

BOOK REVIEWS

STEINMETZ, N. *Rational iteration: complex analytic dynamical systems* (de Gruyter Studies in Mathematics Vol. 16, de Gruyter, Berlin, New York 1993) x + 190 pp., 3 11 013765 8, about £60.

Although work on the iteration of polynomial and rational functions on the complex plane dates back to 1918–19 with the pioneering papers of Fatou and Julia, there has been a tremendous resurgence of interest in this area in the last few years. Partly, at least, this is due to the development of the computer, which has enabled practical examination of iterates and the associated Julia and Mandelbrot sets, with the observation of many beautiful and intricate features inviting theoretical explanation.

This book provides a self-contained account of the theory of rational iteration, including the Fatou–Julia theory, dynamics on the Fatou set (including the existence of rotation domains and Sullivan’s Theorem on the non-existence of wandering domains), the geometry and dynamics of the Julia set, and miscellaneous topics such as the connectivity of the Mandelbrot set and Lyubich’s invariant measure. The book assumes a reasonable knowledge of complex function theory, including Montel’s Theorem on normal families and Carathéodory’s Theorem on the boundary correspondence of conformal mappings. The properties of quasi-conformal mappings required for Sullivan’s theorem are also taken on trust.

The book is readable and mathematically precise, though occasionally the treatment seems a little terse. The text is backed up by well chosen exercises and it is illustrated by many grey-tone illustrations depicting Julia sets and invariance domains of various functions.

The book could be the basis for a postgraduate course on rational iteration and is also suitable for researchers seeking an acquaintance with the area. Inevitably it stops short of the sophisticated methods developed in very recent years such as the Yoccoz puzzle pieces and invariant line families. Nevertheless, it raises the reader to a level of knowledge at which such topics of contemporary research become accessible.

The book should be considered alongside Alan Beardon’s *Iteration of rational functions* (Springer-Verlag, 1991), which covers a remarkably similar range of topics at the same level and with a similar audience in mind. The books differ in style: Beardon’s book is more discursive and there is more motivation for the development by way of specific examples; Steinmetz’s book is mathematically more concentrated, though not at the expense of readability. In their individual ways both books make an excellent job of achieving their aims of providing attractive treatments of rational iteration to bridge the gap between the basic theory of complex functions and the research frontiers of rational iteration theory.

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SCHMIDT, R. *Subgroup lattices of groups* (de Gruyter Expositions of Mathematics Vol. 14, de Gruyter, Berlin, New York 1994) 576 pp., 3 11 011213 2, £139.38.

Early in one’s study of groups one tries to pay less attention to arguments involving elements and concentrate more on the subgroup structure of groups. The study of the lattice $L(G)$ of subgroups of a group G may be thought of as the logical consequence of this approach, investigating a group by looking only at the structure of the lattice of subgroups.

The first paper in the subject was by Röttlander in 1928 and was motivated by the lattice isomorphism described in the Galois correspondence between the subgroups of the Galois group