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Productively Lindelöf Spaces May All Be D

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Abstract. We give easy proofs that (a) the Continuum Hypothesis implies that if the product of X with every Lindelöf space is Lindelöf, then X is a D-space, and (b) Borel's Conjecture implies every Rothberger space is Hurewicz.

1 Introduction

Definition 1.1 A topological space is a *D-space* if for every assignment f from points to open neighborhoods of them there is a closed discrete $D \subseteq X$ such that $\{f(x) : x \in D\}$ covers X.

D-spaces are currently a hot topic in set-theoretic topology (see two recent surveys [13, 15]). For the non-specialist, observe that a T_1 space is compact if and only if it is a countably compact D-space. The primary question of interest is whether every Lindelöf space is a D-space [12]. We shall assume all spaces are T_3.

Productively Lindelöf spaces, *i.e.,* spaces such that their product with every Lindelöf space is Lindelöf, have been studied in connection with two classic problems of E. A. Michael.

Problem 1 *Is the product of a Lindelöf space with the space of irrationals Lindelöf?*

Problem 2 *If X is productively Lindelöf, is X^ω Lindelöf? (We say X is* powerfully Lindelöf*, in this case.)*

For an extensive list of references concerning these problems see [29]. The primary result of note is due to Michael, being implicitly proved in [22]. It is explicitly stated and proved in [2].

Lemma 1.2 *The Continuum Hypothesis implies that productively Lindelöf metrizable spaces are σ-compact.*

Our tools include selection principles and topological games. As a byproduct, we obtain an easy proof of the consistency of every Rothberger space being Hurewicz (see definitions below).

Section 2 gives a self-contained easy proof of the result of the title. Sections 3 and 4 are more for specialists, varying the themes of Section 2. Section 5 contains a short proof of (b) of the abstract. Section 6 is built around a diagram of the relationships among the properties we have discussed.

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2 CH Implies Productively Lindelöf Spaces Are D

We shall give a short, reasonably elementary proof of the following result.

Theorem 2.1 *The Continuum Hypothesis implies productively Lindelöf spaces are D.*

We shall prove Theorem 2.1 by combining Lemma 1.2 with results of Aurichi [5] and Arhangel'skiĭ[4]. Theorem 2.1 is a considerable improvement over [6, 30], in which additional assumptions of separability, first countability, or sequentiality were required.

We require two definitions.

Definition 2.2 ([4]) A space is *projectively σ-compact* if its continuous image in any separable metrizable space is σ-compact.

Definition 2.3 A space X is *Menger* if whenever $\{\mathcal{U}_n\}_{n<\omega}$ are open covers of X, there are finite subsets $\mathcal{V}_n$ of $\mathcal{U}_n$, $n < \omega$, such that $\{\bigcup \mathcal{V}_n : n < \omega\}$ is an open cover.

This latter concept was introduced by Hurewicz [17] and has been studied under various names since then. In particular, some confusion arises because Arhangel'skiĭ calls this property *Hurewicz*. However, our terminology is generally accepted. A breakthrough on the Lindelöf D-problem occurred when Aurichi [5] proved the following lemma.

Lemma 2.4 *Menger spaces are D.*

Arhangel'skiĭ proved the following.

Lemma 2.5 *Projectively σ-compact spaces are Menger.*

Combining Lemmas 2.4 and 2.5, we need only establish that productively Lindelöf spaces are projectively σ-compact. But this follows quickly from Lemma 1.2, since continuous images of productively Lindelöf spaces are easily seen to be productively Lindelöf.

For the convenience of the reader, we sketch the proofs of Lemmas 1.2, 2.4, and 2.5.

Proof of Lemma 1.2 Embed X in $[0,1]^{\aleph_0}$. Since $[0,1]^{\aleph_0}$ has a countable base, by the Continuum Hypothesis (CH) we can take open subsets $\{U_\alpha\}_{\alpha<\omega_1}$ of $[0,1]^{\aleph_0}$ such that every open set about $Y = [0,1]^{\aleph_0} - X$ includes some U_α. By taking countable intersections, we can find a decreasing sequence $\{G_\beta\}_{\beta<\omega_1}$ of G_δ's about Y, such that every open set about Y includes some G_β. If X is not σ-compact, we can assume the G_β's are strictly descending. Pick $p_\beta \in (G_{\beta+1} - G_\beta) \cap X$. Put a topology on $Z = Y \cup \{p_\beta : \beta < \omega_1\}$ by strengthening the subspace topology to make each $\{p_\beta\}$ open. Then Z is Lindelöf, but $X \times Z$ is not, since $\{\langle p_\beta, p_\beta\rangle : \beta < \omega_1\}$ is closed discrete. ■

Proof of Lemma 2.5 By Engelking [14, 5.1.J(e)], given a Lindelöf space X and an open cover $\mathcal{U}$, there is a continuous $f\colon X \to Y$, Y separable metrizable and an open cover $\mathcal{V}$ of Y such that $\{f^{-1}(V) : V \in \mathcal{V}\}$ refines $\mathcal{U}$. Given a sequence $\{\mathcal{U}_n\}_{n<\omega}$ of such covers, find the corresponding f_n's, Y_n's, and $\mathcal{V}_n$'s. Then the diagonal product of the f_n's maps X onto a subspace $\hat{Y}$ of $\prod Y_n$. Then $\hat{Y}$ is σ-compact, hence Menger. So we can take finite subsets of the $\mathcal{V}_n$'s forming a cover and then pull them back to X to find the required finite subsets of the $\mathcal{U}_n$'s. ■

Proof of Lemma 2.4 [15] Suppose X is Menger and f is a neighborhood assignment for X. We play a game in which ONE chooses in the n-th inning an open cover $\mathcal{U}_n$ and TWO chooses a finite $\mathcal{V}_n \subseteq \mathcal{U}_n$. TWO wins if $\{\bigcup \mathcal{V}_n : n < \omega\}$ covers X. Hurewicz [17] proved X is Menger if and only if ONE has no winning strategy.

ONE starts by playing $\{f(x) : x \in X\}$. TWO responds with $\{f(x) : x \in S_0\}$. ONE then plays $\{f(x) : x \in S_0 \cup S : S$ a finite subset of X, $S \cap \bigcup\{f(x) : x \in S_0\} = \varnothing\}$. If TWO replies with $\{f(x) : x \in S_0 \cup S_1\}$, ONE plays

$$\{f(x) : x \in S_0 \cup S_1 \cup S : S \cap \bigcup\{f(x) : x \in S_0 \cup S_1\} = \varnothing\},$$

etc. This defines a strategy for ONE. Since X is Menger, this is not a winning strategy. Let $S_0, \ldots, S_n, \ldots$ be the plays of TWO demonstrating this. Then $\bigcup_{n<\omega} S_n$ is closed and discrete, and $\bigcup\{f(x) : x \in \bigcup_{n<\omega} S_n\}$ covers X. ■

3 Variations on the Theme

We now move on to more specialized results. Since finite powers of productively Lindelöf spaces are productively Lindelöf, we have the following result.

Theorem 3.1 *The Continuum Hypothesis implies that all finite powers of a productively Lindelöf space are Menger and hence D.*

Definition 3.2 A γ-*cover* of a space is a countably infinite open cover such that each point is in all but finitely many members of the cover. A space is *Hurewicz* if, given a sequence $\{\mathcal{U}_n : n \in \omega\}$ of γ-covers, there is for each n a finite $\mathcal{V}_n \subseteq \mathcal{U}_n$, such that either $\{\bigcup \mathcal{V}_n : n \in \omega\}$ is a γ-cover or else for some n, $\bigcup \mathcal{V}_n$ is a cover.

This property was also introduced in [17]. It falls strictly between "Menger" and "σ-compact". Our results can be improved to obtain the following theorem.

Theorem 3.3 *The Continuum Hypothesis implies that finite powers of productively Lindelöf spaces are Hurewicz.*

The proof of Theorem 3.3 is a straightforward modification of what we have done for Menger.

Problem 3 *Are any of our uses of the Continuum Hypothesis necessary?*

The assumption of CH in our results can be weakened somewhat. Let $\mathfrak{b}$ be the least cardinal of a subset B of ${}^\omega\omega$ that is unbounded under eventual dominance. Then CH implies $\mathfrak{b} = \aleph_1$.

Theorem 3.4 $\mathfrak{b} = \aleph_1$ *implies that every productively Lindelöf space is Menger and hence D.*

Proof Defining *projectively Menger* in the obvious way, what Arhangel'skiĭ really proved above was that *Lindelöf projectively Menger spaces are Menger*, which indeed was later proved specifically in [10]. Thus our result follows, since Alas et al. [1] proved that $\mathfrak{b} = \aleph_1$ implies productively Lindelöf metrizable spaces are Menger. ■

Corollary 3.5 *Every productively Lindelöf space that is the union of $\leq \aleph_1$ compact sets is Menger and hence D.*

Proof This was proved for spaces of countable type, hence in particular for metrizable spaces in [1]. Our result follows, since if a space is the union of $\aleph_1$ compact sets, so also is its continuous image. ■

We can remove CH from Theorem 2.1 by strengthening the hypothesis. In [6] we defined a space to be *indestructibly productively Lindelöf* if it remained productively Lindelöf in any countably closed forcing extension.

Theorem 3.6 *Indestructibly productively Lindelöf spaces are projectively σ-compact and hence Hurewicz, Menger, and D.*

Proof Let $f\colon X \to Y$, Y separable metrizable. Collapse $\max(w(X), |X|, 2^{\aleph_0})$ to $\aleph_1$ by countably closed forcing. In the extension, X is productively Lindelöf, Y is separable metrizable, and f is continuous. Therefore Y is σ-compact. Countably closed forcing adds no new closed sets to separable metrizable spaces, so $Y = \bigcup_{n<\omega} F_n$, where the F_n's are in the ground model. The F_n's are countably compact in the ground model, and so they are in fact compact there. No new countable decompositions of Y are added by the forcing, so indeed Y is σ-compact in the ground model. ■

Earlier [30] we had obtained the Hurewicz and Menger conclusions, but this new result is stronger. Similarly, in [6] we proved that $\mathfrak{d} = \aleph_1$ implied that productively Lindelöf metrizable spaces are Hurewicz.

Theorem 3.7 *$\mathfrak{d} = \aleph_1$ implies productively Lindelöf spaces are Hurewicz.*

Corollary 3.8 *Every productively Lindelöf space that is the union of $\leq \aleph_1$ compact sets is Hurewicz.*

The corollary follows since it was proved from $\mathfrak{d} > \aleph_1$ in [30].

A finer analysis leads to the following result.

Corollary 3.9 *Every productively Lindelöf space that is the union of $\leq \aleph_1$ compact sets is projectively σ-compact.*

Proof Since every metrizable space is of countable type, the corollary follows immediately from [1], which stated that every productively Lindelöf space of countable type that is the union of $\leq \aleph_1$ compact sets is σ-compact. ■

Also in [30], we proved that $\mathrm{Add}(\mathcal{M}) = 2^{\aleph_0}$ implies productively Lindelöf metrizable spaces are Hurewicz. Recall $\mathrm{Add}(\mathcal{M})$ is the least κ such that there are κ many first category subsets of $\mathbb{R}$ with union not of first category.

By the usual reasoning, we have the following result.

Theorem 3.10 $\text{Add}(\mathcal{M}) = 2^{\aleph_0}$ *implies productively Lindelöf spaces are Hurewicz.*

Definition 3.11 ([16]) A *Michael space* is a Lindelöf space such that its product with $\mathbb{P}$, the space of irrationals, is not Lindelöf. A space is *K-analytic* if it is the continuous image of a Lindelöf Čech-complete space.

In [30] we asked whether it is consistent that every productively Lindelöf K-analytic space is σ-compact. We now know this holds under CH, but we can considerably weaken that hypothesis and still get that such spaces are projectively σ-compact (and hence D, *etc.*).

Theorem 3.12 *If there is a Michael space, then productively Lindelöf K-analytic spaces are projectively σ-compact.*

Proof Let X be Lindelöf Čech-complete, g map X onto Y, Y productively Lindelöf, and f map Y onto a separable, metrizable Z. Then Z is K-analytic. Then Z is analytic, since K-analytic subspaces of separable metrizable spaces are analytic; see [16, Theorems 2.1(f), 3.1(d)]. But in [29] we proved that if there is a Michael space, then productively Lindelöf analytic metrizable spaces are σ-compact. ■

There is a Michael space if either $\mathfrak{b} = \aleph_1$ ([21]) or $\mathfrak{d} = \text{cov}(\mathcal{M})$ ([24]).

4 Playing with Projectively σ-Compact Spaces

In this section, we assume some acquaintance with topological games, as in [27]. The players will be ONE and TWO, the games will be of length ω, and *strategies* are perfect information strategies.

Telgársky [31] proved the following lemma.

Lemma 4.1 *A metrizable space is σ-compact if and only if TWO has a winning strategy in the Menger game for X.*

We defined the Menger game above in the process of proving Lemma 2.4. Scheepers [26] provided a more accessible proof of Lemma 4.1, noting that metrizability was only needed in the proof for the backward implication. Banakh and Zdomskyy [7] have weakened "metrizable" to "hereditarily Lindelöf.

Problem 4 *Is a Lindelöf space projectively σ-compact if and only if TWO has a winning strategy in the Menger game for X?*

Okunev has constructed an example of a projectively σ-compact Lindelöf space that is not σ-compact [4]. There is also an example in [25]; see below.

We can prove one direction. Suppose X is not projectively σ-compact. Then there is an $f\colon X \to Y$ separable metrizable such that Y is not σ-compact. Suppose there were a winning strategy for TWO in the Menger game on X. We can define a strategy for the Menger game on Y by simply playing given an open cover $\mathcal{W}$ of Y (and previous information) the finite subset $\mathcal{W}'$ of $\mathcal{W}$ such that $\{f^{-1}(W) : W \in \mathcal{W}'\}$ is the move of TWO for the cover $\{f^{-1}(W) : W \in \mathcal{W}\}$ (and the corresponding previous information). Then since the ω-sequence of moves for X would yield a cover, their images would yield a cover of Y. Thus a winning strategy for X entails a winning strategy for Y. But Y is not σ-compact, so there is no such winning strategy for it, and hence none for X.

The diagram in Figure 1 shows the relationships among the properties we have discussed in this article. A more extensive diagram with many more Lindelöf properties can be found in [6], but it does not mention projective σ-compactness, which is our main concern here. Examples showing that implications do not reverse can be found there and below. For projective σ-compactness, there are two relevant examples in addition to Okunev's.

Example 4.2 *A Hurewicz space that is not projectively σ-compact.* Simply take a Hurewicz set of reals that is not σ-compact [18]. One can even get an example where finite products are Hurewicz [32].

Example 4.3 *A projectively σ-compact space that is not productively Lindelöf.* J. T. Moore [25] constructed a Lindelöf space X such that some finite power of X is not Lindelöf [33], but any continuous real-valued function on X has countable range. It follows that any continuous function f on X into any separable metrizable space Y has countable range. To see this, embed Y in $[0,1]^{\aleph_0}$. If $f(X)$ has uncountable projection onto any factor of $[0,1]^{\aleph_0}$, we have a contradiction, so $f(X) \subseteq \prod_{n<\omega} \pi_n(f(X))$, where each factor is countable and hence 0-dimensional, so $\prod_{n<\omega} \pi_n(f(X))$ is 0-dimensional and hence embeds in a Cantor set included in $\mathbb{R}$, so indeed $f(X)$ is countable.

5 Borel's Conjecture Implies Rothberger Spaces Are Hurewicz

As another example of the utility of projective σ-compactness, we shall prove the following theorem.

Theorem 5.1 *Assume Borel's Conjecture. Then every Rothberger space is Hurewicz.*

Rothberger is a strengthening of *Menger* in that picking *one* element from each member of the sequence of open covers suffices to yield a cover.

Definition 5.2 A set of reals X has *strong measure zero* if and only if given any sequence $\{\varepsilon_n\}_{n<\omega}$, $\varepsilon > 0$, X can be covered by $\{X_n : n < \omega\}$, each X_n having diameter less than ε_n.

Borel [11] conjectured that every strong measure zero set is countable. Laver [20] proved the consistency of Borel's Conjecture. Zdomskyy [34] proved that every paracompact Rothberger space is Hurewicz, assuming $\mathfrak{u} < \mathfrak{g}$. We refer either to his paper or to [9] for the definitions of these cardinals. Scheepers and Tall [28] observed that paracompactness could be eased to regularity in Zdomskyy's theorem. The hypothesis of Zdomskyy's theorem is sophisticated and the proof is non-trivial. Our proof of Theorem 5.1 is very easy.

Proof of Theorem 5.1 Suppose X is Rothberger. Then so is every continuous image of X. Rothberger subsets of the real line have strong measure zero (see [23]) and by

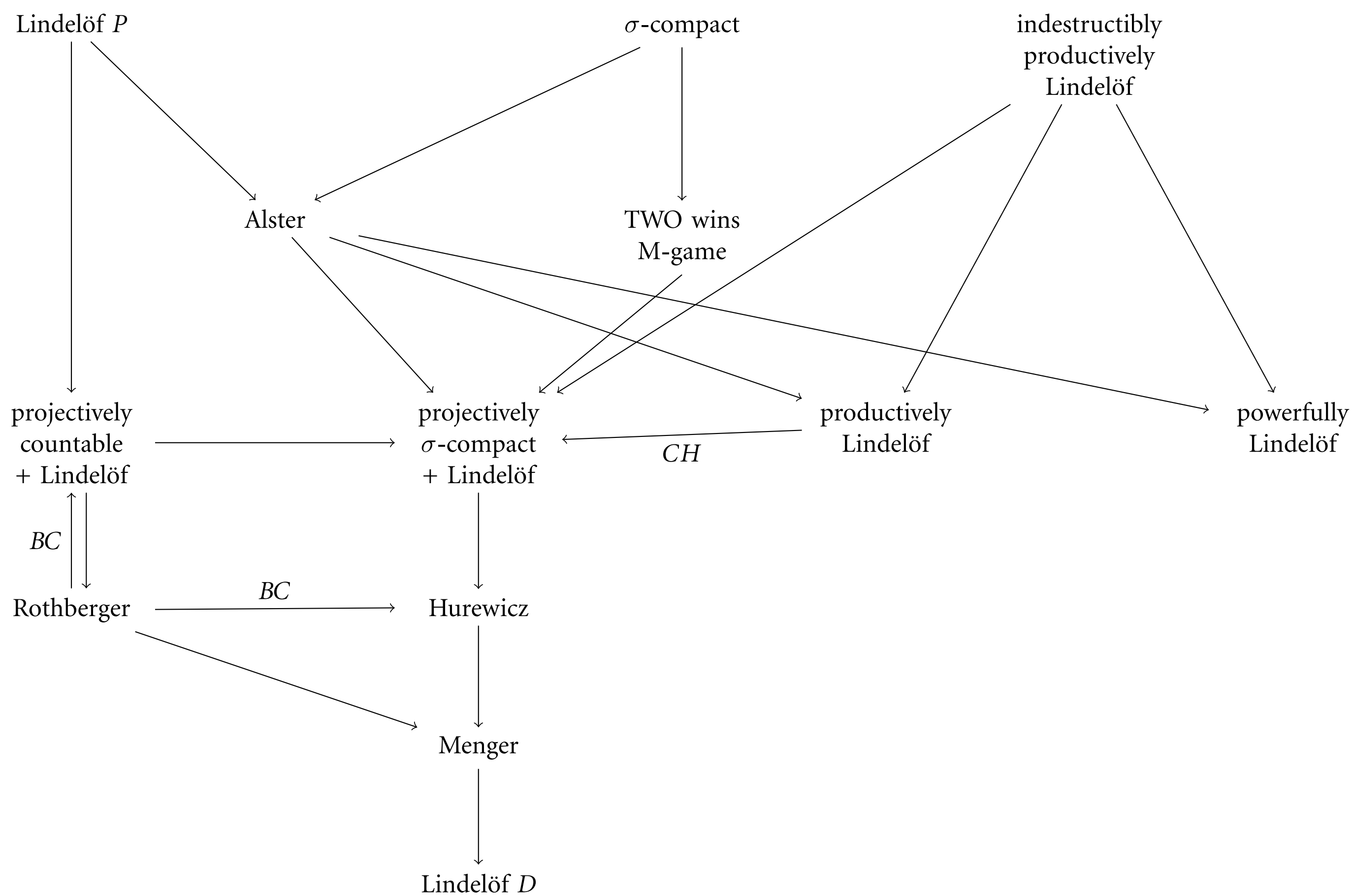

Figure 1: The relationships among various properties discussed.

Borel's Conjecture are therefore countable. By the same argument as for Example 4.3, X is projectively σ-compact. But then it is Hurewicz. ■

Marion Scheepers pointed out that Borel's Conjecture does *not* follow from $\mathfrak{u} < \mathfrak{g}$, since there is a model of Borel's Conjecture in which $\mathfrak{b} = \aleph_1$, which implies there is an uncountable set of reals concentrated about a countable set. Such a set is Rothberger. See [9] for reference to such a model.

Call a space *projectively countable* if its continuous image in any separable metrizable space is countable. We then have the following theorem.

Theorem 5.3 ([10]) *Borel's Conjecture implies a space is Rothberger if and only if it is Lindelöf and projectively countable.*

Proof That Rothberger implies Lindelöf is obvious. We have already proved that Borel's Conjecture implies Rothberger spaces are projectively countable. The converse is proved by the usual technique; indeed Lindelöf projectively Rothberger spaces are Rothberger [19]. ■

6 Implications and Not

Definition 6.1 ([3, 8]) A space X is *Alster* if every cover $\mathcal{G}$ by G_δ's has a countable subcover, provided that for each compact subset K of X, some finite subset of $\mathcal{G}$ covers K.

Alster spaces are important in the study of Michael's problems, since they are both productively Lindelöf and powerfully Lindelöf [3]. Clearly σ-compact spaces are Alster, but not necessarily vice versa [3, 8]. It is not known if productively Lindelöf spaces are Alster or even powerfully Lindelöf; see [3, 6, 30]. Alster spaces in which compact sets are G_δ's are σ-compact, so powerfully Lindelöf spaces need not be Alster. In addition to σ-compact spaces, Lindelöf *P-spaces* (G_δ's are open) are Alster [3, 8]. Alster spaces are projectively σ-compact, but even projective countability is insufficient to imply Alster. To see this, note that Moore's L-space X is projectively countable but not Alster, since some finite power of X is not Lindelöf.

Problem 5 *Does Alster imply TWO has a winning strategy in the Menger game? Is the converse true?*

We have proved or given references already for almost all of the non-obvious implications in the diagram below. That "indestructibly productively Lindelöf" implies "powerfully Lindelöf" is in [30]. To see that Lindelöf P-spaces are projectively countable, observe that if X is P and Y has points G_δ and $f\colon X \to Y$, then the inverse images of points in Y form a disjoint open cover of X.

Moore's L-space is projectively countable but not Alster nor P since closed subsets are G_δ's. As mentioned, it is neither productively Lindelöf nor powerfully Lindelöf. 2^{ω_1} is compact but it is not P and is not indestructibly productively Lindelöf [6]. A Bernstein (totally imperfect) set of reals is powerfully Lindelöf but not productively

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Lindelöf [22]. (See [18, 32] for examples of sets of reals that are Menger, but not Hurewicz, and Hurewicz but not (projectively) σ-compact.) The space of irrationals is Lindelöf *D* but not Menger. It is consistent that there are Rothberger spaces that are not Hurewicz. See the discussion in [28, §3].

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