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The laws of some nilpotent groups of small rank

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It is known ([3], p. 100) that every nilpotent variety of class m is generated by its free group of rank m. This applies in particular to the variety $\underline{\underline{N}}_m$ of all nilpotent groups of class at most m. The question arises which free groups of rank less than m still generate the variety.

A conjecture on this is contained in [3], but this conjecture and some of the supporting evidence offered there have meanwhile been proved false, independently by L.G. Kovács, M.F. Newman, P.F. Pentony [1] and Frank Levin [2]. They prove that if m is an integer greater than 2, then the variety $\underline{\underline{N}}_m$ of all nilpotent groups of class at most m is generated by its free group $F_{m-1}\left(\underline{\underline{N}}_m\right)$ of rank $m-1$ but not by its free group $F_{m-2}\left(\underline{\underline{N}}_m\right)$ of rank $m-2$. Frank Levin has some more information, namely that the variety generated by the free group $F_{k-1}\left(\underline{\underline{N}}_m\right)$ is properly contained in that generated by $F_k\left(\underline{\underline{N}}_m\right)$ for $k \leq m-1$.

From these results we see that the free groups $F_k\left(\underline{\underline{N}}_m\right)$, $2 \leq k \leq m-2$, do not generate $\underline{\underline{N}}_m$. In general little is known of the varieties generated by them. However, we do know that their laws have finite basis ([3], p. 89).

The purpose of the present thesis is to determine the varieties of some free groups of small rank, namely, $F_2(\underline{\underline{N}}_5)$, $F_3(\underline{\underline{N}}_5)$, $F_2(\underline{\underline{N}}_6)$,

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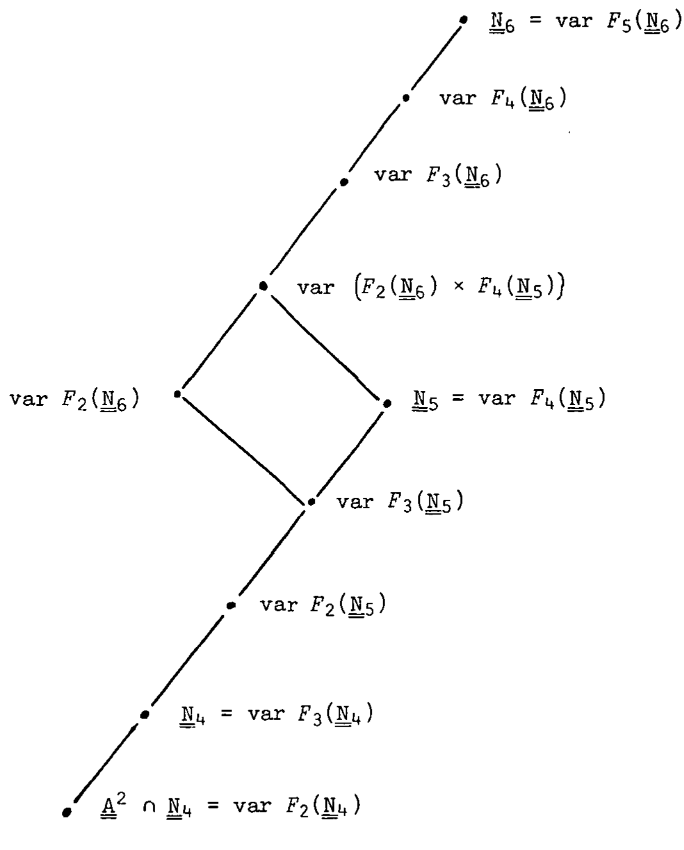
$F_3(\underline{\underline{N}}_6)$, $F_4(\underline{\underline{N}}_6)$, or equivalently, to determine a basis for the laws in these groups. This has been accomplished in all the cases mentioned. It turns out that for each of the free groups $F_2(\underline{\underline{N}}_5)$, $F_3(\underline{\underline{N}}_5)$, $F_3(\underline{\underline{N}}_6)$, $F_4(\underline{\underline{N}}_6)$, a basis consists of laws which are products of commutators of maximal weight, that is of weight five and six respectively. In $F_2(\underline{\underline{N}}_6)$, however, a basis includes a law that is of weight five.

The following diagram depicts the lattice formed by these varieties.



References

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[3] Hanna Neumann, *Varieties of groups* (Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 37, Springer-Verlag, Berlin, Heidelberg, New York, 1967).