ERP (Sovers and Fanselow 1987), K_i, L_k being the components of $\vec{K}$ and $\vec{L}$ in the CRS and the TRS respectively and Q_{ik} being the elements of the matrix Q:

$$\frac{\partial\tau}{\partial\eta} = \frac{K_i}{c}\frac{(\partial Q_{ik})}{\partial\eta} L_k,$$

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The corrections to the ERP can therefore be estimated by a least squares fit among the VLBI observed delays, using the expression for the partial derivatives of the rotational matrix transformation Q with respect to each of the parameter x_p, y_p, UT1, X and Y.

In the proposed transformation referred to the NRO, the coordinates X and Y of the CEP in the CRS appear only in the matrix Q_1, UT1 appears only in the matrix Q_2 and the coordinates x_p and y_p of the CEP in the TRS appear only in the matrix Q_3. Such a form of the rotational transformation Q simplifies the calculation of its partial derivatives with respect to the ERP as compared to the classical case in which the precession and nutation parameters appear in a complicate way both in Q_1 and Q_2.

The rotation matrix Q_2 being independent of the celestial pole coordinates X and Y, the UT1 parameter as estimated from VLBI observables using this transformation would be free from the errors on the model for precession and nutation. This represent an improvement as compared to the classical method in which the estimated UT1 is dependent, due to the used relationship between GST and UT1, on the precession and nutation model in right ascension.

The partial derivatives with respect to UT1 and the pole coordinates are, such that, k being the conversion factor between the stellar angle, θ, and UT1: $\dfrac{\partial Q}{\partial UT1} = -\dfrac{1}{k} Q_1 \cdot \dfrac{\partial R_3(\theta)}{\partial \theta} \cdot Q_3$,

$$\frac{\partial Q}{\partial x_p} = Q_1.Q_2.R_3(-s'+x_p y_p/2).R_1(y_p).\frac{\partial R_2(x_p)}{\partial x_p},\quad \frac{\partial Q}{\partial y_p} = Q_1.Q_2.R_3(s'+x_p y_p/2).\frac{\partial R_1(y_p)}{\partial y_p}.R_2(x_p),$$

the derivatives $\partial R_2(x_p)/\partial x_p$, $\partial R_1(y_p)/\partial y_p$, $\partial R_3(\theta)/\partial\theta$ being easily obtained as the derivative of a rotation matrix of angle η with respect to η.

The partial derivatives with respect to the coordinates of the CEP in the CRS are such that:

$$\frac{\partial Q}{\partial X} = \frac{\partial Q_1}{\partial X}.Q_2.Q_3,\quad \frac{\partial Q}{\partial Y} = \frac{\partial Q_1}{\partial Y} \cdot Q_2 \cdot Q_3 .$$

5.2. Expression of the partial derivatives with respect to the celestial coordinates of the CEP

For computing the partial derivatives of Q_1 with respect to X and Y, the forms (5) or (6) are the most convenient ones, as, firstly X and Y appear explicitly, and secondly, the partial derivatives of δθ or s with respect to X and Y can be neglected.

Such partial derivatives can be written as:

$$\frac{\partial Q_1}{\partial X} = \begin{pmatrix} -X-\frac{X^3}{2}-\frac{XY^2}{4} & -Y+\frac{Y^3}{4} & 1 \\ 0 & -\frac{Y^2}{4}X & 0 \\ -1-\frac{Y^2}{2} & XY+\frac{X^3Y}{2} & -X-\frac{X}{2}(X^2+Y^2) \end{pmatrix} .R_3(\delta\theta) \qquad (10)$$

$$\frac{\partial Q_1}{\partial Y} = \begin{pmatrix} -\frac{X^2Y}{4}(1+X^2+Y^2) & -X-\frac{3XY^2}{4} & 0 \\ 0 & -Y-\frac{Y^3}{2}-Y\frac{X^2}{4} & 1 \\ -XY-\frac{YX^3}{4} & -1+\frac{X^2}{2}+\frac{X^4}{8}+\frac{3X^2Y^2}{8} & -Y-\frac{Y}{2}(X^2+Y^2) \end{pmatrix} .R_3(\delta\theta) \qquad (11)$$

5.3. Interpretation of the estimated quantities dX and dY

The corrections dX and dY which can be estimated from VLBI observables, using the partial derivatives (10) and (11) of the rotational matrix Q_1, correspond to the deficiencies in the conventional model for the coordinates X and Y of the CEP in the CRS. They can be written as:

$dX = \xi_0 + \delta X$, $dY = \eta_0 + \delta Y$, where ξ_0, η_0 are for the constant offset between the pole of the CRS and the pole of the epoch of the model and δX, δY for the deficiencies in the models (including both precession and nutation) at the date of the observation. Using the developments of X and Y as given by Capitaine (1990), the deficiencies δX, δY in the models can be written, with an accuracy better than 5×10^{-5}" after a century, as: $\delta X = d\psi_A \sin\varepsilon_0 + d\Delta\psi \sin\varepsilon_0$, $\delta Y = d\varepsilon_A + d\Delta\varepsilon$, where $d\psi_A$ and $d\varepsilon_A$ are the errors on the model for precession and $d\Delta\psi$, $d\Delta\varepsilon$, the errors on the model for nutation.

dX and dY are the quantities to which the VLBI observations of radio-sources are actually sensitive, as they provide the position of the instantaneous equator with respect to the equator of the CRS. Such quantities are presently derived from VLBI observables on the form $d\psi \sin\varepsilon_0$ and $d\varepsilon$ by using a more complicated procedure involving separately precession and nutation of which the effects are in fact not separable using VLBI observations.

This shows the advantage of using X and Y as the two fundamental parameters for the celestial motion of the equator instead of the large number of the precession and nutation parameters generally considered.

6. CONCLUSION

The coordinate transformation from the TRS to the CRS using both the NRO and the coordinates of the CEP in the CRS has been shown to be numerically consistent with the classical one (when the complete equation of the equinoxes is used) with an accuracy better than 0.1mas. Such a representation, which clarifies the involved concepts, has been shown to simplify the estimates of the Earth Rotation Parameters from VLBI observables and to free the estimated UT1 parameter from the precession and nutation model.

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