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The Right Numeraire or the Just Weights? How to Make BCA Rational and Fair

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Keywords: equity weights; life years; numeraire; social welfare function; subjective well-being

JEL classifications: D61; D63

Abstract

Unweighted benefit–cost analysis (BCA) based on aggregate willingness to pay might be, at long last, falling into disrepute, as it is widely recognized that it exhibits a bias toward the wealthy, and as alternatives are appearing more and more practicable. However, the choice of alternatives is often framed in terms of choosing an alternative metric to willingness to pay in money, such as willingness to pay in healthy life years, or a measure of subjective well-being. It is argued in this paper that (i) a simple summation of individuals’ willingness to pay in any numeraire (e.g., money, healthy life years) is bound to generate non-transitivity issues in a similar way as money-based BCA, and (ii) a metric such as subjective well-being involves distributional value judgments that are too specific to reflect the relevant spectrum in the public debate. The “orthodox” weighted BCA method, which links BCA to an underlying social welfare function, offers more flexibility and guarantees transitive choices. Fortunately, in some relevant cases, these various methods may provide similar results, and the main options currently proposed all give greater weight to the worse off in the population than does unweighted BCA.

1. Introduction

The classical approach to benefit–cost analysis (BCA) adds compensating variations over the relevant population, without any weights, in order to assess whether a reform or policy program is desirable. Variants of the approach rely on equivalent variations, Kaldor and Hicks compensation tests, and the potential Pareto criterion.¹ This classical approach has retained a substantial foothold in economic teaching and in the practice of policy evaluation, but has been severely criticized by specialists of welfare economics, at least since the early publications of Bergson (1938, 1966), Samuelson (1947), Arrow (1963), Drèze and Stern

¹ When using compensating variations, BCA compares the total willingness to pay of individuals who benefit from a project with the total willingness to accept compensation of those who are harmed. Using equivalent variations, BCA compares the total willingness to accept compensation to forgo the project of those who would benefit with willingness to pay to prevent the project of those who would be harmed.

(1987), Blackorby and Donaldson (1990), and others. However, such criticism has not been very impactful, perhaps because practical alternatives were not proposed in this literature. More recently, this approach has again come under fire by authors arguing for specific alternative approaches (e.g., Fujiwara & Campbell, 2011; Adler, 2012, 2016, 2017; Nyborg, 2012; Adler *et al.*, 2014; Fujiwara & Dolan, 2016; Hammitt, 2021; Layard & Oparina, 2021; Adler & Norheim, 2022; Canning, 2023), and this time it appears more likely that practitioners and norms will change, as guidance provided by the UK “Green Book” (HM Treasury, 2018) and similar documents in various countries (including the recently published revised circular A-4 in the USA) is now incorporating the concept of weighted BCA.

However, part of the recent literature (Hammitt, 2021; Layard & Oparina, 2021; Canning, 2023), instead of advocating for weighted BCA, focuses on the idea of changing the numeraire in which the computation of costs and benefits is made, abandoning willingness to pay in money to replace it either with willingness to pay in healthy life years or with happy years (or similar subjective well-being measures). This paper is devoted to a discussion of the role of the numeraire in the unweighted summation of willingness to pay, along the lines of the argument in Hammitt (2021), highlighting some issues that are not developed or addressed in Hammitt’s paper, such as the violations of transitivity with unweighted BCA, or the reasons why unweighted BCA cannot escape some fairness issues. It also discusses the alternative proposal of summing happy years, showing that it is only one specific approach among the wide diversity of normative approaches that can be implemented with weighted BCA. Finally, it examines special cases in which weighted and unweighted BCA produce similar policy conclusions.

Note that the terminology of unweighted and weighted BCA can be misleading. “Unweighted BCA” means using equal weights, not eliminating weights. Moreover, the violations of transitivity that can occur with unweighted BCA can also occur using any set of fixed weights. (Fixed weights may depend on individuals’ fixed characteristics, such as gender or race, but not on their variable circumstances, such as income.) To avoid these violations, weights that are sensitive to circumstances (such as income) that can affect individuals’ rates of substitution between the numeraire and other goods must be used.

The paper is structured as follows: The following section explains why unweighted BCA is vulnerable to violations of rationality in the form of decision cycles in which a sequence of alleged improvements can end up back at the starting point. Section 3 is devoted to the description of fairness problems with unweighted BCA and shows why, whatever the numeraire, the approach is likely to be unfair to individuals who are poorly endowed in the numeraire. Section 4 is devoted to the idea of summing subjective well-being measures rather than willingness to pay in any numeraire. This can be viewed as a variant of weighted BCA rather than a new form of unweighted BCA, since it relies on a type of utilitarian social welfare function, and the section discusses some pros and cons of this interesting approach. In Section 5, the wide variety of normative approaches covered by the social welfare function (SWF) approach, which underlies weighted BCA, is briefly described in order to repel any apprehension that the SWF approach is a moral straitjacket that imposes very specific value judgments. Section 6 examines under what conditions one can obtain a convergence between the policy conclusions of unweighted and weighted BCA, and argues that all the current alternatives that are being proposed tend to shift the moral compass away from the interests of richer social groups, compared to classical unweighted BCA using money as the numeraire. Section 7 concludes this article.

2. Cycling issues with numeraires

In this section, we show that when the evaluation of reforms is made with a sum of willingness to pay defined in terms of compensating variations (or equivalent variations, or both) computed in a given numeraire, serious problems arise for the rationality of social decisions. First, definitions of the basic notions are needed.

Consider a world in which life is made, for every individual i , of two dimensions a_i, b_i . These two dimensions are generally good, and people have better lives when they enjoy greater quantities of each of them. Thus, individuals are usually willing to trade off one good against the other. When an individual is indifferent between

$$(a_i - w, b_i + z) \text{ and } (a_i, b_i),$$

one can call w the compensating variation in good a for the change z in good b , starting from the status quo (a_i, b_i) . A similar definition applies to the compensating variation in good b for a change in good a . The good in which compensating variation is computed is called the numeraire.²

It is more rigorous to think of these notions as functions of the change and the status quo and to extend them to simultaneous changes in all goods:

$$w_i^a(z^a, z^b; a_i, b_i), w_i^b(z^a, z^b; a_i, b_i)$$

are the functions defined by the condition that the individual is indifferent between

$$(a_i + z^a - w_i^a(z^a, z^b; a_i, b_i), b_i + z^b) \text{ and } (a_i, b_i)$$

as well as between

$$(a_i + z^a, b_i + z^b - w_i^b(z^a, z^b; a_i, b_i)) \text{ and } (a_i, b_i).$$

Since both goods are desirable by assumption, the compensating variation is positive whenever the change from (a_i, b_i) to $(a_i + z^a, b_i + z^b)$ is considered an improvement by the individual (willingness to pay for the improvement) and negative when the situation is worsened by the change (willingness to accept compensation for the harm).

The compensating variation is not always defined, as illustrated in [Figure 1](#). A gain may be so valuable that abandoning one good completely would not suffice to “pay” for it. But we will assume, from now on, that such a problem does not occur in the relevant cases, which is plausible when dealing with a taxonomy of goods in which each of them is necessary to reach a minimal level of welfare. This figure also illustrates the fact that compensating variation for a change in a good is always equal to that change when this good is the numeraire.

Another way in which compensating variation may be undefined is when a loss is so bad that no compensation for a particular good would suffice. [Figure 2](#) illustrates this phenomenon. This phenomenon is more worrisome than the previous one, as one, for instance, often hears that “no amount of money” would compensate for a loss. For example, it is plausible that no amount of money would compensate an individual for a large increase in current mortality risk. However, in the sequel, we will also ignore this issue.

² We are dealing here with two goods for simplicity. As is well known, the usual definition of compensating and equivalent variations involves minimum expenditures at reference prices, and the numeraire is then the monetary value of such minimum expenditures.

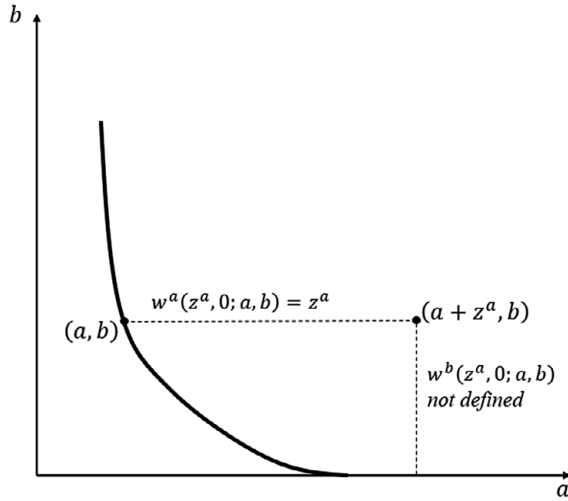


Figure 1. The compensating variation may be undefined, greater than the endowment.

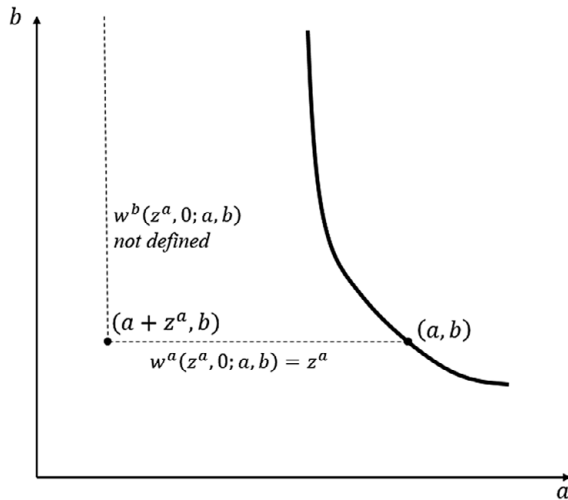


Figure 2. The compensating variation may be undefined, greater than infinite.

A related concept is that of equivalent variation, which is focused on obtaining indifference at the final situation rather than the initial one, that is, is defined by the conditions of indifference between

$$(a_i + z^a, b_i + z^b) \text{ and } (a_i + \bar{w}_i^a(z^a, z^b; a_i, b_i), b_i)$$

as well as between

$$(a_i + z^a, b_i + z^b) \text{ and } (a_i, b_i + \bar{w}_i^b(z^a, z^b; a_i, b_i)).$$

Intuitively, the individual would “accept” to receive $\bar{w}_i^a(z^a, z^b; a_i, b_i)$ or $\bar{w}_i^b(z^a, z^b; a_i, b_i)$ rather than enjoy the new situation $(a_i + z^a, b_i + z^b)$. In a similar fashion as the compensating

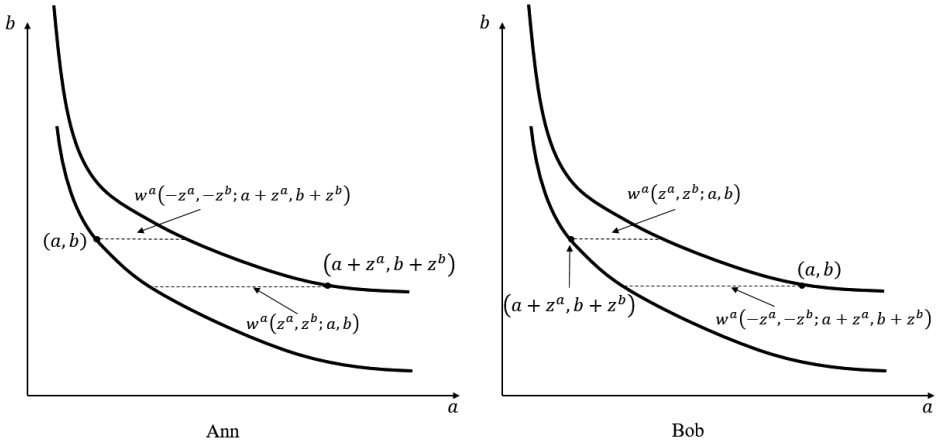


Figure 3. A decision cycle with two situations for compensating variation.

variation, the equivalent variation is positive for a gain (willingness to accept compensation to forgo a gain) and negative for a loss (willingness to pay to prevent a loss), and is equal to the gain or the loss when this occurs for the change is in the numeraire.

A decision cycle is a sequence of moves such that each step is an improvement and the first and last situations are identical. A criterion that can generate decision cycles cannot be used for rational decision-making (it precludes a transitive ranking of situations). At the individual level, no decision cycle ever occurs with compensating (or equivalent) variations; because these notions respect individual preferences, a sequence of improvements cannot bring the individual back to the initial situation.

Things are very different when compensating (or equivalent) variations are aggregated over a population. Consider the criterion according to which a decision is good whenever the sum

$$\sum_i w_i^a(z^a, z^b; a_i, b_i) > 0.$$

This approach is closely linked to the Kaldor compensation test, since when this sum is positive, one can (hypothetically) organize transfers of the numeraire between individuals so that everyone ends up being better off than at the status quo.³

This criterion can produce decision cycles with only two situations. This is illustrated in Figure 3, with two individuals. In this example, the positive compensating variation of the individual who benefits from a move always dominates the negative compensating variation of the individual who loses, and thus either move is good according to the sum of willingness to pay. What is noticeable in this illustration is that the two individuals have identical indifference curves, and the change is simply a swap of their bundles.

This example shows how easy it is to obtain a decision cycle, since it is sufficient to have winners and losers with preferences inducing a greater compensating variation for gains than

³ The actual Kaldor test is more complicated because it involves taking account of possible changes in prices due to the transfers. In the simple framework of this paper, prices and budgets over multiple goods are ignored and we focus on bundles.

for losses. Such a pattern is plausible when the more advantageous situation for any individual makes this individual willing to pay more in terms of the numeraire. As an example, let a be income and b be life expectancy. For Ann, a large increase in income (z^a) may compensate for a small reduction in life expectancy (z^b) and also make her willing to exchange a larger amount of wealth for an increment of life expectancy ($w^a(z^a, z^b, a, b) > w^a(-z^a, -z^b, a + z^a, b + z^b)$). Similarly, for Bob, a large decrease in wealth (z^a) decreases the amount of wealth he will exchange for an increment in life expectancy.

Another observation is that checking the criterion for the two possible numeraires would not eliminate the problem, since when good b is taken as the numeraire, both changes are considered good as well (in the figure, the vertical distance between the two indifference curves is, as the horizontal distance, larger for the right bundle than for the left bundle).

However, when one checks the equivalent variations,⁴ one finds that the sum is now negative for both moves, thus also generating a decision cycle but in the opposite direction. This is a logical consequence of the fact that

$$\bar{w}_i^a(z^a, z^b; a_i, b_i) = -w_i^a(-z^a, -z^b; a_i + z^a, b_i + z^b),$$

which entails that when the sum of compensating variations for a project is positive, the sum of equivalent variations for the reverse move is always negative (and of the same absolute value).⁵

This suggests adopting a more stringent criterion according to which both sums (of compensating variations and of equivalent variations) should be positive to declare a change good.⁶ Unfortunately, this double criterion eliminates cycles with two situations but does not prevent cycles for larger numbers of situations. Figure 4 illustrates this for three situations and two individuals.

In order to save on notation, the compensating variation to move from (a, b) to (a', b') (respectively, (a', b') to (a'', b'') , (a'', b'') to (a, b)) is denoted w (respectively, w' , w''), and the corresponding equivalent variation is denoted \bar{w} (respectively, \bar{w}' , \bar{w}''). The numeraire is good a in this example. Notice that, here again, the two individuals have the same indifference curves.

According to the double criterion, the move from (a, b) to (a', b') , as well as the move from (a', b') to (a'', b'') , are good because Ann's compensating and equivalent variations dominate Bob's, whereas the move back from (a'', b'') to (a, b) is also good because Bob's compensating and equivalent variations dominate Ann's.

Such violations of the transitivity of the evaluation imply that such criteria cannot be rationalized as maximizing a social welfare objective. The root of the problem is that every move is evaluated by measures of compensating or equivalent variation which take a different benchmark for the non-numeraire good when the quantity of this good varies from

⁴ This criterion is linked to the Hicks compensation test.

⁵ Coate (2000) made the interesting observation that, for the evaluation of a set of reforms from a fixed status quo, the equivalent variation approach behaves better than the compensating variation approach. Indeed, it relies on a fixed reference in that case, whereas compensating variations involve different references for the assessment of different reforms, which may yield inconsistent results. Here, we are considering cycles occurring in sequences of successive decisions, and for such a context, equivalent variation also relies on different references and hence may yield inconsistent results.

⁶ This suggestion was made by Scitovsky when considering similar problems with the Kaldor and Hicks compensation tests.

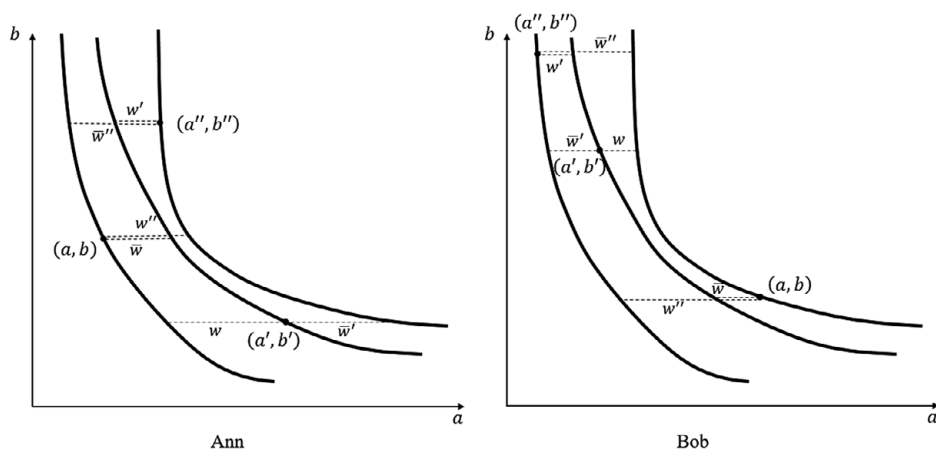


Figure 4. A decision cycle with three situations for the double criterion.

one situation to the next. For the evaluation of individual moves, cycles cannot occur, but the same issue of changing references implies that the sum of compensating variations over several moves is not equal to the compensating variation for the whole sequence of moves, and this is what generates the cycle when summation is made over individuals. Indeed, in the example of Figure 4, Ann's compensating variation is large in the first two moves, but it is not so large in the move back to (a, b) : $|w''| < w + w'$. In contrast, for Bob, one has $w'' > |w + w'|$.

The lesson of this section is that relying on compensating or equivalent variations in a numeraire to measure changes in well-being is not a sound method and should be avoided as much as possible. There is no way to avoid the introduction of weights in the summation if one wants to make sure to avoid decision cycles. Moreover, the weights cannot be fixed (constant for each individual) but must change as the individual's circumstances change, as explained in Section 5.

3. Fairness issues with numeraires

Relying on a numeraire without weights also raises issues relative to fairness in the distribution of costs and benefits. To simplify the presentation, we will focus on compensating variations in this section, but approaches involving equivalent variations have the same issues.

A key property of the sum of compensating variations criterion is the following:

A unit of numeraire is a unit of numeraire: Giving a given increment in the numeraire has the same value no matter who the recipient is.

This property is due to the fact, noted in the previous section, that the compensating variation for a change in the numeraire is exactly equal to the amount of this change for every individual. When money is the numeraire, this expression takes the familiar form "a dollar is a dollar." This property is attractive when one believes that individuals are equally entitled to any increment in the numeraire.

A prominent example of such an attitude involves using life years (or quality-adjusted life years) as the numeraire. When life years are considered to have the same value when granted to any individual, because health and longevity are considered a universal good to which

everyone is equally entitled, the criterion relying on compensating variation computed in terms of life years appears particularly attractive (Canning, 2023).

However, this intuition of the universality of the value of life years is debatable (Fleurbaey & Ponthière, 2022). In particular, the other side of the coin is that, with the sum of compensating variation criterion, one is then indifferent to any distribution of the numeraire, and therefore, one has no aversion to inequality in the numeraire. This is directly contrary to the same intuition that all individuals are equally entitled to the numeraire. For instance, if longevity is considered a most basic good of universal value, one should be averse to an unequal distribution of this good, with some individuals dying prematurely whereas others live very long lives. For a given sum of life years, it appears clearly better if all individuals can enjoy lives of similar longevity, at least when the equal sharing of life years enables them to live reasonably long lives.

There is clearly a tension between the idea of equal entitlements to any increment and the idea of equal entitlements to a sufficient level of the considered good. However, the absence of aversion to inequality in the numeraire can be justified in a different way. Suppose that redistribution of life attributes takes place mostly in terms of the numeraire through public policy or similar collective arrangements. And suppose that the redistribution policy is optimal according to a well-defined social objective. Then, it makes sense to be indifferent to inequality around the status quo because any distributive effect of a small reform will be of second-order magnitude only.

This argument justifies taking money as the numeraire when redistribution takes place in this good and when one believes that the redistributive policy is optimal.⁷ There are some complications when redistribution is constrained by incentive issues because, in this case, the distribution of resources is not fully optimal, and further redistribution by reforms may have a first-order effect (Fleurbaey & Kornek, 2021). A key question, then, is whether the reform can be assessed with sufficient information about the relative priority of the individuals so that the distributive effect can be well estimated. An important literature (Kaplow, 2008 provides a synthesis) studies under what conditions distributive issues can be ignored when redistribution is second-best optimal (i.e., optimal under imperfect conditions due to incentive constraints), as well as the conditions under which one can separate efficiency effects from distributive effects of policies. Positive results are limited to situations in which the contemplated policy affects the distribution only among income groups and not in relation to other characteristics (Hammitt, 2021).

This argument based on the optimality of the distribution of the numeraire, in contrast, can hardly justify taking life years as the numeraire because life years are not redistributed directly by public policy (even if many policies do influence the distribution of longevity), and there is no reason to believe that the actual distribution of life years is optimal for any reasonable social criterion.

Another important property of compensating variation is the following:

To every one who has, more shall be given. If the numeraire is a normal good, the compensating variations for other goods are greater, other things equal, for individuals who are better endowed in the numeraire.

⁷ In a model with only private goods, benefit–cost analysis with unweighted equivalent variations taking money as the numeraire is equivalent to maximizing the sum of money-metric utilities at status quo reference prices. Schlee and Khan (2021) develop a detailed analysis of the conditions under which the sum of money-metric utilities is maximized at a competitive equilibrium, when the equilibrium prices are taken as reference.

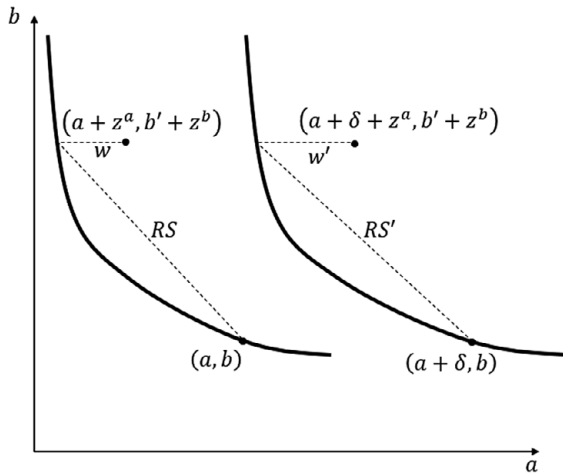


Figure 5. The compensating variation increases with endowment in the numeraire when it is a normal good.

A good is normal when indifference curves display the property that the rate of substitution between two goods increases with the initial endowment of the good that is traded off. This property is illustrated in Figure 5, where the slope RS linking two points on the indifference curve decreases when the endowment in good a increases, and this mechanically implies that the compensating variation increases ($w' > w$ in the figure).

This property implies that richer individuals tend to have a greater compensating variation in money, while individuals with greater life expectancy tend to have a greater compensating variation in life years. The latter fact may not appear obvious because individual attitudes about longevity are mixed with risk considerations, and individuals with a lower life expectancy may, in some cases, have a more casual attitude about life because they do not stand to benefit long from any gain. However, the general tendency to be less stingy about good one is well endowed with seems to be the more robust and relevant phenomenon for the present discussion.

Thus, taking money as the numeraire will tend to be unfair to poor individuals whose willingness to pay is reduced, whereas taking life years as the numeraire will tend to be unfair to individuals with a low life expectancy. It is hard to decide which is the more unfair, but if one considers that longevity is a more fundamental good than money, it may be reasonable to give more priority to avoiding unfairness to short-lived individuals. A plea for taking such inequalities seriously was made by Fleurbaey *et al.* (2014).

4. Subjective well-being

If one abandons the hope of finding a numeraire with which unweighted BCA can be soundly performed, a prominent candidate for a substitute metric to willingness to pay is the subjective well-being (SWB) approach, advocated by an important literature (Fujiwara & Campbell, 2011; Fujiwara & Dolan, 2016; Layard & Oparina, 2021). Sometimes, this is even presented in terms of an alternative numeraire (“WELLBY”), but the idea is to compute

the impact of a policy in terms of a sum of WELLBY units, not in terms of willingness to pay in SWB.

The SWB approach to BCA can be framed as a proposal for particular weights in the summation of standard compensating variations in money, and it is sometimes argued that this provides value-free weights because such weights only reflect the diminishing marginal utility of money, measured as an empirical magnitude thanks to SWB surveys. It is important to emphasize that there are no value-free weights because any set of weights used in weighted BCA can be analyzed as combining two value-laden coefficients: the marginal social value of individual well-being and the marginal value of money for the chosen measure of individual well-being. The key value judgments made in such an approach are, first, the choice of the measure of individual well-being, which has to be interpersonally comparable, and second, the choice of the degree of priority granted to individuals with lower well-being.

It may be tempting to think that by taking SWB as the measure of well-being, one is just tracking people's own views in a neutral way while avoiding any degree of priority for the worse-off is escaping the delicate issue of distributive value judgments. However, this is an illusion because no measure of well-being is value neutral when used for interpersonal comparisons, and zero inequality aversion is not a neutral stance, but rather one of the most controversial positions in this domain.

An important way to understand why SWB measures are not value-free is to observe that SWB measures of well-being do not respect individual preferences, and this constitutes a stark departure from standard principles of individual sovereignty underlying BCA. There are two main SWB measures, and the argument applies differently to each. The measures of emotional happiness (hedonic SWB) do not respect preferences because individuals commonly trade off their emotions against other types of achievements in their lives. Emotional well-being, close to mental health, is a very important dimension of life, and the literature on happiness has greatly contributed to bringing attention to mental health issues. But, this is only a subset of the dimensions of life, and individuals do not care exclusively about their mental health.⁸

The measures of satisfaction with life (evaluative SWB) constitute the other prominent set of SWB measures. These more plausibly reflect individual preferences regarding comparisons of situations for a given individual at a given time, but they may often fail to respect preferences for interpersonal comparisons or for comparisons spread in time. This is because variations in scale use (the way in which respondents assign a score to their life) introduce noise for comparisons between individuals and also between periods for any given individual. In particular, it can easily happen that two individuals with identical preferences and similar life situations will give different scores of satisfaction to their lives simply because they treat the scale differently, for instance, because they compare their lives to different benchmarks or because they understand the scope of the question differently. It remains largely unknown how widespread, in the populations of respondents, such variations in scale use are, and thus how problematic they are for evaluative SWB. Future research will shed light on this issue.

⁸ Unless life satisfaction is synonymous with emotional happiness, individuals who seek to maximize life satisfaction necessarily sacrifice some degree of emotional happiness. For an overview of these issues, see Benjamin *et al.* (2023).

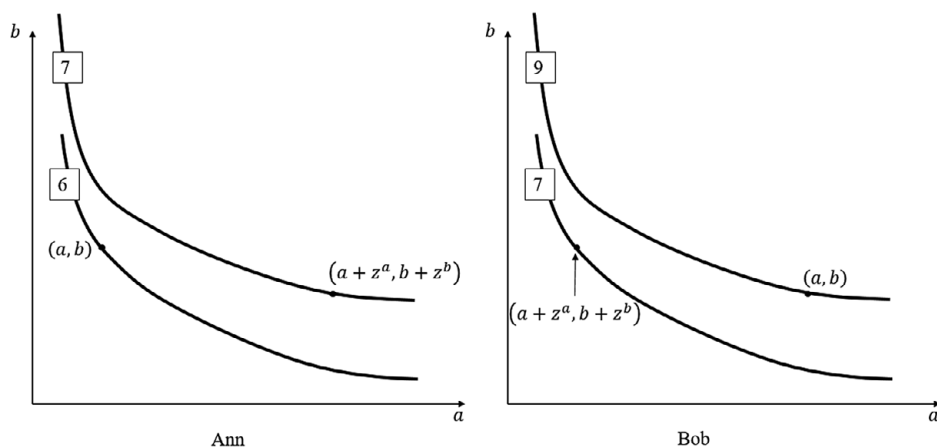


Figure 6. Satisfaction with life under different scale use across individuals.

Figure 6 illustrates this problem with the same configuration as in Figure 3, adding the measures of life satisfaction attached to every indifference curve for each individual. In the figure, Ann has lower SWB and lower sensitivity to the move than Bob, generating a situation in which the sum of SWB is greater in (a, b) (total SWB is $6 + 9 = 15$ while the difference is $9 - 6 = 3$) but inequality is reduced in the other situation (total SWB is $7 + 7 = 14$ while the difference is $7 - 7 = 0$). Since they have the same preferences and this is just a swap of their bundles, one can argue that the social evaluation should be neutral, out of respect for their identical preferences (and despite their different use of the SWB scale).⁹ Notice that in Figure 3, the fact that unweighted BCA using either good as a numeraire was not neutral can also be used as an argument against BCA because it fails to respect preferences in interpersonal comparisons.

Another proposal that relies on a different measure of well-being deserves to be discussed in this section. It consists of relying on von Neumann–Morgenstern (VNM) utility functions (Adler, 2012; Canning, 2023), which can be estimated through surveys eliciting risk attitudes. This approach, therefore, proposes to link the estimation of the diminishing marginal utility of money to individual attitudes toward risks to wealth. More risk-averse individuals will exhibit a greater rate of diminishing marginal utility. Since VNM functions are only estimated up to an affine transform at the individual level, interpersonal comparisons are possible only after a choice of specific scales for these functions. A popular scale in the literature consists of fixing utilities to be 0 and 1 at two benchmark (very bad and very good) situations. This approach implies the problematic consequence that more risk-averse individuals are deemed better off than less risk-averse individuals, thereby giving them less priority when some degree of inequality aversion is introduced in the application of BCA. An alternative scaling (Fleurbaey & Zuber, 2021) involves equalizing utility levels as well as marginal utilities at a reference point, which implies that more risk-averse individuals are

⁹ One might object that when individuals have the same preferences, they may nonetheless have different “cardinal” well-being measures. But if everything they care about is taken into account in their preferences, they would themselves agree on who is better off, all things considered, and invoking another metric would fail to respect their judgments.

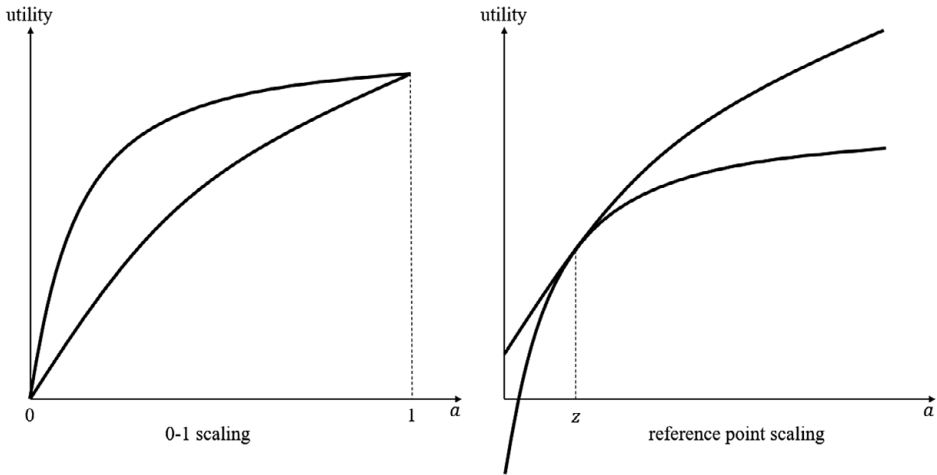


Figure 7. Two scaling methods for VNM utility functions.

nowhere deemed better off, and that redistribution from someone above the reference point to someone below is always a social improvement, even in the absence of inequality aversion over utilities. These two scaling methods are illustrated in Figure 7.

It is not easy to assess the VNM approach in terms of respect for preferences. On the one hand, it introduces numbers as in Figure 6, whatever the scaling method, which will, in general, exclude the neutrality judgment that Figure 6 suggests (identical preferences, a mere swap of situations). On the other hand, excluding risk attitudes from the evaluation of riskless situations leads to difficulties in contexts in which the various options vary in the composition of the future population or in the contents of this population's preferences (Fleurbaey & Zuber, 2022). To illustrate this point, consider a situation in which the considered policy may lead to the creation of a future individual with different risk attitudes, but always submitted to the same prospect and with fixed preferences over riskless options. Since the prospect is fixed, the individual ends up being equally well off, regardless of her risk attitude, in every final situation. On the other hand, she has a lower certainty equivalent when she is more risk averse. If one wants to respect her indifference between the prospect and its certainty-equivalent, she should be deemed worse off in the case she is more risk averse. There is therefore a tension between respecting preferences over riskless options for interpersonal comparisons (or, in this case, comparisons across situations involving the same individual with different risk attitudes) and respecting preferences and risk attitudes over prospects.

In conclusion of this section, the main point is that relying on empirical surveys eliciting SWB or VNM utility functions raises interesting and difficult normative issues. These methods are valuable and avoid the cycling issues depicted in Section 2, but relying on SWB surveys does not escape the necessary value judgments involved in interpersonal comparisons.

5. Flexible weights

Weighted BCA involves, as recalled in the previous section, relying on the weighted sum of compensating variations, with weights that combine, in a product, the marginal social value

of individual well-being and the marginal value of the numeraire for the chosen measure of individual well-being. It is clear that changing the numeraire, which changes the values and the units of measurement of the compensating variation of all individuals, requires adjusting the marginal value of the numeraire for individual well-being.

The rationale behind such weights comes from the analysis of a marginal change for a social welfare function (SWF). Let social welfare be defined as

$$W(U_i(a_i, b_i); i = 1, \dots, n), \quad (1)$$

where $U_i(a_i, b_i)$ represents the chosen measure of interpersonally comparable well-being, and W aggregates the distribution of well-being over the population with a chosen degree of inequality aversion. A small change in the situation can be decomposed as follows, taking good a as the numeraire:

$$dW = \sum_{i=1}^n \frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial a_i} \left(da_i + \frac{\frac{\partial U_i}{\partial b_i}}{\frac{\partial U_i}{\partial a_i}} db_i \right), \quad (2)$$

where one recognizes $\frac{\partial W}{\partial U_i}$, the marginal social value of individual well-being, $\frac{\partial U_i}{\partial a_i}$, the marginal value of the numeraire for individual well-being, and $\frac{\partial U_i}{\partial b_i} / \frac{\partial U_i}{\partial a_i}$, the marginal compensating variation for good b in terms of the numeraire. The expression

$$da_i + \frac{\frac{\partial U_i}{\partial b_i}}{\frac{\partial U_i}{\partial a_i}} db_i$$

measures the compensating variation for the small change in the bidimensional situation.

When the change is not marginal, one obtains an approximate formula

$$\Delta W \simeq \sum_{i=1}^n \frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial a_i} w_i, \quad (3)$$

where w_i is the compensating variation for the change incurred by i , and $\frac{\partial W}{\partial U_i}$, $\frac{\partial U_i}{\partial a_i}$ are estimated in a situation close to the initial or the final situation, or some intermediate situation. One can also take the average of the weights computed at the initial and the final situations.

When the numeraire is changed in BCA, only the terms $\frac{\partial U_i}{\partial a_i}$, w_i need to be modified accordingly, whereas $\frac{\partial W}{\partial U_i}$ remains unchanged, as it only depends on the choice of well-being measure.

The point of this section is to emphasize that the SWF (1) offers a wide array of possibilities. It accommodates any type of measure of well-being and any degree of inequality aversion. When the function U_i represents i 's preferences, then the compensating variation appearing in (2) and (3) is the actual compensating variation of the individual, but when U_i does not represent preferences, these formulae are still valid, but for a form of compensating variation that may be different from individuals' actual one. For instance, when U_i is measured by satisfaction with life, a policy that produces systematic shifts in people's scale use (e.g., making them more difficult to satisfy because of habituation to better conditions) will have effects on well-being that will not be correctly captured by people's "naïve" willingness to pay. For instance, people may express a strong willingness to pay for a park, whereas the policymaker may consider that habituation will reduce the actual impact on their well-being, so that the willingness to pay for life satisfaction is lower than the one reflecting people's preferences.

It is possible to rely on measures of well-being that embody fairness principles, for instance, by taking into account certain characteristics of individuals. One prominent example, reflecting the ideal of equality of opportunity, is to differentiate the weight of individuals depending on their background conditions, in order to give greater priority to those who have disadvantaged roots. Another prominent example is a form of libertarianism or meritocracy, according to which individuals with greater talent (as reflected in market earning power) deserve to keep the fruits of their talent to some extent. Note that incorporating fixed weights that depend on individuals' background characteristics does nothing to eliminate the problem of possible decision cycles described in [Section 2](#).

The equivalence approach to the measurement of well-being deserves mention here. The notion of compensating variation, or even more transparently the notion of equivalent variation, consists in computing a unidimensional variation in the numeraire that is equivalent, in the eyes of the individual (or of a suitably chosen measure of well-being), to the change incurred by the individual in her complex multidimensional situation. This idea of bringing the multidimensional complexity of individual situations to a single dimension by equivalence in terms of individual well-being can also be mobilized for the computation of well-being *levels* rather than *variations*. The idea is to determine a unidimensional path in the set of possible individual situations and estimate which situation on this reference path any given individual deems equally good as her current situation. Once these "equivalent" situations on the reference path are estimated for the population, one deals with a unidimensional world in which interpersonal comparisons are much more straightforward.

One prominent example of such an approach is the equivalent income measure of well-being, which collapses all differences between individuals into differences in income or wealth by taking a reference set of situations in which all other dimensions are assumed to be at a satisfactory level for the individuals whereas all levels of income are possible (see Fleurbaey & Abi-Rafeh, 2016 for a discussion of equivalent income in the context of BCA, and Fleurbaey, 2016 for a more general presentation of the approach). Another prominent example is the VNM approach discussed in the previous section, which, under the 0–1 calibration, measures well-being in terms of simple lotteries in which individuals can obtain either the "0" situation or the "1" situation with any possible probability. Asking people what probability for such simple lotteries they consider equivalent to their current situation is a classical method for the computation of the weights in quality-adjusted life years, and is called the "standard gamble" method.¹⁰

One remarkable feature of the equivalence approach is that it produces measures of well-being satisfying the following property:

¹⁰ The alternative scaling introduced in [Figure 7](#) can also be described in similar terms, with a standard gamble in which the reference point z is the "0" option, and the "1" option is the individual-specific alternative α such that $u(\alpha) = u(z) + \lambda u'(z)$ for any arbitrary VNM function representing the individual's preferences, and $\lambda \geq 0$ large enough so that the current individual situation x lies in-between z and α in the individual's preferences. One then seeks the probability p such that

$$u(x) = pu(\alpha) + (1 - p)u(z),$$

and one finds

$$\lambda p = \frac{u(x) - u(z)}{u'(z)},$$

satisfying the property, when read as a function of x , that $\lambda p(z) = 0$ and $\lambda p'(z) = 1$.

Respect for common interpersonal comparisons: When individuals have identical preferences (and identical risk attitudes in the case of the VNM approach), the interpersonal comparison of their situations will always respect their own judgment about such comparisons.

That is, if both Ann and Bob consider that Ann is better off with their common preferences, then a measure of well-being based on the equivalence approach will agree with their judgment and obtain a greater level for Ann's well-being. The reason for this property is that when they have identical preferences, individuals will link any arbitrary situation to the same situation on the reference path. Therefore, if they both consider Ann's situation to be better, Ann will identify a reference situation equivalent to her own that is better on the reference path than the reference situation identified by Bob as equivalent to his own situation. It remains an open question whether there exist other methods than the equivalence approach that satisfy such a property. As explained in the previous section, SWB measures of well-being do not satisfy this property.

It must be stressed that the choice of numeraire in (weighted) BCA and the choice of a reference path to measure individual well-being are completely independent. One can use a life-year measure of well-being in a weighted BCA approach in terms of money, and conversely. The choice of equivalent income to measure well-being does not require the use of money as a numeraire in BCA.

The main point of this section is that weighted BCA is very flexible and can accommodate any social evaluation that relies on a SWF of the (1) type, and this is a very large class, especially when one allows the function W to treat individuals differently depending on any characteristic that is deemed morally relevant, such personal background or talents. What is not accommodated by such a function is a type of evaluation that relies on other considerations than individual well-being, however measured. For instance, a libertarian approach that focuses on procedures and the establishment of rights without looking at consequences for the population does not enter into the considered class.

6. Convergences

Weighted BCA may sometimes come close to unweighted BCA for specific numeraires, even when the distribution of the numeraire is not optimal. But we now show why optimality of the distribution is close to necessary.

A first, extreme, case to consider is when both terms $\frac{\partial W}{\partial U_i}$, $\frac{\partial U_i}{\partial a_i}$ are constant and equal across individuals. This happens when utility is quasi-linear

$$U_i(a, b) = a + v(b)$$

and when W is the utilitarian sum. In this case, $\frac{\partial W}{\partial U_i} = \frac{\partial U_i}{\partial a_i} = 1$ and the weights are indeed equal across individuals in all situations—but it also holds that in this case, any distribution of the numeraire is socially optimal provided no resource is wasted. We will now see that this is essentially the only case in which this happens, up to some transformations yielding the same ordinal ranking of social situations.

Consider the case of a SWF (1) of the Atkinson type

$$W(U_i(a_i, b_i); i = 1, \dots, n) = \sum_{i=1}^n \varphi(U_i(a_i, b_i)), \quad (4)$$

where φ is an increasing, concave function. One then has

$$\frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial a_i} = \phi'(U_i) \frac{\partial U_i}{\partial a_i}.$$

For weighted BCA to mimic unweighted BCA (with a as the numeraire) for all distributions, this expression must be equal to the same constant for all i . That is, for all i and some k , one must have

$$\frac{\partial U_i}{\partial a_i} = \frac{k}{\phi'(U_i)}.$$

This implies

$$\int_0^{a_i} \phi'(U_i) \frac{\partial U_i}{\partial a_i} da = \int_0^{a_i} k da = ka_i.$$

By a change of variables, let us write

$$\int_0^{a_i} \phi'(U_i) \frac{\partial U_i}{\partial a_i} da = \int_{U_i(0, b_i)}^{U_i(a_i, b_i)} \phi'(u) du = \phi(U_i(a_i, b_i)) - \phi(U_i(0, b_i)).$$

Inserting this into the previous equation, one obtains

$$\phi(U_i(a_i, b_i)) = ka_i + \phi(U_i(0, b_i)) = ka_i + v(b_i),$$

for $v(b_i) = \phi(U_i(0, b_i))$, and thus

$$U_i(a_i, b_i) = \phi^{-1}(ka_i + v(b_i)).$$

But this means that the SWF can be rewritten as

$$W(U_i(a_i, b_i); i = 1, \dots, n) = \sum_{i=1}^n ka_i + v(b_i),$$

and this is equivalent to the quasi-linear case, implying that all distributions of the numeraire are socially optimal. In this case, $\frac{\partial W}{\partial U_i}, \frac{\partial U_i}{\partial a_i}$ are not constant and not equal across individuals in general, but their product is, and the SWF is ordinally equivalent to combining utilitarian aggregation with quasi-linear utilities.

These are cases in which equal weights are obtained in the chosen numeraire. It is also possible to obtain a situation in which weighted BCA in one numeraire is equivalent to unweighted BCA in the *other* numeraire. The familiar case involves assuming that the willingness to pay for good b is, in the population, roughly proportional to a power function of the numeraire a :

$$w_i \simeq \beta a_i^\gamma$$

for $\beta, \gamma > 0$. It is, for instance, common to assume that the value of a statistical life year (VSLY) is roughly proportional to income. If the weights are a power function with the opposite coefficient:

$$\frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial a_i} = \mu a_i^{-\gamma},$$

then the marginal social value of providing one unit of good b to any individual is roughly equal across individuals:

$$\frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial a_i} w_i = \beta \mu.$$

This provides the same evaluation criterion as unweighted BCA with good b as the numeraire. For instance, a frequent shortcut for the computation of weights consists of taking weights equal to the inverse of income. Multiplied by a VSLY that is proportional to income, one obtains that an additional life year has the same value no matter who obtains it in the population. This can be justified by assuming that utility is proportional to the logarithm of income, and hence, the marginal utility of income is inversely proportional to income.

More rigorously, this result can be obtained with an Atkinson SWF (4) when the utility function is

$$U_i(a_i, b_i) = \varphi^{-1}(kb_i + v(a_i)),$$

because one then obtains that the marginal social value of good b , namely $\frac{\partial W}{\partial U_i} \frac{\partial U_i}{\partial b_i}$, is equal across individuals.

For practical purposes, a more relevant point is the following: If one takes unweighted BCA with money as the numeraire as the historical starting point, a reform consisting either in taking life years as an alternative numeraire, or in weighted BCA (in any numeraire) with weights based on a SWF with some inequality aversion over utilities (that are concave in wealth), will divert the social evaluation in the direction of giving less weight to rich individuals.

The SWF approach has the advantage of precluding decision cycles and allowing for a wide set of fairness considerations to enter the W function and the choice of well-being measures, while avoiding unfairness to individuals who are poorly endowed in the numeraire. But in practice, in many applications, similar evaluations may be produced by a range of methods, and the proposals that are currently made all consist in taking account of the distribution of economic resources in a more inequality-averse way than the traditional practice.

7. Conclusion

This paper argues against unweighted BCA, whatever the chosen numeraire, and in favor of flexibly weighted BCA, which guarantees rational decisions (no decision cycles) and allows for a wide variety of approaches to substantive fairness. As analyzed by Hammitt (2021) and Canning (2023), the choice of numeraire can have substantial consequences on project evaluation and involve key normative considerations. But, overall, it clearly appears that the weighted BCA approach is superior on all counts except perhaps simplicity.

The question of simplicity cannot be downplayed because practitioners grappling with data and policymakers trying to understand the results of a BCA study are very sensitive to it. This is why it could be helpful to make weighted BCA analysis easier by providing ready-made tables of weights, based on transparent normative assumptions and allowing for a variety of parameter choices, that practical BCA studies could directly borrow from (Fleurbaey & Abi-Rafeh, 2016). Weights in such tables can be computed once for a suitable period (of perhaps 3 to 5 years) on the basis of a general population survey, and the tables of weights can be directly used at the stage of summing compensating variations over the population. The survey would have to estimate the two components of the weights (marginal

social priority of utility and marginal utility of the numeraire), for a suitable range of degrees of inequality aversion and an appropriate variety of measures of utility, in order to provide the relevant weights for users with a variety of ethical preferences over SWF and measures of well-being. Moreover, the tables would display the relation between weights and characteristics of the population (such as income, age, gender, education, health, and location) that would typically be identified in applied BCA.

Of course, this requires that BCA practitioners using such tables estimate more than the average compensating variation over the population, since heterogeneity in compensating variation correlated with unequal weights may be a key element of a sound-weighted BCA study. In particular, when social groups have clashing interests (e.g., between rich and poor, urban and rural, skilled and unskilled workers), recognizing differences in compensating variation between groups can help identify potential political conflicts. Thus, weighted BCA is not hard primarily because of the weights, since the weights can be made available as an open-access public resource, but primarily because it forces the analyst to track important differences in interests among the population affected by the project. Another source of complexity with weighted BCA is the need to track the distribution of the financial costs of projects, and not just their other impacts (mostly benefits). But again, ignoring the distribution of costs is a fundamental oversight that should be avoided, and weighted BCA should be considered positively in part because it forces the analyst not to overlook this aspect of impacts.

Simplicity may be a key factor in the choice of a numeraire. Once a weighted BCA approach is followed, the choice of the numeraire is a matter of indifference from a theoretical point of view, since the policy conclusions will rely on the same SWF variation no matter what numeraire is used in the computation of the individuals' marginal utility and willingness to pay. But not all numeraires are equally intuitive. Money is convenient because willingness to pay in money is so commonplace a notion in a market economy that the expression "willingness to pay" is usually understood as referring to money necessarily. Similarly, marginal utility of money is a very familiar notion for economists. In contrast, using life years may be harder because the idea of trading longevity for other goods is less familiar, even if people routinely endanger their own health in the pursuit of other goods. Another advantage of money is that it can lend itself to BCA applied to well-being over a period (like a year), whereas life years can be used as a numeraire only when lifetime well-being is taken as the measuring rod for individual well-being. Although evaluating lifetime well-being is in theory preferable because that is what ultimately matters to people, it makes the analysis more complex.

Overall, the final point of this paper is that all the current proposals to reform and improve on the outdated unweighted BCA practices point in the same direction of avoiding giving priority to the desires of richer populations and taking better account of distributional issues. This is a most interesting and promising time for BCA.

Acknowledgments. This paper builds on a 2022 conference held by the Brocher Foundation, "Healthy, Wealthy, and Wise - The Ethics of Health Valuation," organized by Nir Eyal (Rutgers University), Samia Hurst (University of Geneva), Lisa A. Robinson (Harvard University), and Daniel Wikler (Harvard University). We thank the Brocher Foundation and conference participants for their support and helpful comments. This special issue was supported by the Brocher Foundation, with supplemental funding from the Rutgers University Center for Population-Level Bioethics, the University of Geneva Institute for Ethics, History, and the Humanities, and the University of Bergen Centre for Ethics and Priority Setting. More information on the Brocher Foundation is available here: <https://fondation-brocher.ch/>. This paper has also benefited from inspiring conversations and feedback at the European

SBCA Conference in September 2022, in particular with David Canning and John Broome. It was further improved by helpful comments from two anonymous reviewers.

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