IMPACT OF THE QUADRUPOLE MOMENT OF THE SUN ON THE DYNAMICS OF THE EARTH-MOON SYSTEM

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Abstract. Range of values of the Sun's mass quadrupole moment of coefficient J_2 arising both from experimental and theoretical determinations enlarge across literature on two orders of magnitude, from around 10^{-7} until to 10^{-5}. The accurate knowledge of the Moon's physical librations, for which the Lunar Laser Ranging data reach an outstanding precision level, prove to be appropriate to reduce the interval of J_2 values by giving an upper bound of J_2. A solar quadrupole moment as high as $1.1\ 10^{-5}$ given either from the upper bounds of the error bars of the observations, or from the Roche's theory, is not compatible with the knowledge of the lunar librations accurately modeled and observed with the LLR experiment. The suitable values of J_2 have to be smaller than $3.0\ 10^{-6}$.

As a consequence, this upper bound of $3.0\ 10^{-6}$ is accepted to study the impact of the Sun's quadrupole moment of mass on the dynamics of the Earth-Moon system. Such an effect (with $J_2 = 5.5 \pm 1.3 \times 10^{-6}$) has been already tested in 1983 by Campbell & Moffat using analytical approximate equations, and thus for the orbits of Mercury, Venus, the Earth and Icarus. The approximate equations are no longer sufficient compared with present observational data and exact equations are required. As if to compute the effect on the lunar librations, we have used our BJV relativistic model of solar system integration including the spin-orbit coupled motion of the Moon. The model is solved by numerical integration. The BJV model stems from general relativity by using the DSX formalism for purposes of celestial mechanics when it is about to deal with a system of n extended, weakly self-gravitating, rotating and deformable bodies in mutual interactions.

The resulting effects on the orbital elements of the Earth have been computed and plotted over 160 and 1600 years. The impact of the quadrupole moment of the Sun on the Earth's orbital motion is mainly characterized by variations of $\dot{\Omega}$, $\dot{\omega}$, and $\dot{E}$. As a consequence, the Sun's quadrupole moment of mass could play a sensible role over long time periods of integration of solar system models.

Key words: Sun, Quadrupole, Moon, Libration, Earth, LLR data
Abbreviations: LLR: Lunar Laser Ranging; **GR:** general relativity; **PN:** post-Newtonian; **mas:** milliarcsecond.

1. Introduction

Despite recent advances in space (SOHO) or ground-based observations (helioseismology networks) of solar oscillations, we are still unaware of the real value of the Sun's mass quadrupole moment of coefficient J_2. Range of values of $J_{2\odot}$ arising both from experimental and theoretical determinations enlarge across literature on two orders of magnitude, from around 10^{-7} until to 10^{-5}. An assessment of various theoretical values compared to available observations has been recently carried out by Rozelot and Bois (1998). The theoretical value strongly depends on the solar model used whereas accurate measurements are very difficult to obtain. If $J_{2\odot}$ exceeds a specific value, then GR alone cannot make up the remainder of the precession rate of the Mercury's perihelion. As an error of 0.1 % is generally admitted on the observed rate, it turns out that this test is not enough revealing.

In the other hand, most of the current observational experiments access to a solar oblateness resulting from combined effects in interactions such as the global

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gravitational field, the internal magnetic field, the general internal rotation rate, or the mass and angular velocity distributions and the surface rotation of the sun. The $J_{2\odot}$ values derived from an oblateness Δr or inferred by inverting some theoretical equations have not to be mistake for a dynamical oblateness directly measured by experiments of gravitational dynamics. Let us stress that a J_2 quadrupole moment of mass is a dynamical coefficient strictly related to the gravitational figure. Now, the understanding of the gravitational figure of the Sun is also a way to infer its internal structure and to approach the core rotation problem.

The accurate knowledge of the Moon's physical librations, for which the LLR data reach an outstanding precision level (i.e. 1 cm for the distance Earth-Moon, 1 mas for the lunar librations) prove to be appropriate to reduce the interval of $J_{2\odot}$ values by giving an upper bound of $J_{2\odot}$. The method is explained in the present paper. We find that the suitable values of $J_{2\odot}$ have to be smaller than 3.0 10^{-6}.

As a consequence, this upper bound (coinciding with the largest value that GR can accommodate by fitting to planetary data) is accepted to study the impact of the Sun's quadrupole moment of mass on the dynamics of the Earth-Moon system. A first attempt in this way has been carried out by Campbell & Moffat (1983) who computed the effects obtained by taking into account a $J_{2\odot}$ in the analytical equations used to determine the orbital elements ϖ, Ω and i of Mercury, Venus, the Earth and Icarus (the experiment was performed with J_2 equal to $5.5 \pm 1.3 \times 10^{-6}$). Calculations didn't show any significant discrepancy. This was mainly due to a lack of sufficiently accurate observational data in the determination of the planetary orbits, in particular the Icarus' one. However, as predicted by the authors, if the accuracy of the observational data were to improve enough, then using the approximate equations would produce incorrect results. Since the analytical equations are linear in J_2, any increase in J_2 would magnify this shift. Consequently, we have used a solar system model of complete equations solved by numerical integration and built in accordance with the requirements of current observational accuracy given by the LLR experiment.

2. The Model *BJV* of Relativistic Integration of the Solar System

In order to study the impact of $J_{2\odot}$ on the Earth-Moon system dynamics, we have used a gravitational model of the solar system including the Moon's spin-orbit motion. This model, called BJV, was previously constructed by Bois, Journet and Vokrouhlický in accordance with the requirements of LLR observational accuracy (see previous papers: Bois et al. 1992; Bois & Journet 1993; Bois & Vokrouhlický 1995; Bois et al. 1996).

The approach consists in integrating the n - body problem on the basis of the gravitation description given by the Einstein's general relativity theory. The BJV model stems from GR by using the DSX formalism presented in a series of papers by Damour, Soffel and Xu (Damour et al. 1991, 1992, 1993, 1994). It is the most suitable formulation of the post-Newtonian theory of motion of a system of n weakly self-gravitating extended bodies for purposes of celestial mechanics. The DSX formalism is derived from the first post-Newtonian approximation level. Gravitational fields of the extended bodies are parameterized in multipole moment expansions: (M_L^A, S_L^A) define the mass and spin Blanchet-Damour multipoles characterizing the PN gravitational field of the extended bodies while (G_L^A, H_L^A) are tidal gravitoelectric and gravitomagnetic PN fields. Because we do not dispose of dynamical equations for the quadrupole moments M_{ab}^A, and although the notion of rigidity faces conceptual problems in the theory of relativity, we have adopted the 'rigid-multipole' model of the extended bodies as known from the Newtonian approach. Practically this is acceptable since the relativistic quadrupole contributions are very small. Consequently and because it is conventional in geodynamical research to use spherical harmonics analysis of the gravitational fields with the corresponding notion of harmonic coefficients (C_{lm}^A, S_{lm}^A), the quadrupole moments M_{ab}^A have been expressed in those terms, according to reasons and assumptions given in Bois & Vokrouhlický (1995). Moreover, internal structures of solid deformable bodies, homogeneous or with core-mantle interfaces, are represented by several terms and parameters arising from tidal deformations of the bodies (both elastic and anelastic). More details and references on these topics are given in the above quoted papers.

The simultaneous numerical integration of the Moon and planets uses a global reference system given by the solar-system barycenter. The lunar rotational motion is evaluated relative to a local dynamically non-rotating reference system whose a slow (de Sitter) rotation is calculated with respect to the global reference frame (kinematically non-rotating). It should be noticed that the Moon's reference frame undergoes a similar de Sitter precession to the Earth one. An alternative way of representing the two effects is to introduce the de Sitter precession of the common Earth-Moon barycentric reference frame, as it can be easily verified that the principal effects originate in the solar action. However, due to the Earth-Moon mutual action, the de Sitter precession of the two reference frames differs slightly. A detailed inspection had shown that the lunar reference frame undergoes an additional precession of the order of 30 mas/cy (Bois & Vokrouhlický 1995).

The model is solved by modular numerical integration and controlled in function of the different physical contributions and parameters taken into account. The n-body problem (for the motions of the planets, the Sun and the Moon), the lunar spin motion and the figure-figure and tidal interactions are simultaneously integrated with the choice of the contributions and truncations at our disposal. For instance, the upper limits of the extended figure expansions and mutual interactions may be chosen as follows: up to $l = 5$ in the Moon case, 4 for the Earth, 2 for the Sun while only the Earth-Moon quadrupole-octupole interaction is taken into account.

The model has been especially built to favor a systematic analysis of all the effects and contributions. In particular, it permits the separation of various families of lunar librations. One of the aims in building the model was to include all phenomena up to the precision level resulting from the LLR data (Dickey et al. 1994), and if possible better for reasons of consistency (i.e. at least 1 cm for the distance, 1 mas for the librations). In particular, several phenomena capable of producing effects of at least 0.1 mas in the lunar physical librations have been modeled and analyzed (the resulting libration may be at the observational accuracy level). Other libration effects smaller than this threshold of accuracy have been nevertheless included and studied because of their qualitative interest. Results can be found in previous papers (Bois et al. 1992; Bois & Journet 1993; Bois & Vokrouhlický 1995; Bois et al. 1996). Let us precise that the Earth-Moon figure-figure interaction, i.e. the Earth's quadrupole moment of mass acting as a gravitational torque on the rotational motion of the Moon, produces lunar physical librations with amplitudes of 45 mas over periods of 18.6 and 80.1 years. In the same way, Venus as monopole produces the major impact resulting from direct planetary actions; the resulting lunar libration may reach 3 mas over 18.6 years. The relativistic contributions due to explicit PN terms in the quasi-Newtonian torque related to the Earth as monopole induce lunar librations whose amplitudes reach the observational level (one mas).

3. Impact on the Moon

The two modes of lunar motion, spin and orbital, being simultaneously integrated, the spin-orbit couplings of the Moon are naturally taken into account. They play a part in the present calculus. Some particular spin-orbit couplings of the Moon are described in Bois et al. (1996). The solar quadrupole moment modifies the solar gravitational field with respect to a spherical Sun usually reduced in dynamics to a point mass. As a consequence, the orbital motions of the planets are no longer the same (in particular, even without mutual interactions, the semi-major axes of the planets have to take other values according to the classic problem of J_2). The motions are then simultaneously integrated in the new barycenter of the solar system. The orbital motion of the Moon is also disturbed but its relative motion with respect to the Earth remains globally of same geometric structure. The gravitational action of the solar quadrupole moment may be therefore considered as a perturbation on the lunar motion. Let us notice that the impact is not taken into account as a gravitational torque directly acting on the rotational motion of the Moon. It should be quite negligible. However, all the figure-figure interactions between the Sun (up to $l = 2$), the Earth (up to $l = 4$), the Moon (up to $l = 5$) and the other planets as monopoles are integrated. Finally, the gravitational action of the solar quadrupole moment of mass is evaluated with the spin motion of the Moon by the way of its lunar spin-orbit coupling. As it is generally the case, the indirect effects on the lunar rotational motion are relatively significant with respect to direct effects (cf. Moons 1984).

The Moon's rotational motion is represented by the classical Eulerian angles (3-1-3 angular sequence). The local reference system is given by the terrestrial equatorial frame (J2000). The indirect signature of the solar quadrupole moment on the Moon's rotational motion has been computed with different values of $J_{2\odot}$,

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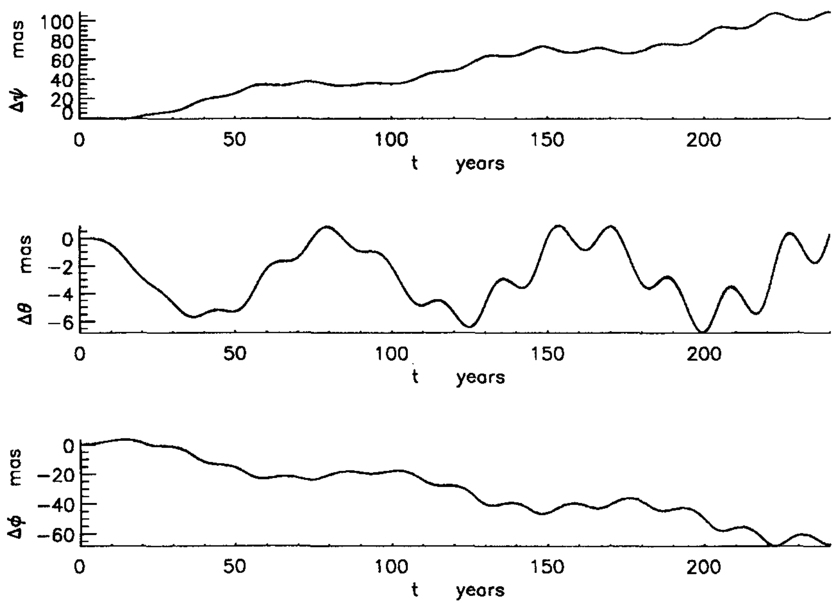

Fig. 1. Indirect effect of the solar quadrupole moment on the lunar physical librations. The integration is performed with $J_2 = 1.1\ 10^{-5}$. Milliarcseconds are on the vertical axis and years on the horizontal axis (initial date is July/01/1969).

namely $1.1\ 10^{-5}$ in Figure 1 and $3.0\ 10^{-6}$ in Figure 2. Computations have been performed in quadruple precision (32 digits). The differences $(\Delta\psi, \Delta\theta, \Delta\phi)$ in Figures 1 and 2 are obtained with respect to the solution free of the gravitational modification related to $J_{2\odot}$.

The non-linearity features of the differential equations, the degree of correlation of the studied effect with respect to its neighbours (in the Fourier space) and the spin-orbit resonance, in the lunar case, make it hardly possible to speak about 'pure' effects with their proper behaviour (even after fitting of the initial conditions). The effects are not absolutely de-correlated but relatively isolated. However, the used technique (modular and controlled numerical integration, differentiation method and frequency analysis) gives the right qualitative behaviour of an effect and a good quantification of this effect relative to its neighbours. When a rotational effect is simply periodic, a fit of the initial conditions for a set of given parameters only refines without changing completely the effect's behaviour (the amplitude variation is lower than five per cent). The amplitudes of librations plotted on Figures 1 and 2 are then slightly upper bounds.

Let us observe $\Delta\theta$ in Figure 1. It represents the nutation variation of the Moon's polar axis relative to the Earth's one due to the indirect effect of the solar quadrupole moment. It shows a periodic dominant term of 80.1 years with an amplitude around

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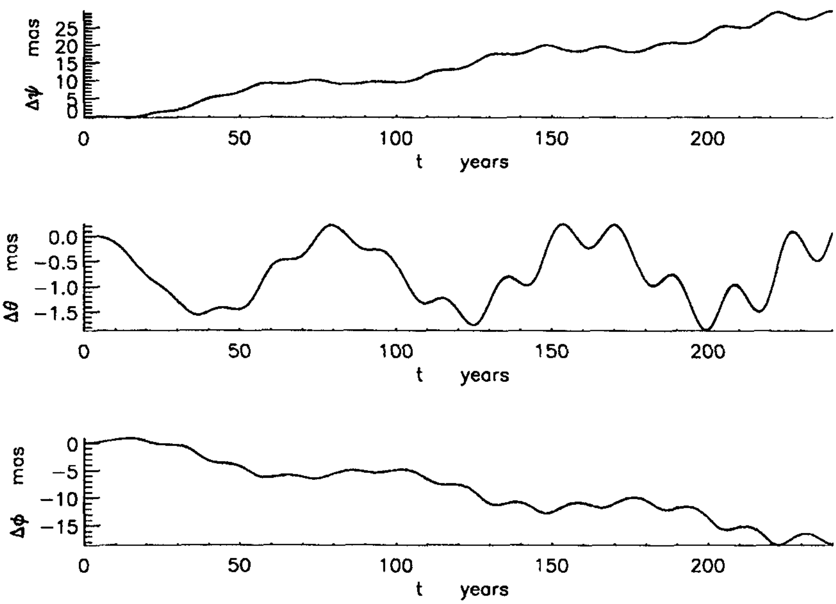

Fig. 2. Indirect effect of the solar quadrupole moment on the lunar physical librations. The integration is performed with $J_2 = 3.0\ 10^{-6}$. Milliarcseconds are on the vertical axis and years on the horizontal axis (initial date is July/01/1969).

6 mas within the first period. The usual modulation of 18.6 years, related to the nodal precession, takes an amplitude around one mas. The ordinary resonant frequency of 2.9 years for physical librations in longitude is also detectable, by frequency analysis, in the θ angle with an amplitude around few tenths of mas. Let us note that in the present simulation, our model uses the dynamical parameters of the JPL DE303 ephemeris and the initial conditions come also from DE303 (the initial date is July/01/1969).

Except for the JPL latest ephemeris, DE405, where $J_{2\odot}$ is affected of a low non zero value (2. 10^{-7}), the recent JPL ephemerides including lunar librations have been adjusted to LLR observations up to the milliarcsecond level of accuracy without taking into account a solar quadrupole moment (Dickey et al. 1994) (in the other hand, Williams, Newhall and Dickey (1995) do not comment on the solar J_2 in their determination of relativistic parameters from LLR data analysis). Anyway, the JPL DE303 ephemeris is adjusted to the LLR observations with a zero value of $J_{2\odot}$ and its accuracy is around one mas ($\sigma = 1$ mas). As a consequence, the lunar physical librations being observed up to 1 mas in accuracy, an amplitude of 6 mas, even over a period of 80 years, is too large so that its cause be neglected in the ephemeris. Consequently, Figure 1 shows that the upper envelope determined by the upper limits of the previous mentioned error bars (1.1 10^{-5}) is not compatible with the knowledge of the Moon's physical librations.

We accept now that an upper bound of $J_{2\odot}$ has to be suitable with $\frac{3}{2}\sigma$ of the LLR residuals, taking into account a possible reasonable shift in the relevance of residuals derived from a least-squares process; let be a signature lesser than 1.5 mas. In Figure 2, obtained with $J_2 = 3.0\ 10^{-6}$, the periodic dominant term of 80.1 years takes an amplitude of 1.5 mas within the first period. The other amplitudes are respectively just lesser than 0.5 and 0.1 mas within the periods of 18.6 and 2.9 years. More precisely, the three amplitudes have decreased according to a factor 4 with respect to those of Figure 1. Consequently, taking into account a faint surcharge estimated to few tenth of mas in our simulations performed without fitting in the initial conditions, the present experiment leads in return to give a limit value of J_2 equal to $3.0\ 10^{-6}$. This result coincides with the largest J_2 value that general relativity can accommodate by fitting to planetary data (Campbell & Moffat, 1983). In the other hand, an average of all the available data arising from experimental determinations of the solar oblateness yields to $3.6 \pm 2.8 \times 10^{-6}$ (Rozelot and Bois, 1998).

Rather than to isolate the differential signature of $J_{2\odot}$ on the lunar librations as we have done it, an alternative way would consist in performing a big least-squares fit to LLR data in order to try to get $J_{2\odot}$ as an absolutely solve-for parameter. Müller et al. (1996) have carried out this investigation but end up with an upper bound of the value: a realistic error on $J_{2\odot}$ equal to $5.\ 10^{-6}$.

4. Impact on the Earth

$J_2 = 3.0\ 10^{-6}$ being an upper bound, it is an opportunity to evaluate the resulting maximal impact of the Sun's quadrupole moment of mass on the orbital motion of the Earth. Such an effect (with $J_2 = 5.5 \pm 1.3 \times 10^{-6}$) has been already tested in 1983 by Campbell & Moffat using analytical approximate equations, and thus for the orbits of Mercury, Venus, the Earth and Icarus. Their conclusion was that if the planetary data became known with enough accuracy, the exact equations would become necessary. Starting from this, the present calculations have been performed with an accurate model of the solar system, the BJV model as described in the second section.

The resulting effects on the orbital elements of the Earth have been computed and plotted over 160 and 1600 years. The solutions arising from the numerical integration are turned into heliocentric system and expressed in geometrical elements $a, e, i, \Omega, \omega, E$ (respectively the semi-major axis, the eccentricity, the inclination, the longitude of the ascending node, the perihelion relative to the line of nodes, and the eccentric anomaly; the elements refer to the equator and mean equinox J2000). Figures 3 and 4 show the results plotted on 160 years. The differences Δa, Δe, Δi, $\Delta\Omega$, $\Delta\omega$, ΔE are obtained with respect to the solution free of the gravitational variation related to $J_{2\odot}$. The maximal variations of the Earth's orbital elements related to the solar quadrupole moment give then the following major changes:

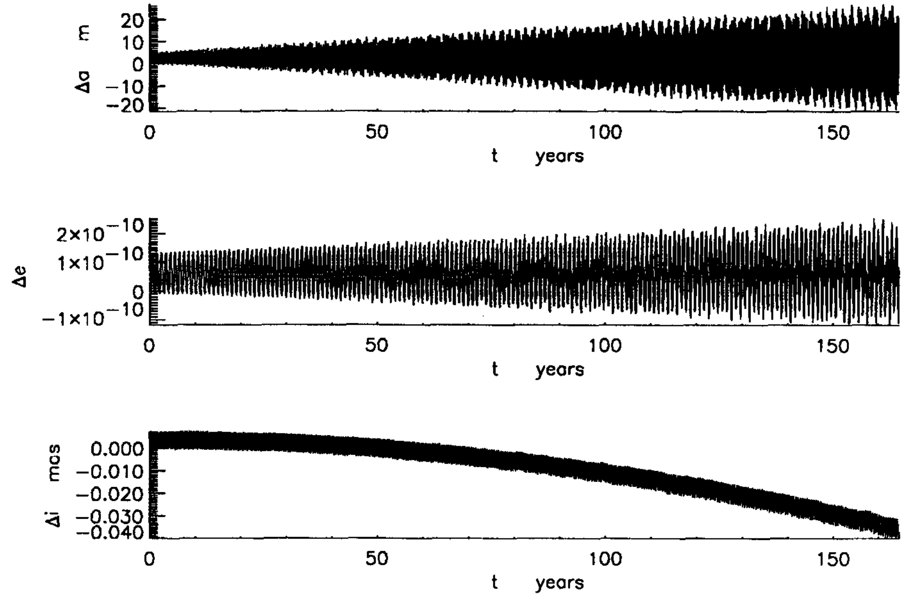

Fig. 3. Maximal variations of the orbital elements a, e, i of the Earth due to a solar quadrupole moment J_2 equal to 3.0 10^{-6}. Δa is in meters and Δi in mas.

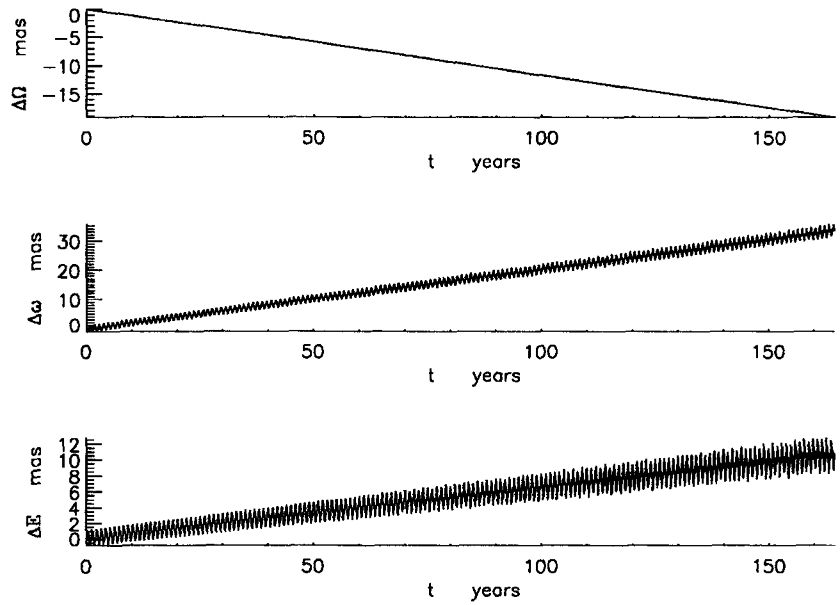

Fig. 4. Maximal variations of the orbital elements Ω, ω, E of the Earth due to a solar quadrupole moment J_2 equal to 3.0 10^{-6}. The three variables are expressed in mas.

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$\Delta a \leq 5$ m, i.e. the new mean semi-major axis of the Earth is in fact increased of 2.5 m; $\Delta e = 6.\ 10^{-11}$; the secular variation of the inclination of the Earth's orbital plane is very faint [$\Delta(di/dt) = -0.02$ mas/cy] but it presents a quadratic term [$\Delta(di/dt) = -0.2$ mas/cy when computed over 1600 years] (at this point, it would be worthwhile to take into account the inclination of the solar polar axis); $\Delta(d\Omega/dt) = -12$ mas/cy (which is on the opposite sign of $d\Omega/dt$); $\Delta(d\omega/dt) = 20$ mas/cy; $\Delta(dE/dt) = 6.5$ mas/cy.

Let us note that the nominal values such as $di/dt = -47.5$"/cy, $d\Omega/dt = 11.5$"/cy, and $d\omega/dt = 1250$"/cy deriving from our calculations represent essentially the mutual effects of planets on the orbital motion of the Earth. $\dot{\Omega}$ is then the most disturbed element by the impact of $J_{2\odot}$, let be three orders of magnitude lesser than the planetary effects. It is nevertheless of the same order of magnitude that the impact of the Moon on $\dot{\Omega}$.

5. Conclusion

The initial point to emphasize is that values of $J_{2\odot}$ enlarge across literature on two orders of magnitude. It has been shown that a solar quadrupole moment as high as $1.1\ 10^{-5}$ (such as given either from the upper bounds of the error bars of the observations, or from the Roche's theory) is not compatible with the knowledge of the lunar physical librations accurately modeled and observed with the LLR experiment. The suitable values of $J_{2\odot}$ have to be smaller than $3.0\ 10^{-6}$. Let us note that this value would be rather slightly an overvalued upper bound. However, the interval of available values of solar J_2 is certainly reduced. In the other hand, the recent value, namely $2.18 \pm 0.06 \times 10^{-7}$, inferred from helioseismology by Pijpers (1998), is very probably a minimal realistic value of J_2. As a consequence, the interval of possible values of $J_{2\odot}$ should be from now on [$2.\ 10^{-7}$, $3.\ 10^{-6}$]. This interval is in fact maybe more reliable than a simple fine value owing to a possible time dependence of $J_{2\odot}$.

Using our BJV relativistic model of solar system integration (including notably the mutual body-body interactions and the spin-orbit coupled motion of the Moon), we have precisely calculated the impact of the quadrupole moment of the Sun on the Earth's orbital motion. The major changes are as follows: $\Delta a = 2.5$ m, $\Delta e = 6.\ 10^{-11}$, $\Delta(d\Omega/dt) = -12$ mas/cy (which is on the opposite sign of $d\Omega/dt$), $\Delta(d\omega/dt) = 20$ mas/cy, and $\Delta(dE/dt) = 6.5$ mas/cy. The variation of di/dt is very faint but it presents a quadratic term. The most disturbed element in relation to its nominal value is $\dot{\Omega}$, let be three orders of magnitude lesser than the effects due to planetary interactions. The impact of $J_{2\odot}$ is globally slight but nevertheless because of the variations of $\dot{\Omega}$, $\dot{\omega}$, and $\dot{E}$, it has to be taken into account in models of long period evolution of the terrestrial orbit and more generally of the solar system. As a matter of fact, the Sun's quadrupole moment of mass (and some more related to its rotational motion) could play a sensible role in the determinations of prediction limits of the solar system stability.

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