

The predominant feature in the organization of results scattered in the literature is the presentation of a variety of methods which have come to be associated with the names of their initiators. Thus, of the fifteen chapters, ten are devoted to the methods and techniques developed by: Hildreth and d'Esopo, Theil and Van de Panne, Beale, Wolfe, Barankin and Dorfman, Frank and Wolfe, Rosen, Frisch, Zoutendijk, and Houthakker. The utility of the book as a text is considerably enhanced by the addition of carefully worked out numerical examples.

The interested reader may also be directed to another recent book: *Nonlinear Programming* (J. Abadie, Editor, North-Holland, 1967) based on a series of lectures given at a NATO Summer School (Menton, France, 1964).

H. Kaufman, McGill University.

Generalized Functions, Volume 5, Integral Geometry and Representation Theory, by I. M. Gel'fand, M. I. Graev and N. Ya Vilenkin. Translated from the Russian by Eugene Saletan. Academic Press, New York and London, 1966. xviii + 449 pages.

This, the fifth of a series of six volumes on generalized functions and their applications, by Gel'fand and various other authors, is devoted to problems of integral geometry and to their connection with the theory of representations of certain of the classical Lie groups, particularly of the Lorentz group and of matrix groups such as the complex unimodular group which are associated with it.

This statement gives little conception of the wealth of ideas, and especially of new ideas, which this book contains. The approach to integral geometry is among these. The authors see the basic problem of integral geometry to be that of expressing a function in terms of its integrals over some suitable family of manifolds in the space in which it is defined: a connecting idea of the book is the linkage between integral transforms over a suitably chosen family

of differentiable manifolds in a group and the Fourier transform on the group: this, in turn, is crucial in the theory of group representations.

The first chapter studies this connection in real n -dimensional space: the family of manifolds is the family of hyperplanes: the integrals of a function over these give the Radon transform, which in turn gives the multidimensional Fourier transform by means of a one-dimensional Fourier transformation. The theory of the Radon transform for indefinitely differentiable functions and for generalized functions is developed in considerable detail: and there are shown to be interesting connections with generalizations of the hypergeometric functions.

The second chapter studies integral geometry, in this sense, for a certain class of line complexes in three dimensional complex space, for the lines on a complex surface in complex 4-space, and for planes in complex affine space. It is shown later that these studies lead to a geometrical view of the theory of representations of the Lorentz group.

The third chapter studies the representations of the group of second order complex unimodular matrices, using as the main tool the transformations induced by this group in spaces of homogeneous functions of two complex variables. Through a study of invariant bilinear functionals on these spaces and expressions for these in terms of generalized functions, the irreducible representations of this group and of the Lorentz group are found. The next chapter studies Fourier transformations on this group, relating them to the geometrical studies of Chapter 2. The following Chapters deal with integral geometry on spaces of constant curvature, and on harmonic spaces connected with the Lorentz group; and the final chapter studies the group of real unimodular matrices, producing a new approach to the results of Bargmann on the real Lorentz group.

The authors assume that the reader has some knowledge of the theory of generalized functions as expounded in the first two volumes of the series: in particular of the appendix on generalized functions of a complex variable, which has been shifted by the translator from the present volume to the first volume of the English series; it naturally requires a competent knowledge of the techniques of classical

analysis; some acquaintance with the traditional geometry of lines and quadrics is helpful; but the theory of group representations is developed pretty well ab initio. Considering the variety of topics and the wealth of ideas it contains, new and newly presented, this book is remarkably readable: and for this English readers need to be grateful not only to the authors but also to the translator who has produced a clear, idiomatic and accurate version of the original.

J. L. B. Cooper, University of London