GRAVITATIONAL RADIATION BY A COLLAPSING ROTATING STELLAR CORE

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ABSTRACT. The reduction of the ratio between specific angular momentum and mass (a/m) due to gravitational radiation has been investigated for a model of collapsing rotating stellar core in a post-newtonian scheme. Results show that if the initial value of the ratio is significantly greater than one, it tends towards one and if the initial ratio is nearly one, no sharp reduction happens as the core collapses to the neutron star or black hole size.

It was pointed out[1] that if one believes that neutron stars are the end product of massive stars of spectral type Be and O, then one needs to explain a reduction of the ratio between specific angular momentum and mass (a/m) of 3 or 4 orders of magnitude in the course of evolution from the main sequence to the neutron star stage. Recently we have investigated[2-4] the effect of gravitational radiation to the reduction of the ratio a/m during the collapse of the stellar core.

By the definition of the ratio, we have

$$(1) \qquad \frac{d}{dt}\left(\frac{a}{m}\right)=\frac{d}{dt}\left(\frac{J}{m^2}\right)=\left(\frac{a}{m}\right)\left(\frac{1}{J}\frac{dJ}{dt}-\frac{2}{m}\frac{dm}{dt}\right)$$

where J is the angular momentum. Here c=G=1 unit is used. For quadrupole gravitational wave emission, we have[5]

$$(2) \qquad \frac{dm}{dt}=-\frac{1}{5}\langle \dddot{D}_{jk}\dddot{D}^{jk}\rangle$$

$$(3) \qquad \frac{dJ_i}{dt}=-\frac{2}{5}\epsilon_{ijk}\langle \ddot{D}^{jl}\dddot{D}_l{}^k\rangle$$

where D_{ij} is the reduced quadrupole moment of the radiator, a dot denotes differentiation with respect to t and $\langle\cdots\cdots\rangle$ denotes a suitable averaging process. By defining $J=(J^iJ_i)^{1/2}$, formula (3) can be rewritten as

$$(4) \qquad \frac{dJ}{dt}=-\frac{2}{5}z^i\epsilon_{ijk}\langle \ddot{D}^{jl}\dddot{D}_l{}^k\rangle$$

where z^i is the unit direction vector of the angular momentum.

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We consider the collapsing stellar core as a homogeneous ellipsoid with semiaxes $a_1 > a_2 > a_3$ which is rotating around the z-axis with uniform angular velocity ω . It is reasonable to think the rotator is a fast one in the sense that the rotating period is shorter than the collapsing time scale. Keeping the post-newtonian approximation, we got a formula

$$(5) \qquad \frac{d\left(\frac{a}{m}\right)}{d\left(\frac{r}{r_g}\right)} = \frac{\frac{25}{32}\left(\frac{a}{m}\right)^4\left(\frac{r}{r_g}\right)^{-15/2}\chi^2\beta\left[4\left(\frac{r}{r_g}\right)^2 - 5\left(\frac{a}{m}\right)^2\right]}{1 - \frac{125}{64}\chi^2\beta\left(\frac{a}{m}\right)^6\left(\frac{r}{r_g}\right)^{-13/2}}$$

where $\chi = (a_1^2 - a_2^2)/(a_1^2 - a_2^2)$ is the ellipticity, $r = \left[(a_1^2 + a_2^2)/2\right]^{1/2}$ is the effective radius, $r_g = 2m$ and

$$(6) \qquad \beta = -\left(\frac{r_g}{r}\right)^{1/2}\frac{1}{\dot{r}}$$

denotes the ratio between the free fall velocity of a non rotating sphere having the same mass and r, and the actual velocity of collapse.

From the Eq.(3) we see that the value of $L = \chi^2\beta$ is relevent only for the last period of collapse. Therefore we can solve Eq.(5) numerically by taking L as a constant for the convenience in the calculations. The integration was started at $r/r_g = 300$ and various initial values of a/m have been used. The qualitative behavior of the solution is not sensitive to the parameter L. The following figure shows the results for a medial value L=5.0.

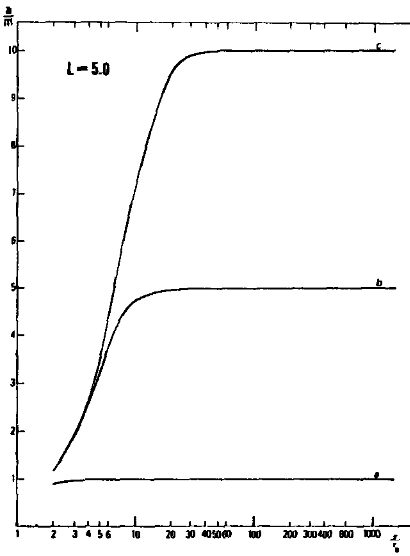

Figure 1. The reduction of the ratio a/m due to the gravitational radiation during the collapse period.

The implication of the calculational results can be summarized

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as follows:

1) whenever the initial ratio a/m is significantly greater than one, the gravitational radiation will reduce the ratio towards about one. As a post-newtonian result, the solution for small value of r/r_g is not consistent with our approximation. However, we feel that it may well give a qualitative indication of the behavior to be expected in the fully relativistic regime.

2) if the initial ratio a/m is only just greater than one, no sharp reduction of the ratio happens. When our configuration reaches neutron star or black hole size, the value of the ratio a/m still remains nearly one. This appears as a curious stiffness of the ratio a/m at the value of one.

REFERENCES

1) F. de Felice and Yu Yunqiang, J. Phys. A15 3341 (1982).
2) Yu Yunqiang, Ph.D Thesis in SISSA (Trieste) (1982) unpublished.
3) Yu Yunqiang, Journal of Peking Observatory, 5 45 (1983).
4) F. de Felice, J. Miller and Yu Yunqiang, SISSA preprint 39/84/A (1984); soon be published in Ap. J. (1985).
5) K. Thorne, Rev. Mod. Phys. 52 299 (1980).

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