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## ERRATA

### VOLUME 34

Page 108, line 10 from below. For «*foundation*», read «*foundations*».

### VOLUME 38

Page 720, column 2, line 18. For «*moda*», read «*modal*».

### VOLUME 40

Page 602, line 19 from below. For «L1» at the beginning of the line, read «**L1**».

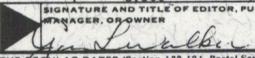
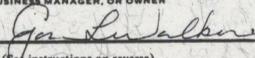
Page 692, column 1, line 10. For «XXXIV 320», read «XXXIV 310».

Page 713, column 2, line 2 from below. For «XXXIX 257», read «XXXIX 357».

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